

BEEE203L

CIRCUIT THEORY



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MODULE – III

Three Phase Systems

Contents

Network Theorems for AC Circuits

- Review of Balanced System
- Unbalanced System
 - Delta connected loads
 - Three wire star connected loads
 - Four wire star connected loads
- Analysis of unbalanced 3 wire star load
 - Kirchhoff's law
 - Loop current method
 - Star delta conversion method
 - Millman's theorem

Introduction

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Consider a residential load = 5000 W (single phase loads)

What is the total current drawn by this residential load at 1 Φ , 230 V supply?

Approximately = $5000/230 = 21.74$ A

If the same residential load is equally shared to 3 Φ supply, then what is the current drawn by each phase?

Power/phase = $5000/3 = 1667$ W (approximately)

Current/phase = 7.25 A

Now, compare the conductor size....

Introduction to Polyphase AC Circuits

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- The domestic AC supply is single phase AC supply with 230V/50Hz.
- But this AC supply is not suitable for certain applications. Some need a poly phase AC supply.
- Poly phase AC supply is the one which produces many phases simultaneously.

Advantages of Three Phase Systems over Single Phase System

1. More output
2. Smaller size
3. Three phase motors are self starting
4. More power is transmitted

Generation of a polyphase ac supply

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- The single phase ac voltage is generated by using a single phase alternator. Single phase alternator consists of only one armature winding.
- But in order to generate a poly-phase voltage, we have to use many armature winding. The number of windings is equal to the number of phases.
- The armature winding is divided into three groups which are displaced by 120° from each other, then it is possible to generate three alternating voltages.

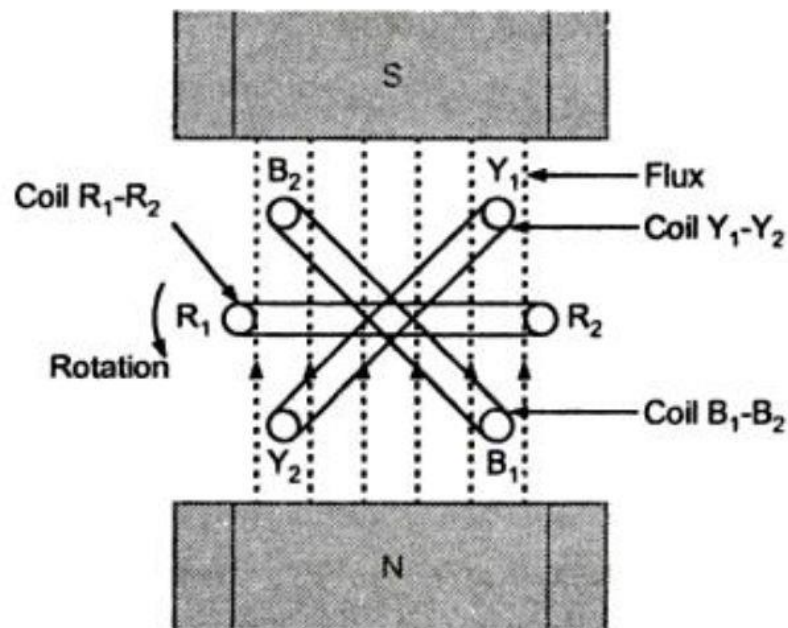
Construction:

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- The voltages induced in these windings are of same amplitude and frequency, but they are displaced by 120° with respect to each other
- As shown in fig.(1) the armature winding is divided into three groups. The three coils are R-R', Y-Y' and B-B'.
- All these coils are mounted on the same shaft and are physically placed at 120° from each other.
- When these coils rotate in the flux produced by the permanent magnet, emf is induced into these coils. As shown in fig.(2), these emf are sinusoidal, of equal amplitudes and equal frequency but they are displaced from each other by 120° .
- V_R , V_Y and V_B are three phase voltages.

Generation

Generation of Three phase voltage system



Generation of 3 phase

Fig.(1): Generation of a 3-phase voltages

Wave form

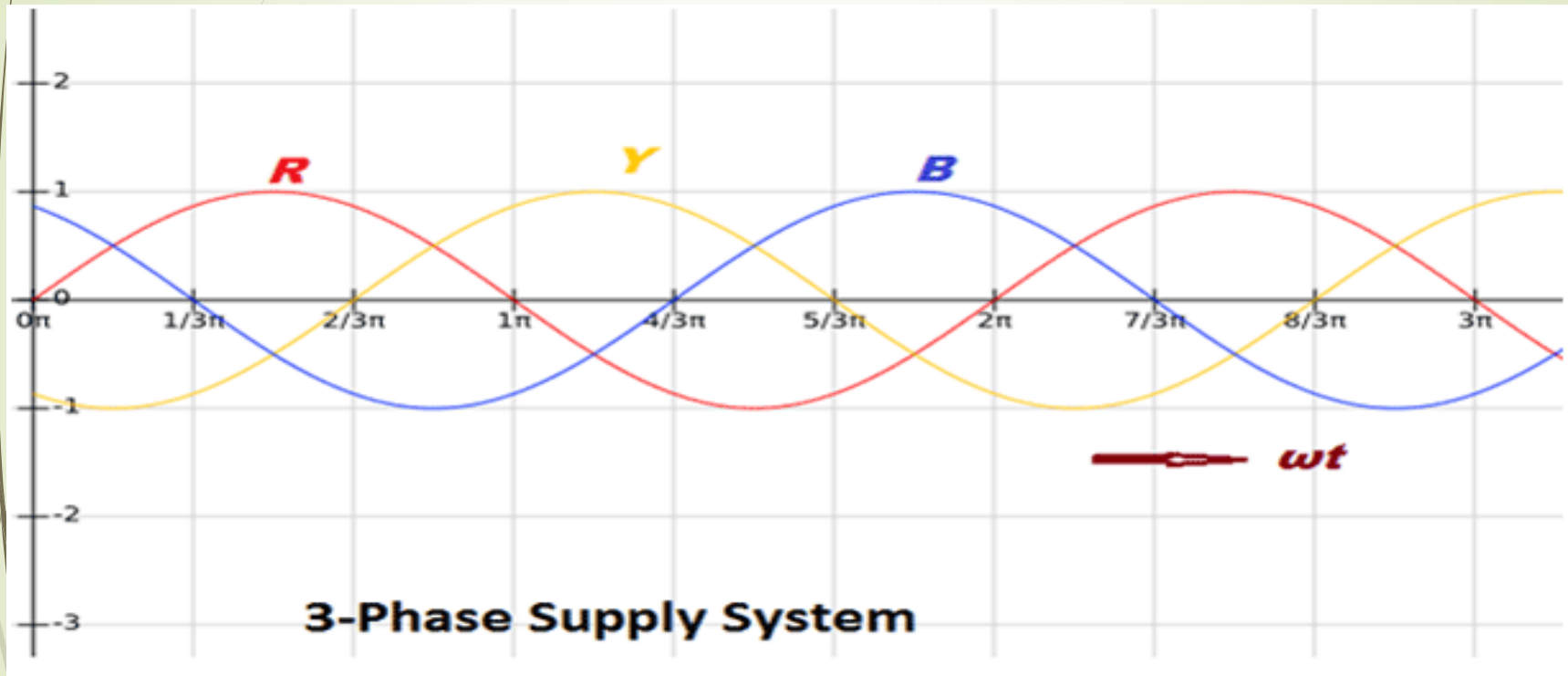


Fig.(2): Voltages induced in the three coils

Phasor

- If V_R is considered as the reference, then we conclude that,
1. V_Y lags V_R by 120° .
 2. V_B lags V_Y by 120° .
 3. In other words, V_B lags V_R by 240° .

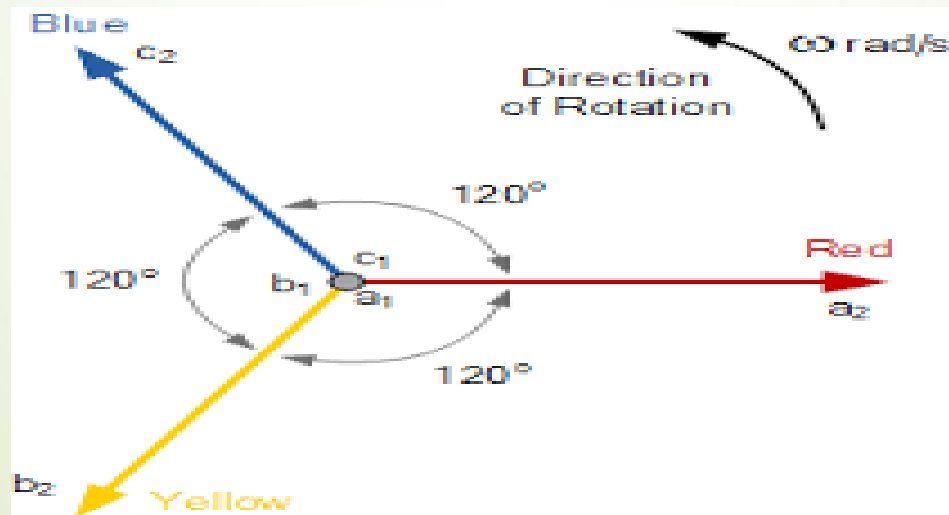


Fig.(3): Phasor representation of the three phase voltages

Mathematical

- The mathematical expression for the three induced voltages are given by:

$$V_R = V_m \sin \omega t$$

$$V_Y = V_m \sin (\omega t - 120^\circ)$$

$$V_B = V_m \sin (\omega t - 240^\circ) = V_m \sin (\omega t + 120^\circ)$$

Phase Sequence

- **Phase sequence:**
- The phase sequence is defined as the sequence in which the three phases reach their maximum values. Normally the phase sequence is R-Y-B.
- **Importance of phase sequence:**
- The direction of rotation of three phase machines depends on the phase sequence.
- If the phase sequence is changed e.g. R-B-Y then the direction of rotation will be reversed.
- In order to avoid such things, the phase sequence of R-Y-B is always maintained.

Comparison of Single Phase & Three Phase Systems

Sr. No.	Parameter	Single phase system	Three phase system
1.	Voltage	Low(230 V)	High (415V)
2.	Transmission efficiency	Low	High
3.	Size of machines to produce same output	Larger	Smaller
4.	Cross sectional area of conductors	Large	Small
5.	Usage	Domestic, small power applications	Industrial, large power applications

Three Phase supply Connection

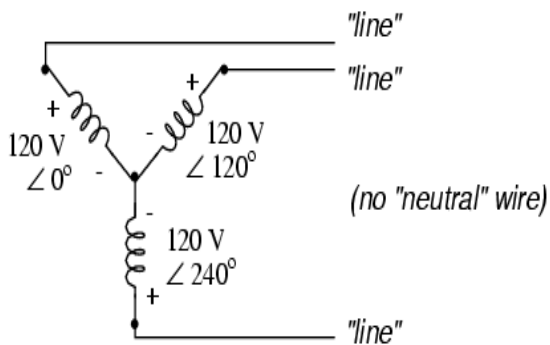
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- In three phase ac system, the three phase alternator has three separate windings one per each phase.
- Hence the power generated in each phase can be transmitted independently to the load using 2 wires per winding. But this will requires 6- different wires.
- Eventhough such a system is practically possible, it makes the system complicated and expensive.
- Hence in practice, the three windings of the alternator are interconnected in two different ways to reduced the number of wires required for the connections

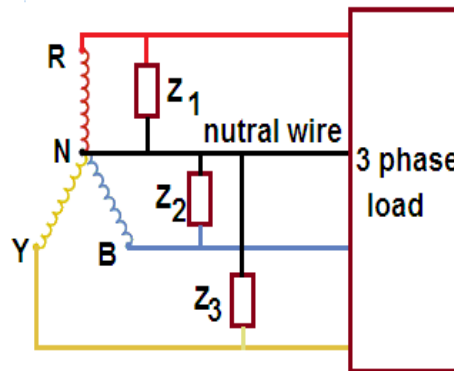
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Types of connection

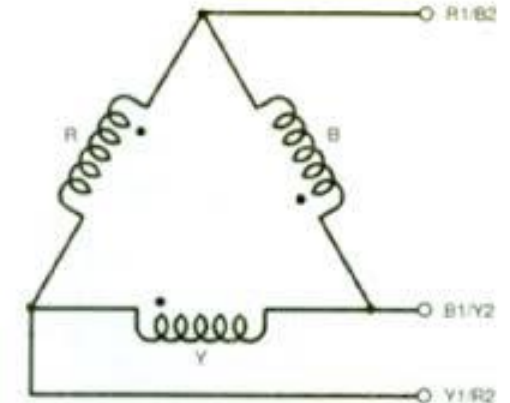
3-phase, 3-wire "Y" connection



**Three phase three wire
star (wye) connection**



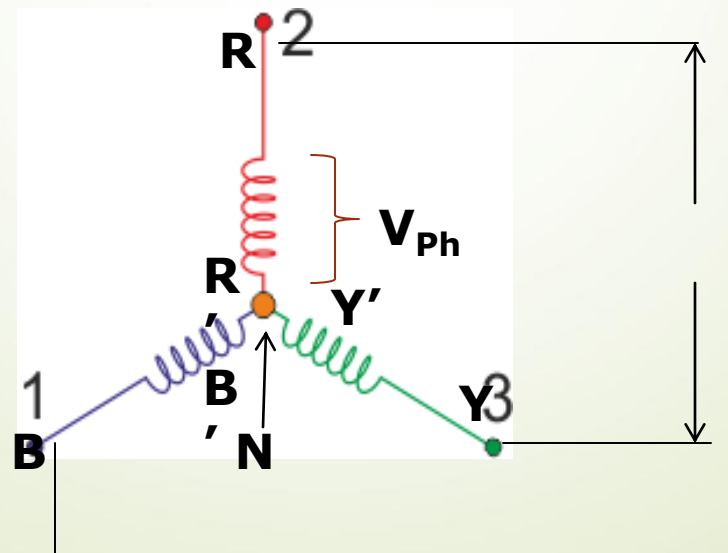
**Three phase four wire
star connection**



**Three phase three wire
delta connection**

Star Connection

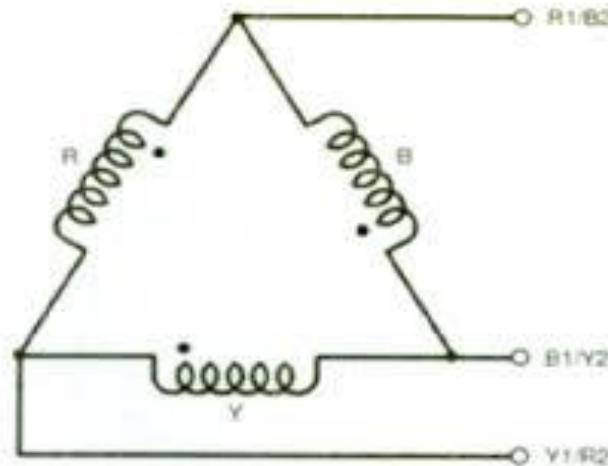
- This configuration is obtained by connecting one end of the three phase winding together.
- We can connect either R Y B (Starting end) or R' Y' B' (Finishing end) together. This common point is called as the Neutral Point.



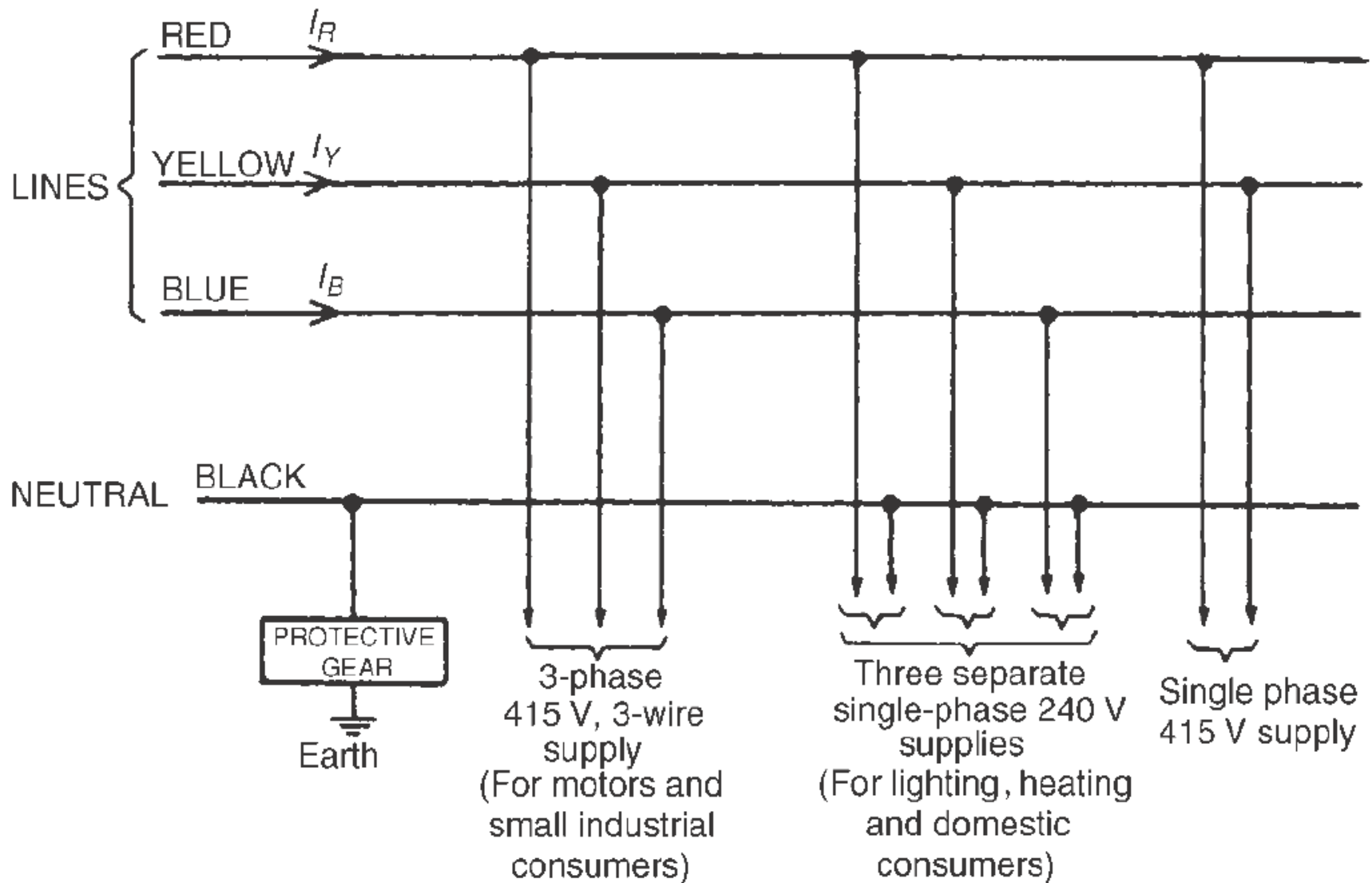
**Terminals
brought
out for
external
connection**

Delta Connection

- Delta or mesh configuration is obtained by connecting Starting end of one winding to the finishing end of the other winding such that it produces a closed loop.



Three phase four wire 415 V, 50Hz supply



Types of Loads

- The two types of load connection are:
 1. Star connection of load
 2. Delta connection of load

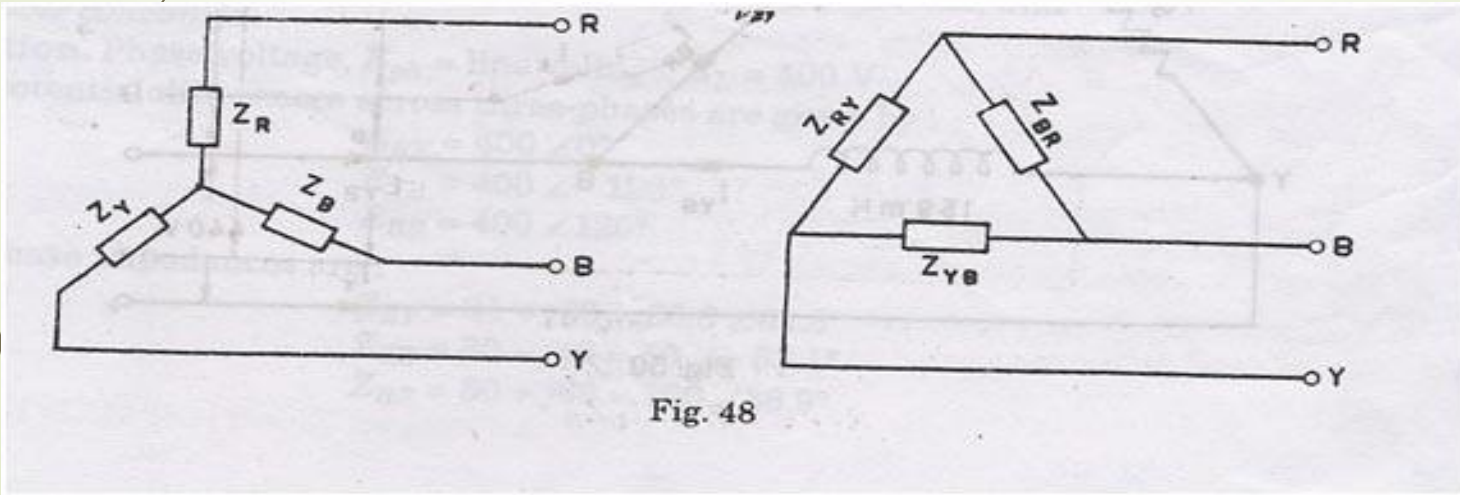
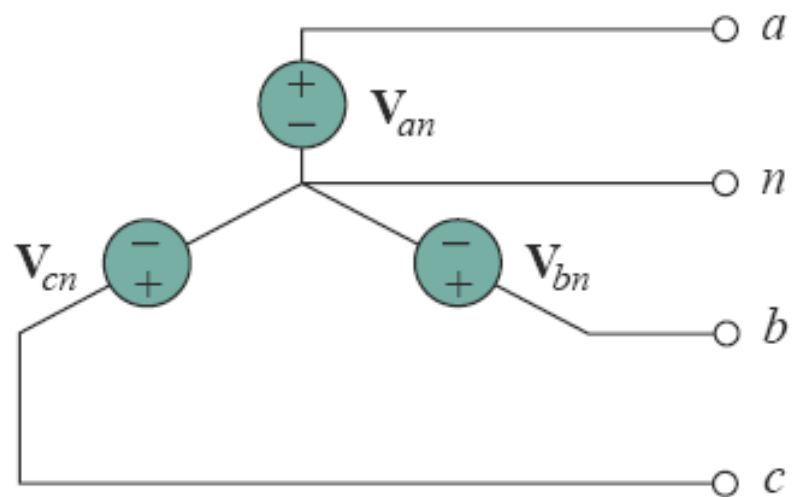
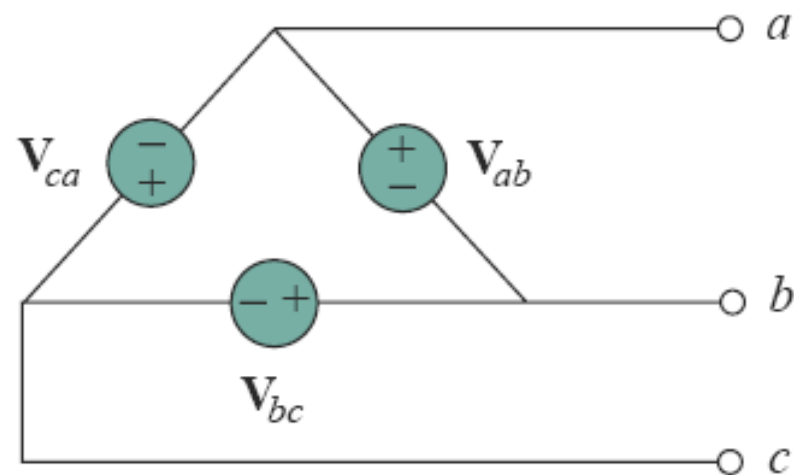


Fig. (1): types of loads

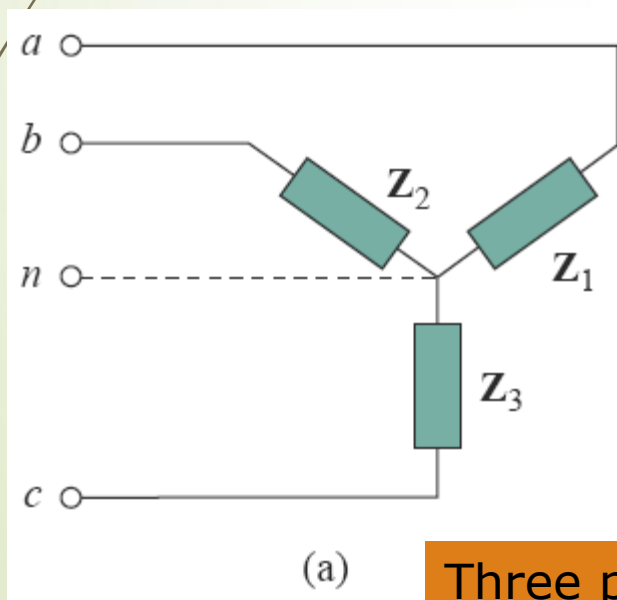


(a)

Three phase source

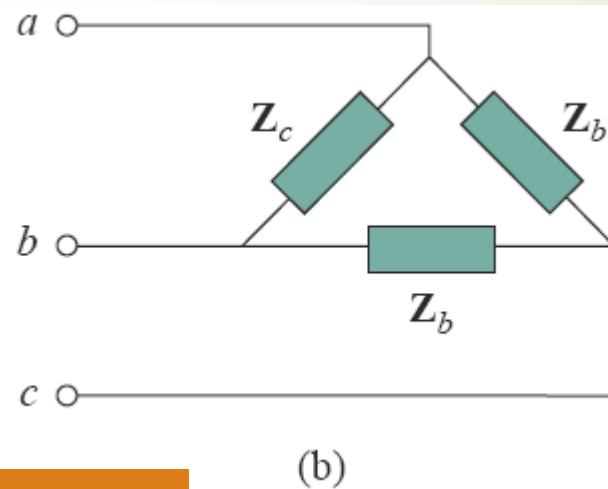


(b)



(a)

Three phase load



(b)

Balanced and Unbalanced Load

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➤ **Balanced load:**

A balanced load is that in which magnitudes of all impedances connected in the load are equal and the phase angles of them also are equal and of same type (inductive, resistive or capacitive).

➤ **Unbalanced load:**

If load doesn't satisfy the condition of balanced, then it is called as the unbalanced load.

The magnitudes and phase angles of the unbalanced loads are differ from each other.

Balanced Star Load

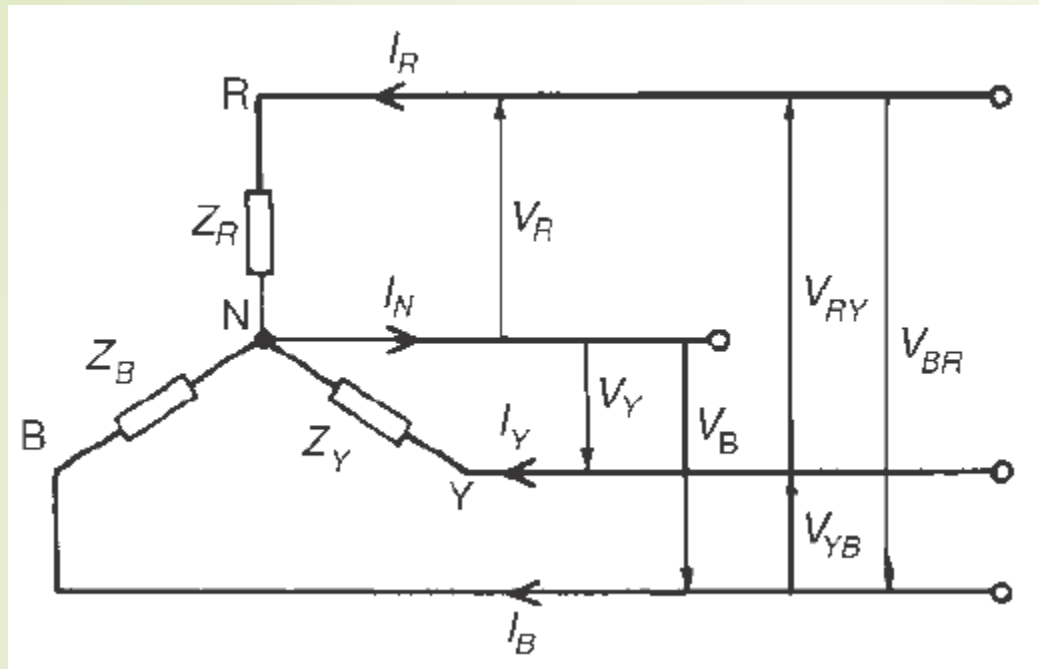
- **Line voltages :**
- If R, Y and B are called as the supply lines, then the potential difference between any two lines is known as the line voltage.
- V_{RY} , V_{RB} , V_{YB} , V_{YR} , V_{BR} and V_{BY} are six possible line voltages.
- All the line voltages are sinewaves of 50 Hz frequency and the phase shift between the adjacent line voltage is 60° .

Balanced Star Load

Phase voltages:

- the voltage measured across a single winding or phase is called as phase voltage.
- All the phase voltages are sinewaves and the phase difference between the adjacent phase voltages is 120° .

Star connection



$$V_R = V_P \angle 0^\circ$$

$$V_Y = V_P \angle -120^\circ$$

$$V_B = V_P \angle -240^\circ$$

- (i) The voltages, V_R , V_Y and V_B are called **phase voltages** or line to neutral voltages. Phase voltages are generally denoted by V_p .
- (ii) The voltages, V_{RY} , V_{YB} and V_{BR} are called **line voltages**.
- (iii) **For a balanced system, All load impedances are equal and hence all phase voltages are equal and hence all the phase currents at the load is also equal**

$$I_R = I_Y = I_B, \quad V_R = V_Y = V_B$$

$$V_{RY} = V_{YB} = V_{BR}, \quad Z_R = Z_Y = Z_B$$

(iv) The phase currents (generally denoted by I_p) are equal to their respective line currents I_R , I_Y and I_B , i.e. for a star connection:

$$I_L = I_p$$

(vi) The line voltage, V_{RY} ,

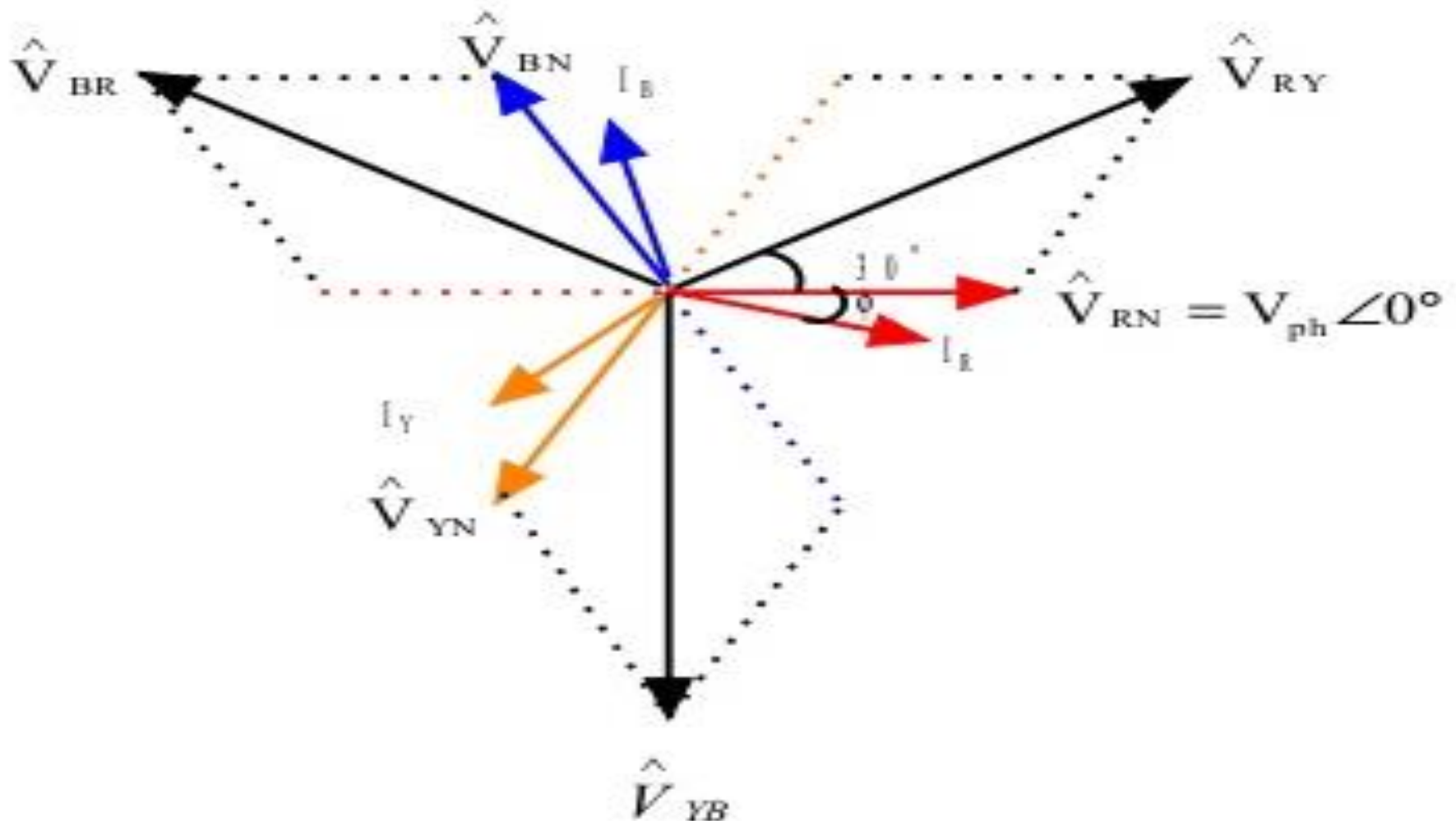
$V_{RY} = V_R - V_Y$ (V_Y is negative since it is in the opposite direction to V_{RY}).

$$= V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$= V_p \left(1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right) = \sqrt{3} V_p \angle 30^\circ$$

$$V_L = \sqrt{3} V_p$$

The complete phasor diagram



Continued....

Conclusion from the phasor diagram:

1. Phase currents lags behind the corresponding phase voltages by ϕ radians respectively as the load is inductive.
2. The line voltages are displaced by 120° from each other.
3. The line voltages leads their respective phase voltages by 30° .

Continued....

➤ **Equations for three phase power:**

➤ In single phase ac circuit, the power consumed in each phase is

$$P_{ph} = V_{ph} I_{ph} \cos \phi \quad \dots(1)$$

$$P_T = 3 P_{ph} = 3 V_{ph} I_{ph} \cos \phi \quad \dots(2)$$

substituting $V_{ph} = V_L/\sqrt{3}$ and $I_{ph} = I_L$, we get,

$$\text{Total power } P_T = 3 \times V_L/\sqrt{3} \times I_L \cos \phi$$

$$\therefore P_T = \sqrt{3} V_L I_L \cos \phi$$

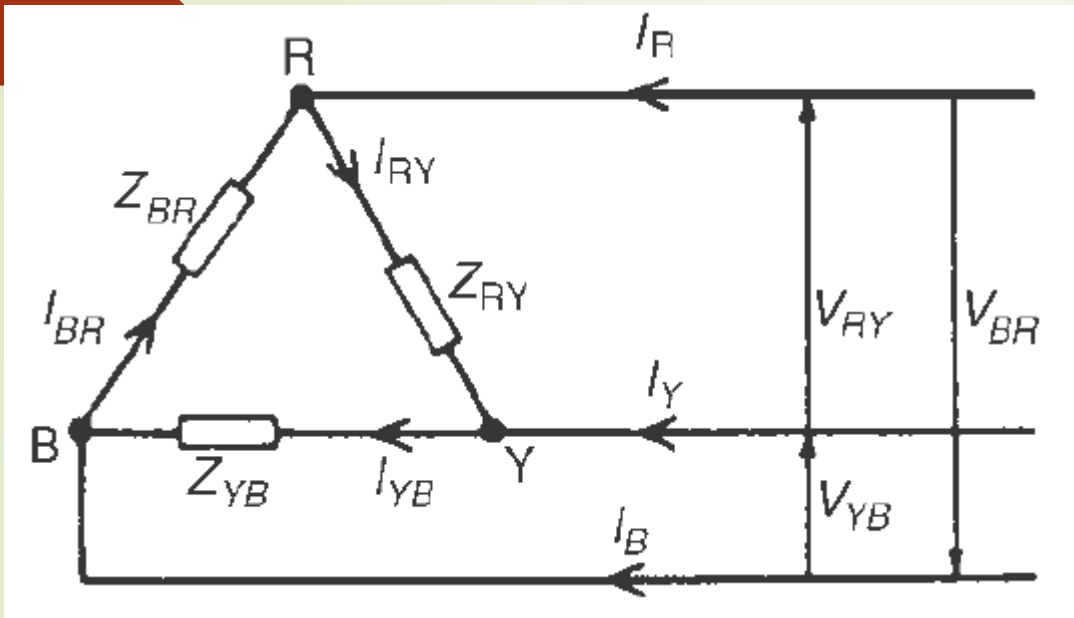
➤ **Power factor for a star load:**

The load power factor for a 3 phase balanced star load is equal to the power factor of each phase in the load.

$$\therefore \text{overall P.F.} = \cos \phi$$

where ϕ = angle between the phase voltage and phase current

Delta connection



$$I_{RY} = I_P \angle 0^\circ$$

$$I_{YB} = I_P \angle -120^\circ$$

$$I_{BR} = I_P \angle -240^\circ$$

(i) It can be seen that the line voltages V_{RY} , V_{YB} and V_{BR} are the respective phase voltages, i.e. for a delta connection:

$$V_L = V_p$$

(ii) $I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$

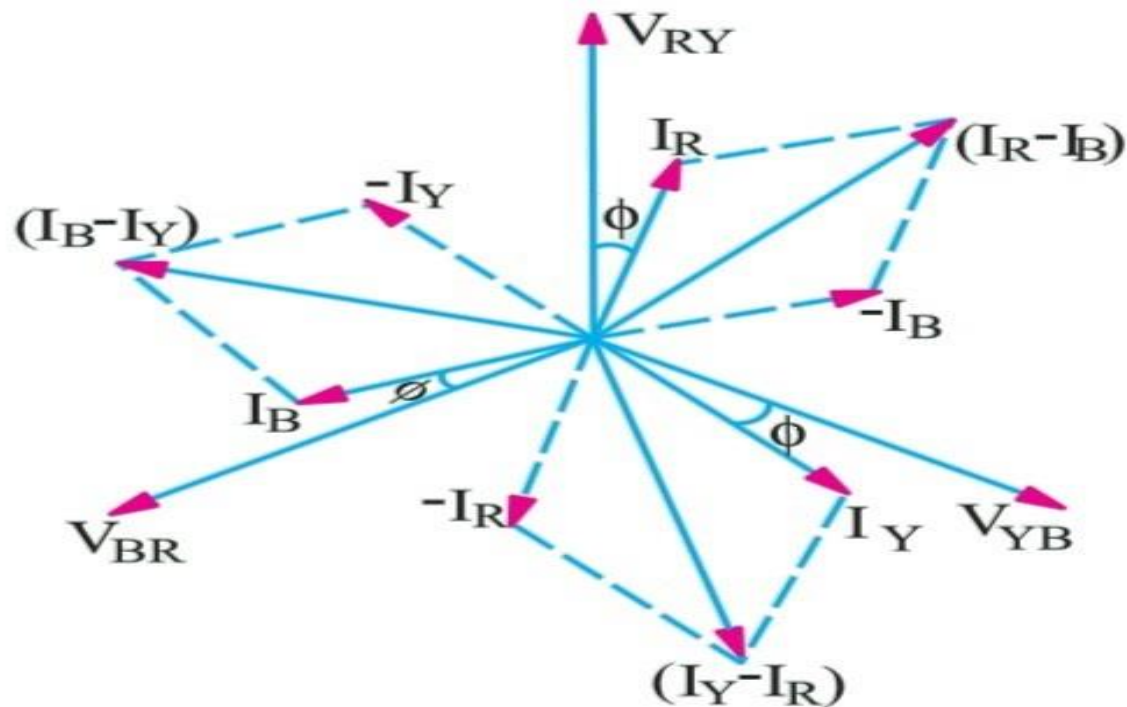
$$= I_P \angle 0^\circ - I_P \angle -240^\circ = \sqrt{3} I_P \angle -30^\circ$$

$$I_L = \sqrt{3} I_p$$

Continued....

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➤ The complete phasor diagram:



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**Line & Phase Current and Line & Phase
Voltage in Delta (Δ) Connection**

Continued....

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Conclusion from the phasor diagram:

1. Phase currents lags behind the corresponding phase voltages by ϕ radians respectively as the load is inductive.
2. Every line current lags the respective phase current by 30° .

Continued....

➤ **Equations for three phase power:**

➤ In single phase ac circuit, the power consumed in each phase is

$$P_{ph} = V_{ph} I_{ph} \cos \phi \quad \dots(1)$$

$$P_T = 3 P_{ph} = 3 V_{ph} I_{ph} \cos \phi \quad \dots(2)$$

substituting $V_{ph} = V_L$ and $I_{ph} = I_L / \sqrt{3}$, we get,

$$\text{Total power } P_T = 3 \times V_L \times I_L / \sqrt{3} \cos \phi$$

$$\therefore P_T = \sqrt{3} V_L I_L \cos \phi$$

➤ **Power factor for a star load:**

The load power factor for a 3 phase balanced delta load is equal to the power factor of each phase in the load.

$$\therefore \text{overall P.F.} = \cos \phi$$

where ϕ = angle between the phase voltage and phase current

Comparison of star & delta connection

Sr. No.	Parameter	Star connection	Delta connection
1.	Connection	See fig.	See fig.
2.	Neutral point	Present	Absent
3.	Relation between phase and line voltages	$V_L = \sqrt{3} V_{ph}$	$V_L = V_{ph}$
4.	Relation between phase and line currents	$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$
5.	Total active power	$P = \sqrt{3} V_L I_L \cos \phi$	$P = \sqrt{3} V_L I_L \cos \phi$
6.	Total reactive power	$Q = \sqrt{3} V_L I_L \sin \phi$	$Q = \sqrt{3} V_L I_L \sin \phi$

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\underline{V_{an}} = V_p \angle 0^\circ$ $\underline{V_{bn}} = V_p \angle -120^\circ$ $\underline{V_{cn}} = V_p \angle +120^\circ$ Same as line currents	$\underline{V_{ab}} = \sqrt{3} V_p \angle 30^\circ$ $\underline{V_{bc}} = \underline{V_{ab}} \angle -120^\circ$ $\underline{V_{ca}} = \underline{V_{ab}} \angle +120^\circ$ $\underline{I_a} = \underline{V_{an}} / \underline{Z_Y}$ $\underline{I_b} = \underline{I_a} \angle -120^\circ$ $\underline{I_c} = \underline{I_a} \angle +120^\circ$
Δ - Δ	$\underline{V_{ab}} = V_p \angle 0^\circ$ $\underline{V_{bc}} = V_p \angle -120^\circ$ $\underline{V_{ca}} = V_p \angle +120^\circ$ $\underline{I_{AB}} = \underline{V_{ab}} / \underline{Z_{\Delta}}$ $\underline{I_{BC}} = \underline{V_{bc}} / \underline{Z_{\Delta}}$ $\underline{I_{CA}} = \underline{V_{ca}} / \underline{Z_{\Delta}}$	Same as phase voltages $\underline{I_a} = \underline{I_{AB}} \sqrt{3} \angle -30^\circ$ $\underline{I_b} = \underline{I_a} \angle -120^\circ$ $\underline{I_c} = \underline{I_a} \angle +120^\circ$

Connection	Phase voltages/currents	Line voltages/currents
Δ -Y	$V_{ab} = V_p \angle 0^\circ$	Same as phase voltages
	$V_{bc} = V_p \angle -120^\circ$	
	$V_{ca} = V_p \angle +120^\circ$	
	Same as line currents	
Y- Δ		$I_a = \frac{V_p \angle -30^\circ}{\sqrt{3} Z_Y}$
		$I_b = I_a \angle -120^\circ$
		$I_c = I_a \angle +120^\circ$
	$V_{an} = V_p \angle 0^\circ$	$V_{ab} = V_{AB} = \sqrt{3} V_p \angle 30^\circ$
	$V_{bn} = V_p \angle -120^\circ$	$V_{bc} = V_{BC} = V_{ab} \angle -120^\circ$
	$V_{cn} = V_p \angle +120^\circ$	$V_{ca} = V_{CA} = V_{ab} \angle +120^\circ$
	$I_{AB} = V_{AB} / Z_\Delta$	$I_a = I_{AB} \sqrt{3} \angle -30^\circ$
	$I_{BC} = V_{BC} / Z_\Delta$	$I_b = I_a \angle -120^\circ$
	$I_{CA} = V_{CA} / Z_\Delta$	$I_c = I_a \angle +120^\circ$

Unbalanced Three-Phase Systems

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An unbalanced system is caused by two possible situations

- The source voltages are not equal in magnitude and/or differ in phase by angles that are unequal
- Load impedances are unequal

Unbalanced system are difficult to deal, In practice, there are three unbalanced 3-phase loads

- Four-wire star-connected unbalanced load
- Unbalanced Δ -connected load
- Unbalanced 3-wire, Y-connected load.

Four-wire star-connected unbalanced load

- In 3-phase, 4-wire unbalanced load, the loads are connected in three phases and the supply neutral (N) and load neutral (N') are connected
- The current in the neutral wire will be the phasor sum of the three line currents

$$I_N = I_R + I_Y + I_B$$

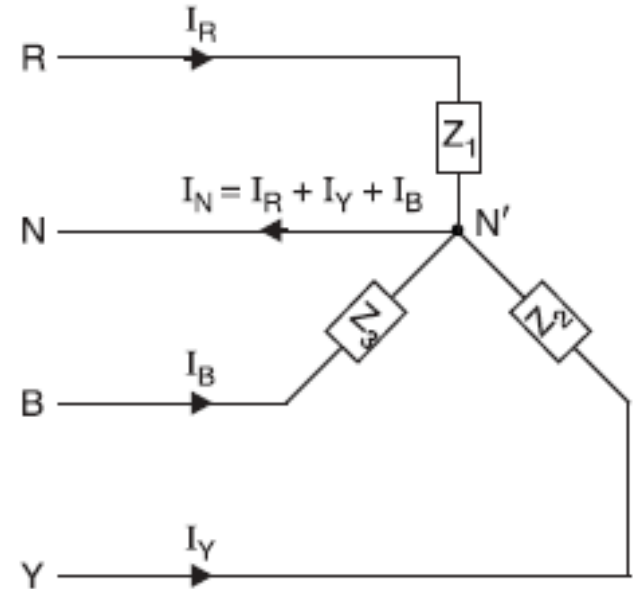


Figure: Unbalanced star connected load

- Since the load is unbalanced Z_1 , Z_2 and Z_3 , The line currents are calculate by Ohms law as

$$I_R = V_R / Z_1, I_Y = V_Y / Z_2, I_B = V_B / Z_3$$

- This set of unbalance currents produces current in neutral line, which is not zero as in case of balanced system. Applying KCL at node N gives

$$I_N = I_R + I_Y + I_B$$

- The power calculation are done as per-phase unbalanced system.

$$\text{Active power } P_{ph} = V_{ph} I_{ph} \cos \phi$$

$$\text{Reactive power } Q_{ph} = V_{ph} I_{ph} \sin \phi$$

$$\text{Apparent power } S_{ph} = V_{ph} I_{ph}$$

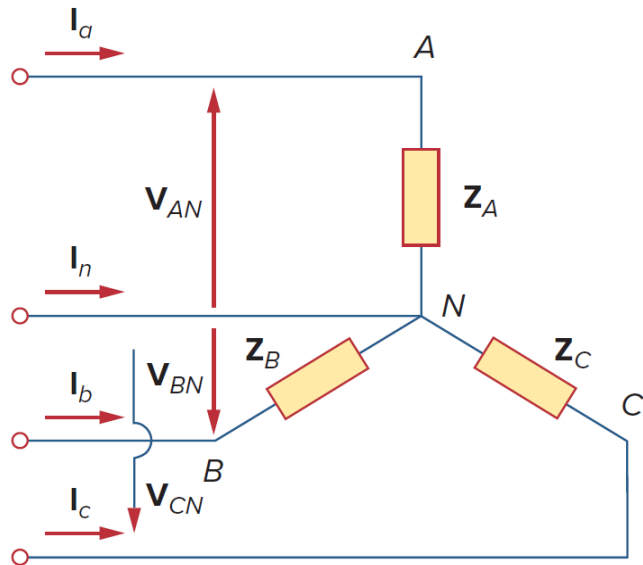
The following points may be noted carefully

- ▶ Since the neutral wire has negligible impedance, supply neutral N and load neutral N will be at the same potential.
- ▶ So voltage across each impedance is equal to the phase voltage of the supply
- ▶ However, current in each phase (or line) will be different due to unequal impedances
- ▶ The amount of current flowing in the neutral wire will depend upon the magnitudes of line currents and their phase relation.
- ▶ In most circuits, the neutral current is equal to or smaller than any one of the line currents

Problems

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P3.1 The unbalanced Y-load of Fig. has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $\mathbf{Z}_A = 15 \Omega$, $\mathbf{Z}_B = 10 + j5 \Omega$, $\mathbf{Z}_C = 6 - j8 \Omega$.



$$\mathbf{I}_a = \frac{100 \angle 0^\circ}{15} = 6.67 \angle 0^\circ \text{ A}$$

$$\mathbf{I}_b = \frac{100 \angle 120^\circ}{10 + j5} = \frac{100 \angle 120^\circ}{11.18 \angle 26.56^\circ} = 8.94 \angle 93.44^\circ \text{ A}$$

$$\mathbf{I}_c = \frac{100 \angle -120^\circ}{6 - j8} = \frac{100 \angle -120^\circ}{10 \angle -53.13^\circ} = 10 \angle -66.87^\circ \text{ A}$$

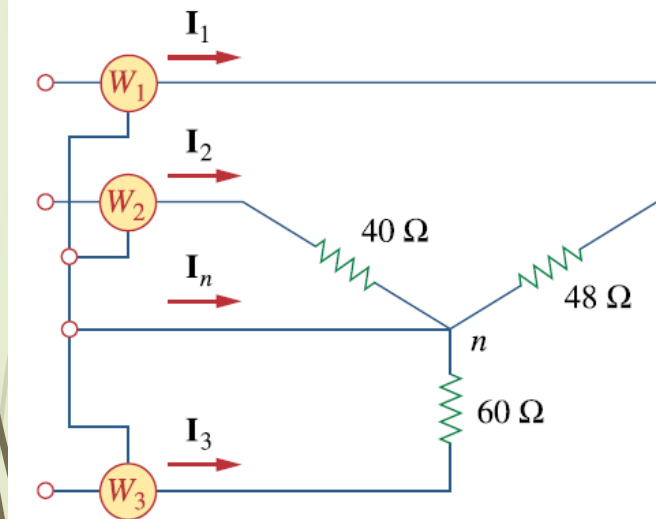
$$\begin{aligned} \mathbf{I}_n &= -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2) \\ &= -10.06 + j0.28 = 10.06 \angle 178.4^\circ \text{ A} \end{aligned}$$

P3.2

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A three-phase, four-wire system operating with a 208-V line voltage is shown in Fig. The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method. Calculate:

- (a) the voltage to neutral
- (b) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_n$
- (c) the readings of the wattmeters
- (d) the total power absorbed by the load



(a)
$$V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = \underline{\underline{120\text{ V}}}$$

(b)
$$\begin{aligned}\mathbf{I}_1 &= \frac{120\angle 0^\circ}{48} = 2.5\angle 0^\circ\text{ A} \\ \mathbf{I}_2 &= \frac{120\angle -120^\circ}{40} = 3\angle -120^\circ\text{ A} \\ \mathbf{I}_3 &= \frac{120\angle 120^\circ}{60} = 2\angle 120^\circ\text{ A}\end{aligned}$$

$$\mathbf{I}_N = -\left[\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3\right] = 0.866\angle 90^\circ \text{ A}$$

Hence,

$$\mathbf{I}_1 = \underline{\mathbf{2.5 \text{ A}}}, \quad \mathbf{I}_2 = \underline{\mathbf{3 \text{ A}}}, \quad \mathbf{I}_3 = \underline{\mathbf{2 \text{ A}}}, \quad \mathbf{I}_N = \underline{\mathbf{0.866 \text{ A}}}$$

$$(c) \quad P_1 = I_1^2 R_1 = (2.5)^2 (48) = \underline{\mathbf{300 \text{ W}}}$$

$$P_2 = I_2^2 R_2 = (3)^2 (40) = \underline{\mathbf{360 \text{ W}}}$$

$$P_3 = I_3^2 R_3 = (2)^2 (60) = \underline{\mathbf{240 \text{ W}}}$$

$$(d) \quad P_T = P_1 + P_2 + P_3 = \underline{\mathbf{900 \text{ W}}}$$