

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions
(10 X 10 = 100 Marks)

1. Find the general solution of the second order differential equation ✓

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \left(\frac{\log x}{x}\right)^2$$

2. An RCL circuit connected in series has a resistance of 5 ohms, an inductance of 0.05 henry, a capacitor of 4×10^{-4} farad, and an applied alternating emf of $200 \cos(100t)$ volts. Find an expression for the current flowing through this circuit if the initial current and the initial charge on the capacitor are both zero. ✓

3. Find the complete integral of the equation $z^2 = p^2 + q^2 + 1$. ✓

4. Solve the equation $(3z - 4y)p + (4x - 2z)q = 2y - 3x$. ✓

5. Find the Laplace transform of the square wave function

$$f(t) = \begin{cases} k, & 0 \leq t \leq a \\ -k, & a \leq t \leq 2a \end{cases} \text{ with } f(t+2a) = f(t) \text{ for all } t. \checkmark$$

6. Use convolution theorem to evaluate the integral

$$\int_0^t u^2 e^{-a(t-u)} du.$$

7. Using Laplace transform, solve the differential equation ✓

$$x''(t) + 3x'(t) + 2x(t) = H(t-2) \text{ with } x(0) = x'(0) = 0.$$

8. Find the Fourier series expansion of $f(x)$ given by $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ and also find the value of $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$.

9. Find the half range sine series for $f(x) = \begin{cases} x \ln\left(0, \frac{\pi}{2}\right) \\ \pi - x \ln\left(\frac{\pi}{2}, \pi\right) \end{cases}$ in $(0, \pi)$ and deduce that the sum of the series $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$. ✓

10. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & \text{in } |x| \leq 1 \\ 0, & \text{in } |x| > 1 \end{cases}$. Also prove that

$$\int_0^{\frac{\pi}{2}} \frac{\sin s - s \cos s}{s^3} \cos \frac{s}{2} ds = \frac{3\pi}{16}.$$

11. Using convolution theorem find $Z^{-1} \left\{ \frac{z^3}{(z-2)^2(z-3)} \right\}$

12. Using Z-transform solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$. ✓