Welcome

Difference Equations

Difference Equations :

Def : A difference eqn. is a relation b/w the differences of a function at one or more general values of the argument.

Ex : (i) $U_{n+2} - 2U_{n+1} + U_n = 0$ ——— (1)

(ii) $U_{n+2} + U_{n+1} - 2U_n = 2^n$ ——— (2)

(iii) $U_{n+1} - 2U_n + 3U_{n-1} = 0$ ——— (3)

→ Difference equations are met under the name of recurrence relations.

→ The difference between the largest & the smallest arguments occuring in the difference equation divided by the unit of increment is the ORDER of a Di. Eq.

$$\frac{\text{Largest argument} - \text{Smallest argument}}{\text{Unit of increment}}$$

order of eqn ① is $\frac{(n+2)-(n-1)}{1} = 3$

order of eqn ② is $\frac{(n+2)-n}{1} = 2$

Formation of difference Eqn's

① Form the D.E from $U_n = A.2^n + B\,3^n$ where A & B are arbitrary constants.

Sol.

$$U_n = A(2)^n + B(3)^n$$

$$U_{n+1} = A(2)^{n+1} + B(3)^{n+1}$$

$$U_{n+2} = A(2)^{n+2} + B(3)^{n+2}$$

Eliminating A, B, we get

$$\begin{vmatrix} U_n & 1 & 1 \\ U_{n+1} & 2 & 3 \\ U_{n+2} & 4 & 9 \end{vmatrix} = 0 \Rightarrow 6U_n - 5U_{n+1} + U_{n+2} = 0$$

(or) $U_{n+2} - 5U_{n+1} + 6U_n = 0.$

② Form the D.E's from

(i) $U_n = A(2)^n + B(-3)^n$

(ii) $U_n = (A + Bn)(3)^n$

③ Form the D.E from $y_n = c_1(3)^n + c_2(-1)^n$ — (i)

Sol. $y_{n+1} = c_1(3)^{n+1} + c_2(-1)^{n+1}$ ---- (ii)

$y_{n+2} = c_1(3)^{n+2} + c_2(-1)^{n+2}$ ...... (iii)

Eliminating c_1 & c_2 from (i), (ii) & (iii)

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 3 & -1 \\ y_{n+2} & 9 & 1 \end{vmatrix} = 0 \Rightarrow \boxed{y_{n+2} - 2y_{n+1} - 3y_n = 0}$$

④ Form the DE from $y_n = A(2)^n + B(5)^n$ .... (i)

Sol. $y_{n+1} = A(2)^{n+1} + B(5)^{n+1}$ .... (ii)

$y_{n+2} = A(2)^{n+2} + B(5)^{n+2}$ .... (iii)

Eliminate A & B from (i), (ii) & (iii)

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 2 & 5 \\ y_{n+2} & 4 & 25 \end{vmatrix} = 0 \Rightarrow \boxed{y_{n+2} - 7y_{n+1} + 10y_n = 0}$$

(5) From the D.E from $y_n = a\cos n\theta + b\sin n\theta$ where a & b are arbitrary constants. .....(i)

Sol. $y_{n+1} = a\cos(n+1)\theta + b\sin(n+1)\theta$ .....(ii)

$y_{n+2} = a\cos(n+2)\theta + b\sin(n+2)\theta$ .....(iii)

Eliminate a & b from (i), (ii) & (iii), we get

$$\begin{vmatrix} y_n & \cos n\theta & \sin n\theta \\ y_{n+1} & \cos(n+1)\theta & \sin(n+1)\theta \\ y_{n+2} & \cos(n+2)\theta & \sin(n+2)\theta \end{vmatrix} = 0$$

$$y_n \sin\theta - y_{n+1} \sin 2\theta + y_{n+2} \sin\theta = 0$$

$$y_n - 2 y_{n+1} \cos\theta + y_{n+2} = 0$$

⑥ Form the D.E from $y_n = an^2 + bn$ where a & b are arbitrary constants. ....(i)

Sol

$$y_{n+1} = a(n+1)^2 + b(n+1) \quad \text{....(ii)}$$

$$y_{n+2} = a(n+2)^2 + b(n+2) \quad \text{....(iii)}$$

Eliminate a & b from (i), (ii) & (iii)

$$\begin{vmatrix} y_n & n^2 & n \\ y_{n+1} & (n+1)^2 & n+1 \\ y_{n+2} & (n+2)^2 & n+2 \end{vmatrix} = 0$$

$$\Rightarrow \quad n(n+1)\,y_{n+2} - 2y_{n+1}\,n(n+2) + 2(n+1)(n+2)\,y_n = 0.$$

Sol. of a difference eqn :

An expression for the unknown function y_n which satisfies the given difference equation,

In the general solution, the no. of A.c's = order of the eqn.

Linear Difference Equations :

An eqn of the form

$$y_{n+r} + a_1 y_{n+r-1} + a_2 y_{n+r-2} + \cdots\cdots + a_r y_n = f(n) \quad \text{——— (1)}$$

where $a_1, a_2, \ldots a_r$ are constants, is called a linear difference eqn. with constant coeff's.

Symbolic form of Eqn (1) is

$$\left[E^r + a_1 E^{r-1} + \cdots + a_r\right] y_n = f(n) \quad \text{——— (2)}$$

General sol. of eqn (2) is

$$y_n = C.F + P.I$$

__Note.__ $E^2 y_n = y_{n+1}$; E : Shift operator.

__Note:__ $\phi(E) = 0 \longrightarrow$ A.E. of eqn 2

__Note:__ If $y_1(n), y_2(n), \dots y_r(n)$ are 'r' l.i solutions then it's g.s. is $u_n = c_1 y_1(n) + c_2 y_2(n) + \cdots + c_r y_r(n)$

__Note:__ If v_n is a P.S. then the C.S. is $y_n = u_n + v_n$.

If $\alpha_1, \alpha_2 \dots$ are distinct roots of A.E

then C.F : $C_1 \alpha_1^n + C_2 \alpha_2^n + \dots$

If $\alpha_1 = \alpha_2$ (Repeated root), C.F $= (C_1 + C_2 n)\alpha^n$

If the roots are imaginary $\alpha_1 = \alpha + i\beta$; $\alpha_2 = \alpha - i\beta$

then $y_n = C.F = r^n \left[A\cos n\theta + B\sin n\theta\right]$

Where $r = \sqrt{\alpha^2 + \beta^2}$; $\theta = \tan^{-1}\left[\frac{\beta}{\alpha}\right]$.

Ex ① Solve $u_{n+2}+5u_{n+1}+6u_n=0$

Sol. Symbolic form of the given difference eqn is

$$(E^2+5E+6)u_n=0 \qquad \text{①}$$

The auxiliary eqn is $E^2+5E+6=0$

$$(E+2)(E+3)=0$$

$$\Rightarrow E=-2;\ E=-3$$

(or)

$$\alpha=-2;\ \alpha=-3$$

$$\therefore u_n = c_1(-2)^n + c_2(-3)^n$$

① Solve $y_{n+2} + 5y_{n+1} + 6y_n = 0$

Sol: Let $y_n = \alpha^n$ be the trial sol.

The ch. eqn becomes $\alpha^2 + 5\alpha + 6 = 0 \Rightarrow \alpha_1 = -3 ; \alpha_2 = -2$.

The general sol. is $y_n = c_1(-3)^n + c_2(-2)^n$.

② Solve $y_n - 4y_{n-1} + 4y_{n-2} = 0$.

Sol: Let $y_n = \alpha^n$ be the trial sol.

The ch. eqn. becomes $\alpha^n - 4\alpha^{n-1} + 4\alpha^{n-2} = 0$

$$\Rightarrow 1 - \frac{4}{\alpha} + \frac{4}{\alpha^2} = 0$$

$$\Rightarrow \alpha^2 - 4\alpha + 4 = 0$$

$$\Rightarrow (\alpha-2)^2 = 0 \Rightarrow \alpha = 2, 2$$

$\therefore$ The general sol. is $y_n = (c_1 + nc_2)(2)^n$.

(3) $y_{n+2} - 5y_{n+1} + 6y_n = n^2 + n + 1$

Sol: C.F: $\alpha^{n+2} - 5\alpha^{n+1} + 6\alpha^n = 0$

$\Rightarrow \alpha^2 - 5\alpha + 6 = 0 \Rightarrow \alpha = 2 ; \alpha = 3$

$\therefore y_{nc} = c_1(2)^n + c_2(3)^n$

P.I: Let $y_{np} = An^2 + Bn + C$

put this in the given eqn.

$A(n+2)^2 + B(n+2) + C - 5\left[A(n+1)^2 + B(n+1) + C\right] + 6\left[An^2 + Bn + C\right] = n^2 + n.$

Comparing the Coeff. of different powers of 'n', we get

$A - 5A + 6A = 1 \Rightarrow A = 1/2$

'n'

$$4A+B-10A-5B+6B=1$$

$$\rightarrow B=2$$

"Constant"

$$4A+2B+C-5A-5B-5C+6C=1$$

$$\Rightarrow C=15/4.$$

$\therefore$ The Complete sol. is $y_n = C.F+P.I = c_1(2)^n + c_2(3)^n + \frac{1}{2}n^2 + 2n + \frac{15}{4}$

④ Solve $y_{n+1} - 2y_n + y_{n-1} = 8$

Sol: C.F : $\alpha^{n+1} - 2\alpha^n + \alpha^{n-1} = 0$

$\Rightarrow \alpha - 2 + \frac{1}{\alpha} = 0 \Rightarrow \alpha^2 - 2\alpha + 1 = 0$

$\Rightarrow (\alpha - 1)^2 = 0 \Rightarrow \alpha = 1, 1$

C.F is $y_{nc} = (A + Bn)(1)^n = A + Bn$

Let $y_{np} = cn^2$

put this value in the given eqn

$$c(n+1)^2 - 2cn^2 + c(n-1)^2 = 8$$

'n^2' $\quad c - 2c + c = 0$

'n' $\quad 2c - 2c = 8$.

'Const' $\quad c + c = 8 \Rightarrow c = 4$

$$\therefore y = y_{nc} + y_{np} = A + Bn + 4n^2$$

Solve $y_{n+2} - 2\cos\theta\, y_{n+1} + y_n = 0$; $y_0 = 1$; $y_1 = \cos\theta$ ——— ①

Sol: Let $y_n = (\alpha)^n$ be the sol. of ① ——— ②

From ① & ②, $(\alpha)^{n+2} - 2\cos\theta(\alpha)^{n+1} + (\alpha)^n = 0$

$$\Rightarrow \alpha^2 - 2\alpha\cos\theta + 1 = 0$$

$$\Rightarrow \alpha = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \frac{2\cos\theta \pm 2\sqrt{-\sin^2\theta}}{2}$$

$$= \frac{2\cos\theta \pm 2i\sin\theta}{2}$$

$$= \cos\theta \pm i\sin\theta = 1e^{\pm i\theta}$$

$\therefore y_{nc} = (\rho)^n [A\cos n\phi + B\sin n\phi]$; $\rho = \sqrt{x^2+y^2} = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left[\frac{\sin\theta}{\cos\theta}\right] = \theta$

$(1)^n [A\cos n\theta + B$

Also given that $y_0 = 1$; $y_1 = \cos\theta$

$\therefore 1 = (1)\left[A + B(0)\right] \Rightarrow 1 = A$

$\cos\theta = (1)^1\left[A\cos\theta + B\sin\theta\right] \Rightarrow \cos\theta = \cos\theta + B\sin\theta$

$\Rightarrow 0 = B\sin\theta$

$\Rightarrow B = 0$

$\therefore y_n = (1)^n\left[\cos n\theta\right]$

$\boxed{y_n = \cos n\theta}$ req. sol

Fibonacci Sequence:

0 1 1 2 3 5 8 13 21 34 ......

$$x_n = x_{n-1} + x_{n-2}$$

Seq: $y_n = y_{n-1} + y_{n-2}$ — (1)

with $y_0 = 0$; $y_1 = 1$

Let $y_n = \alpha^n$ be the sol of (1)

$$\therefore \alpha^n = \alpha^{n-1} + \alpha^{n-2}$$

$$1 = \frac{1}{\alpha} + \frac{1}{\alpha^2}$$

$$\Rightarrow \alpha^2 - \alpha - 1 = 0$$

$$\alpha = \frac{1 \pm \sqrt{1+4}}{2}$$

$$\alpha = \frac{1 \pm \sqrt{5}}{2}$$

$$\therefore y_n = c_1 \left[\frac{1+\sqrt{5}}{2}\right]^n + c_2 \left[\frac{1-\sqrt{5}}{2}\right]^n$$

i) Solve the difference eqn of the fibonacci sequence of numbers

Sol: We know that $y_n - y_{n-1} - y_{n-2} = 0$ is the Fibonacci sequence of numbers with initial conditions ——— ①

$y_0 = 0$ & $y_1 = 1$

Let $y_n = (\alpha)^n$ be the sol of ① ——— ②

From ① & ② $(\alpha)^n - (\alpha)^{n-1} - (\alpha)^{n-2} = 0$

$1 - \frac{1}{\alpha} - \frac{1}{\alpha^2} = 0 \Rightarrow \alpha^2 - \alpha - 1 = 0$

$\Rightarrow \alpha = \frac{+1 \pm \sqrt{1+4}}{2} = \frac{+1 \pm \sqrt{5}}{2}$

$$\therefore y_n = c_1\left[\frac{1+\sqrt{5}}{2}\right]^n + c_2\left[\frac{1-\sqrt{5}}{2}\right]^n$$

given that $y_0 = 0$ & $y_1 = 1$ $\Rightarrow$ $c_1 + c_2 = 0$

$$c_1\left(\frac{1+\sqrt{5}}{2}\right) + c_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

(or) $c_1 + c_2 = 0$ ——— (3)

$c_1(1+\sqrt{5}) + c_2(1-\sqrt{5}) = 2$ ——— (4)

Solving (3) & (4), we get $c_1 = \frac{1}{\sqrt{5}}$; $c_2 = \frac{-1}{\sqrt{5}}$

$$\therefore \boxed{y_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$

which is the req. sol.

8\. Solve the difference equation

$$y_{n+2} - 4y_{n+1} + 4y_n = 0 \quad \text{where } y_0 = 1 ; y_1 = 0 \qquad (1)$$

Sol: Let $y_n = (\alpha)^n$ be the trial sol of (1) $\qquad (2)$

From (1) & (2) $\quad (\alpha)^{n+2} - 4(\alpha)^{n+1} + 4(\alpha)^n = 0$

$$\alpha^2 - 4\alpha + 4 = 0$$

$$\Rightarrow (\alpha - 2)^2 = 0 \Rightarrow \alpha = 2, 2$$

$$\therefore y_n = (c_1 + c_2 n)(2)^n \qquad (3)$$

Also given that $y_0 = 1 ; y_1 = 0 \qquad (4)$

From (3) & (4), $\quad 1 = c_1 \quad ; \quad 0 = (c_1 + c_2)2$

$$\Rightarrow c_2 = -1$$

$$\therefore \boxed{y_n = (1 - n)(2)^n} \quad \text{req. sol.}$$

☆ Solve $y_{n+2} + y_n = 1$; $y_0 = y_1 = 0$ —— ①

Sol:

Let $y_n = (\alpha)^n$ —— ②

From ① & ② $(\alpha)^{n+2} + (\alpha)^n = 1$

$\alpha^2 + 1 = 1 \Rightarrow \alpha = 0, 0$

$\therefore y_n = (c_1 + c_2 n)(0)^n = (c_1 + c_2 n)\left[\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right]$

$y_0 = y_1 = 0 \rightarrow \quad 0 = c_1 (0)^n$

solve $y_{n+2} - y_n = 2^n$; $y_0 = 0$; $y_1 = 1$

Sol : Let $y_n = (\alpha)^n$

$\therefore (\alpha)^{n+2} - (\alpha)^n = 0 \quad \Rightarrow \quad \alpha^2 - 1 = 0 \Rightarrow \alpha^2 = 1$
$\Rightarrow \alpha = \pm 1$

$\therefore y_{nc} = c_1(-1)^n + c_2(1)^n$

Let $y_{np} =$ $K(2)^n$

$K(2)^{n+2} - K(2)^n = 2^n$

$\Rightarrow \quad K(4) - K = 1$

$\Rightarrow \quad K = 1/3$

$\therefore y = y_{nc} + y_{np} = c_1(-1)^n + c_2(1)^n + \frac{1}{3}(2)^n$

given that $y_0 = 0$; $y_1 = 1$

$$\therefore 0 = c_1 + c_2 + \frac{1}{3} \Rightarrow c_1 + c_2 = -\frac{1}{3}$$

$$1 = -c_1 + c_2 + \frac{2}{3} \Rightarrow -c_1 + c_2 = 1 - \frac{2}{3} = \frac{1}{3}$$

$$c_1 + c_2 = -\frac{1}{3}$$
$$-c_1 + c_2 = \frac{1}{3}$$
$$2c_2 = 0$$
$$c_2 = 0$$
$$c_1 = -\frac{1}{3}$$

$$\therefore y = -\frac{1}{3}(-1)^n + \frac{1}{3}(2)^n$$

$$\Rightarrow \boxed{y = \frac{1}{3}\left[2^n - (-1)^n\right]}$$

→ req. sol.

Ex ② Solve $U_{n+2} - U_{n+1} + U_n = 0$

Sol. $E^2 - E + 1 = 0$

$$E = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$\rightarrow U_n = r^n [c_1 \cos n\theta + c_2 \sin n\theta]$$

where $r = \sqrt{\alpha^2 + \beta^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right) = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \tan^{-1}(\sqrt{3}) = \pi/3$$

$$\therefore U_n = (1)^n \left[c_1 \cos \frac{n\pi}{3} + c_2 \sin \frac{n\pi}{3}\right]$$

Ex 3 Solve $U_{n+2} - 2U_{n+1} + U_n = 0$

Sol. $E^2 - 2E + 1 = 0 \Rightarrow (E-1)^2 = 0 \Rightarrow E = 1, 1$

G.S. $\therefore U_n = (c_1 + c_2 n)(1)^n$ (or $U_n = c_1 + c_2 n$.

Ex 4 Solve $U_{n+2} - U_{n+1} - 6U_n = 0$

Ex 5 Solve $U_{n+2} + 11U_{n+1} + 24U_n = 0$

Finding P.I by using MUC :

① Find the P.I. for $u_{n+2} - u_{n+1} - 6u_n = 2^n$ —— ①

Sol. Let $u_n^{(p)} = A(2)^n$

then $u_{n+1} = A(2)^{n+1}$

$u_{n+2} = A(2)^{n+2}$

$$\left.\begin{aligned} u_n^{(p)} &= A(2)^n \\ u_{n+1} &= A(2)^{n+1} \\ u_{n+2} &= A(2)^{n+2} \end{aligned}\right\} \text{—— ②}$$

From ① & ② $A(2)^{n+2} - A(2)^{n+1} - 6A(2)^n = 2^n$

$\Rightarrow \quad 4A(2)^n - 2A(2)^n - 6A(2)^n = 2^n$

$\Rightarrow \quad 4A - 8A = 1$

$\Rightarrow \quad -4A = 1 \quad \Rightarrow \boxed{A = \frac{-1}{4}}$

$\therefore$ P.I is $u_n^{(p)} = \left(\frac{-1}{4}\right)(2)^n$.

② Solve $u_{n+2} - 2u_{n+1} + u_n = 5$

Sol. A.E: $E^2 - 2E + 1 = 0$

$\Rightarrow \quad E = 1, 1$

C.F: $(c_1 + c_2 n)(1)^n$

Let $u_n^{(P)} = d$, then $u_{n+1} = d$; $u_{n+2} = d$

$\therefore d - 2d + d = 5 \quad \Rightarrow \quad 0 = 5$ (Not possible)

$\therefore$ Let $u_n = dn$, then $u_{n+1} = d(n+1)$

$u_{n+2} = d(n+2)$

$\Rightarrow \quad \cancel{n d} + \cancel{2d} - \cancel{2n} d - \cancel{2d} + \cancel{n d} = 5$

$\Rightarrow \quad 0 = 5 \quad$ (Not possible)

Let $u_n^{(p)} = d\, n^2 \;;\; u_{n+1} = d(n+1)^2 \;;\; u_{n+2} = d(n+2)^2$

$\therefore (n+2)^2 d - 2(n+1)^2 d + d n^2 = 5$

$\Rightarrow \quad \cancel{n^2 d} + 4d + \cancel{4n d} - \cancel{2n^2} d - 2d - \cancel{4n d} + \cancel{d n^2} = 5$

$\Rightarrow \quad 2d = 5 \quad \Rightarrow \quad d = 5/2$

$\therefore \quad u_n^{(p)} = \frac{5}{2} n^2$.

$\therefore$ Hence the general sol. of the given eqn is

$$\boxed{u_n = c_1 + c_2 n + \frac{5}{2} n^2}$$

(3) Form the difference eqn for the Fibonacci and hence solve it.

(4) Solve $u_{n+2} - u_{n+1} + 6u_n = 5^n$

(5) Solve $u_{n+2} + 5u_{n+1} + 6u_n = 1 + n$

(Hint: $u_n^{(p)} = A + Bn$)

(6) Find the P.I for $u_{n+2} - 4u_{n+1} + u_n = 30$

Sol. AE: $E^2 - 4E + 1 = 0$

Let $u_n^{(p)} = d$ (constant)

then $u_{n+1} = d$; $u_{n+2} = d$

$\therefore \quad d - 4d + d = 30$

$\rightarrow \quad d = -15$

Hence P.I $= -15$.

Theorem $Z[f_{n+k}] = z^k \left[F(z) - f_0 - \frac{f_1}{z} - \cdots - \frac{f_{k-1}}{z^{k-1}} \right]$

Proof: $$Z[f_{n+k}] = \sum_{n=0}^{\infty} f_{n+k} z^{-n}$$

$$= \sum_{n=0}^{\infty} f_{n+k} z^{-(n+k)} z^k$$

$$= z^k \sum_{n=0}^{\infty} f_{n+k} z^{-(n+k)}$$ Put $n+k=m$

$$= z^{k} \sum_{m=k}^{\infty} f_m \, z^{-m}$$

$$= z^{k} \left[\sum_{m=0}^{\infty} f_m \, z^{-m} - \sum_{m=0}^{k-1} f_m \, z^{-m} \right]$$

$$= z^{k} \left[F(z) - f_0 - \frac{f_1}{z} - \cdots - \frac{f_{k-1}}{z^{k-1}} \right]$$

<u>Note</u>

$$Z[f_n] = F(z)$$

$$Z[f_{n+1}] = z[F(z) - f_0]$$

$$Z[f_{n+2}] = z^2\left[F(z) - f_0 - \frac{f_1}{z}\right]$$

Solution of difference equation using Z-transform

① Solve $u_{n+2} - u_{n+1} - 6u_n = 2^n$

given that $u_0 = 0$; $u_1 = 0$

Sol. Given that $u_{n+2} - u_{n+1} - 6u_n = 2^n$ —— ①

Taking Z-transform on both sides of eqn ①, we get

$$Z\{u_{n+2}\} - Z\{u_{n+1}\} - 6Z\{u_n\} = Z\{2^n\}$$

$$\Rightarrow \quad z^2\left\{Z\{u_n\} - u_0 - \frac{u_1}{z}\right\} - z\left\{Z\{u_n\} - u_0\right\} - 6\,Z\{u_n\} = \frac{z}{z-2}$$

put $u_0 = 0$; $u_1 = 0$

$$\Rightarrow \quad (z^2 - z - 6)\,Z\{u_n\} = \frac{z}{z-2}$$

$$\Rightarrow \quad Z\{u_n\} = \frac{z}{(z-2)(z^2-z-6)}$$

$$\Rightarrow \quad \frac{Z\{u_n\}}{z} = \frac{1}{(z-2)(z-3)(z+2)} \quad \text{———} \quad (2)$$

Now $\dfrac{1}{(z-2)(z-3)(z+2)} = \dfrac{A}{z-2} + \dfrac{B}{z-3} + \dfrac{C}{z+2}$

put $z = 2$, $A = -1/4$

put $z = 3$, $B = 1/5$

put $z = -2$, $C = 1/20$

$$\therefore \frac{Z\{u_n\}}{z} = -\frac{1}{4}\frac{1}{z-2} + \frac{1}{5}\frac{1}{z-3} + \frac{1}{20}\frac{1}{z+2}$$

$$\Rightarrow Z\{u_n\} = -\frac{1}{4}\frac{z}{z-2} + \frac{1}{5}\frac{z}{z-3} + \frac{1}{20}\frac{z}{z+2}$$

$$\Rightarrow u_n = -\frac{1}{4}Z^{-1}\left\{\frac{z}{z-2}\right\} + \frac{1}{5}Z^{-1}\left\{\frac{z}{z-3}\right\} + \frac{1}{20}Z^{-1}\left\{\frac{z}{z+2}\right\}$$

$$\Rightarrow u_n = -\frac{1}{4}(2)^n + \frac{1}{5}(3)^n + \frac{1}{20}(-2)^n$$

Ex. Solve $u_{n+2} + 5u_{n+1} + 6u_n = 5^n$

given that $u_0 = 0$; $u_1 = 0$.

Problems Solve the following difference equations using Z-transform.

(a) $u_{n+1} + u_n = 1, \quad u_0 = 0$

(b) $y_{n+1} - 3y_n = 2^n, \quad y_0 = 1$

(c) $y_{n+2} - 3y_{n+1} + 2y_n = 0, \quad y_0 = 0, \; y_1 = 1$

(d) $y_{n+2} = y_{n+1} + y_n, \quad y_0 = 0, \; y_1 = 1$

(or) Form difference equation for fibonacci seq. and then solve using Z-transform.

Thank you