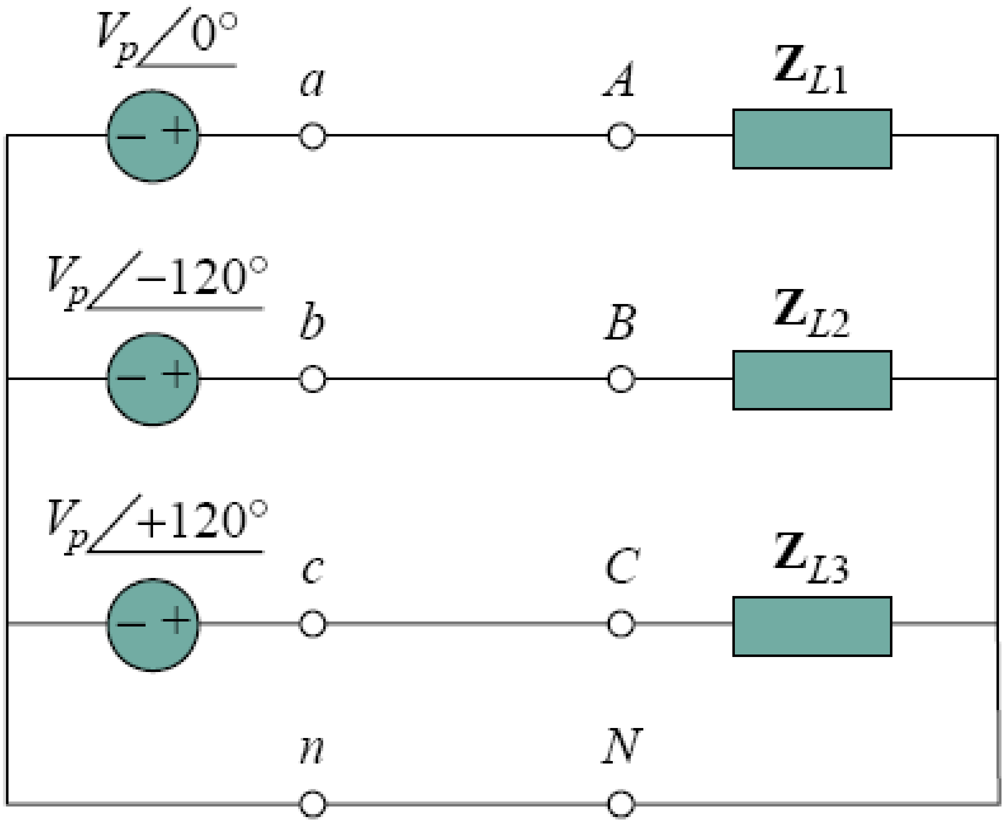
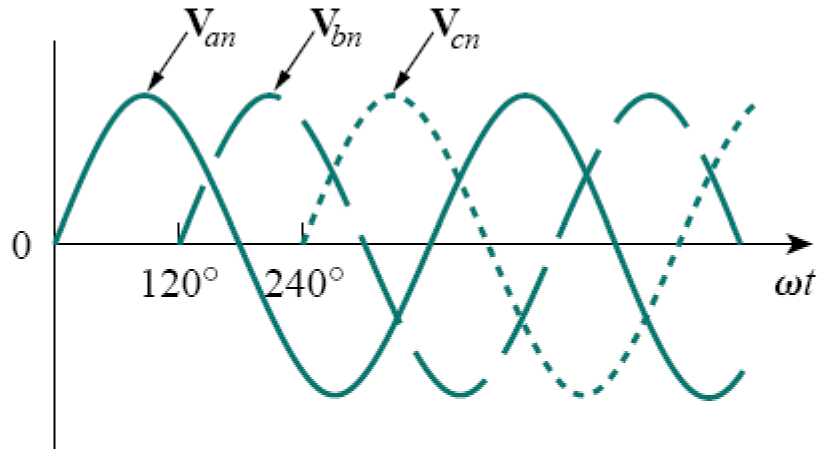
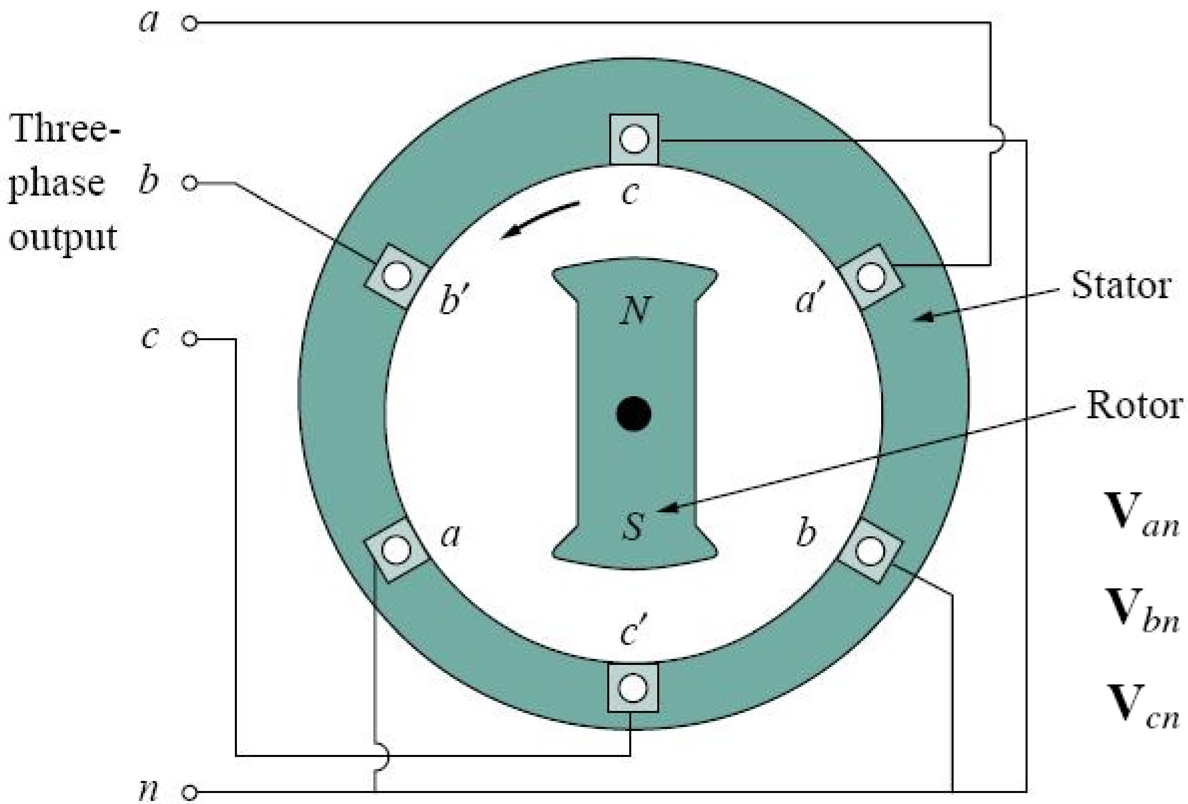
3 phase circuits

Three-phase four-wire system.





A three-phase generator.



$$\mathbf{V}_{an} = V_p \angle 0^\circ$$

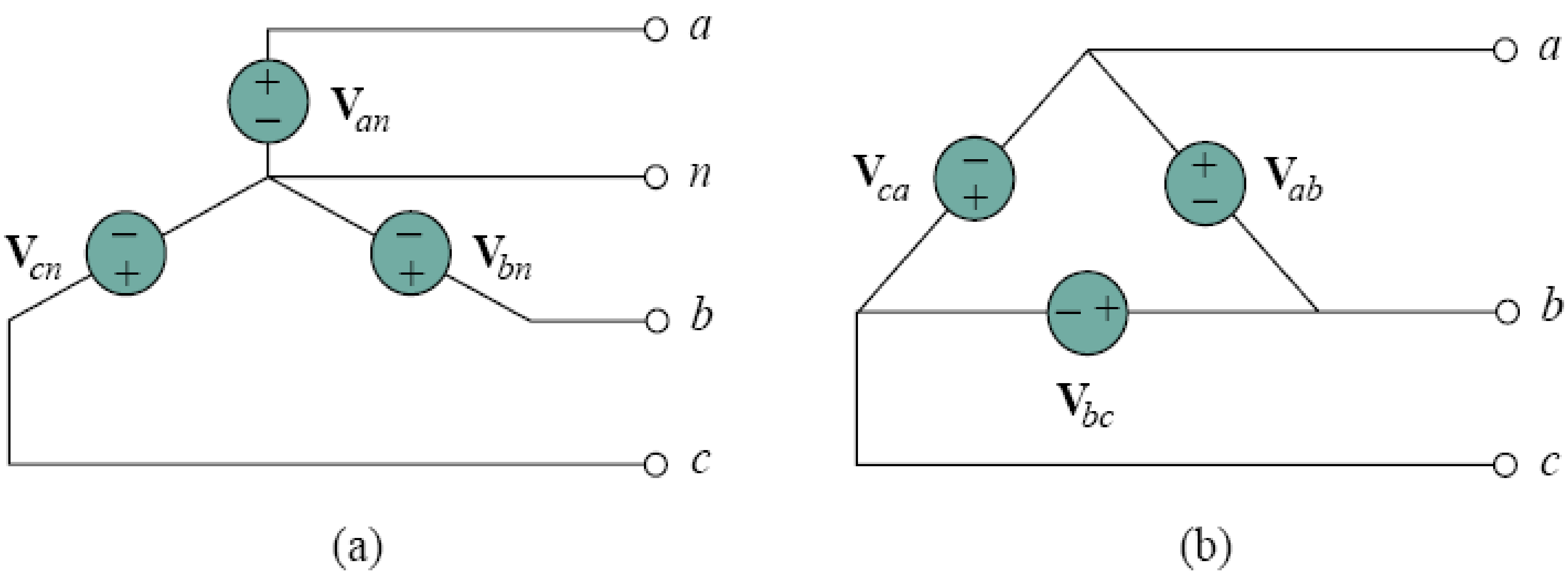
$$\mathbf{V}_{bn} = V_p \angle -120^\circ$$

$$\mathbf{V}_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0$$

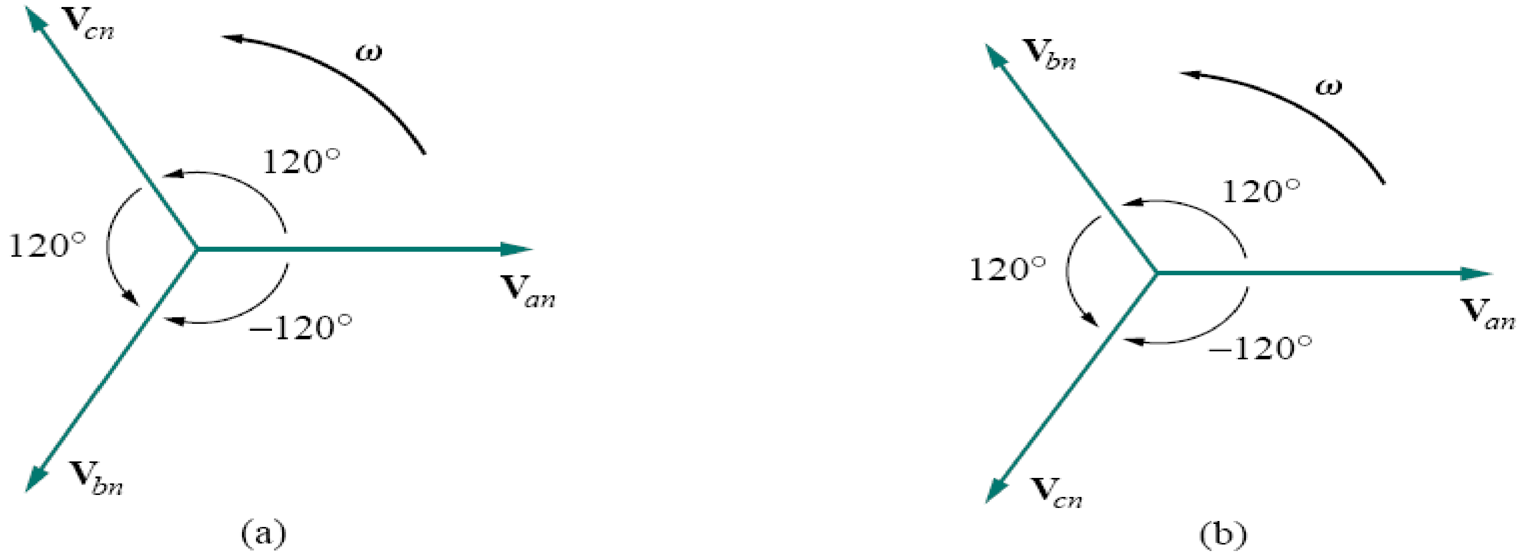
$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°.

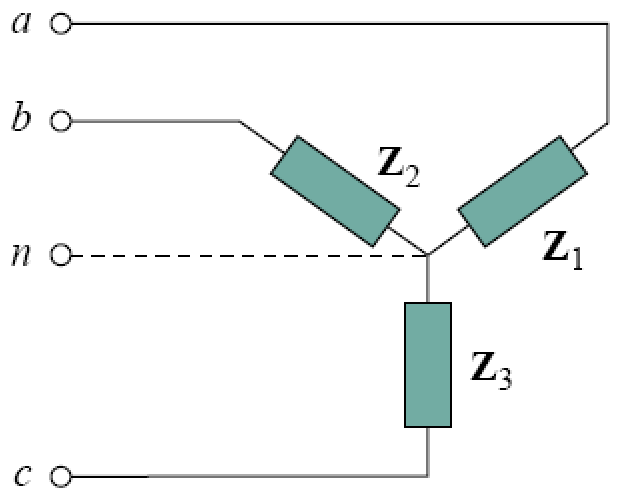


Three-phase voltage sources: (a) Y-connected source, (b) Δ-connected source.

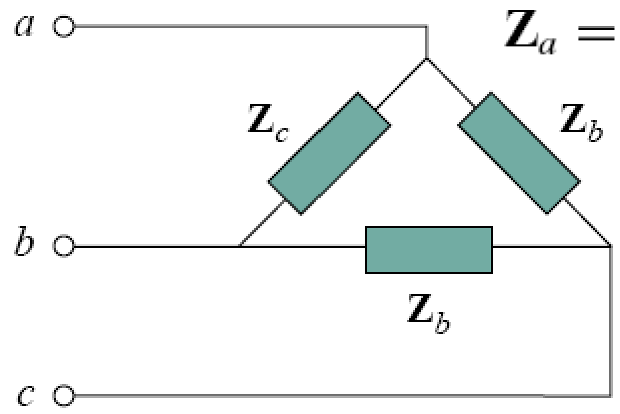
Phase sequences: (a) *abc or positive sequence*, (b) *acb or negative sequence.*



A balanced load is one in which the phase impedances are equal in magnitude and in phase.



$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y$$



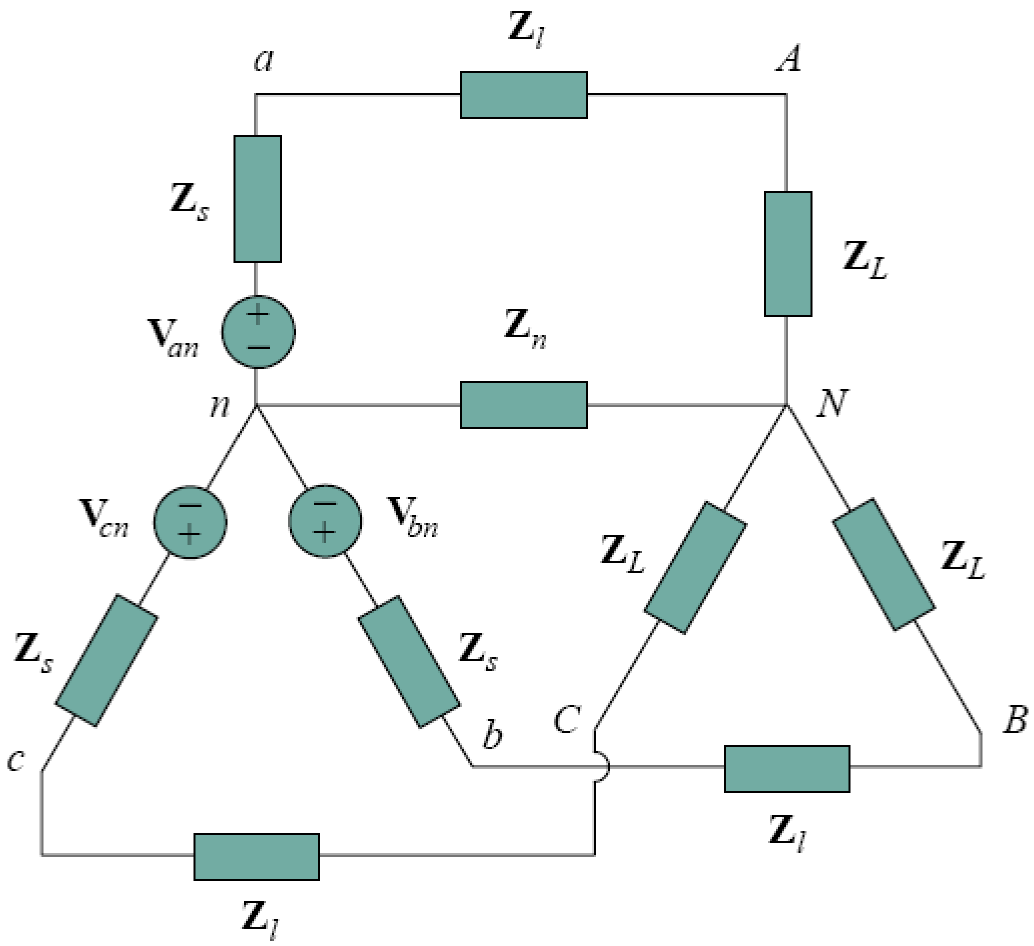
$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta$$

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

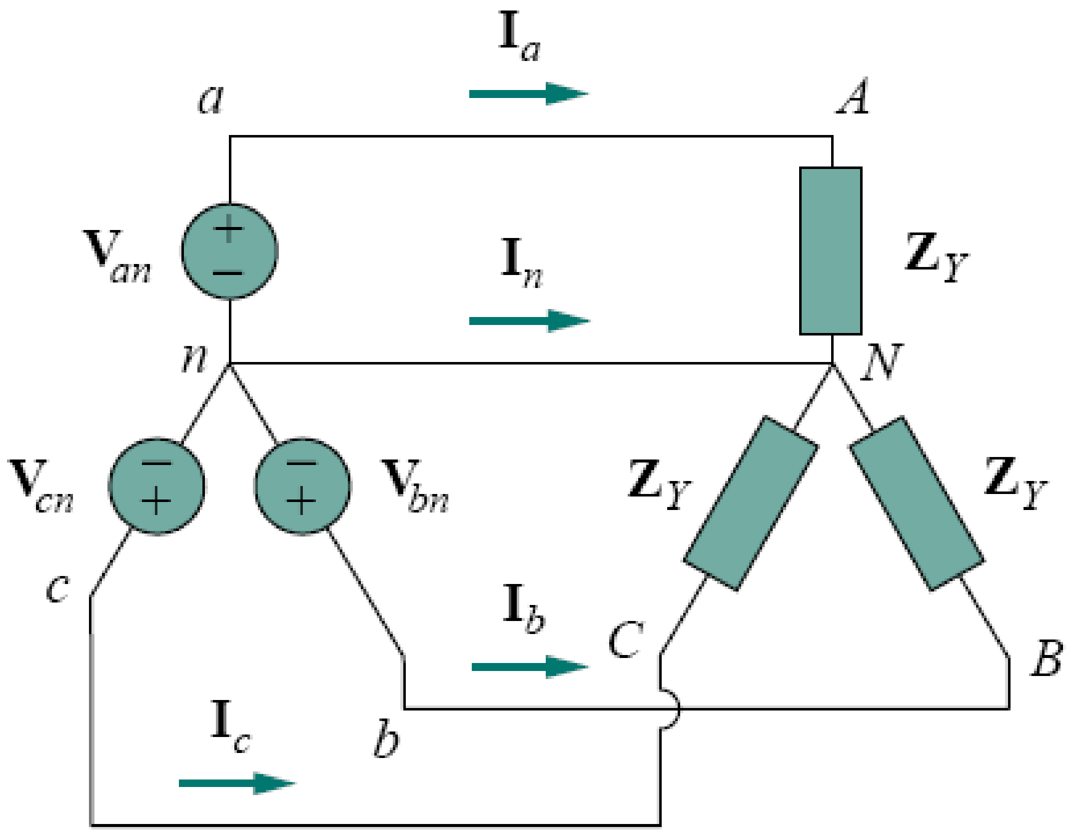
Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

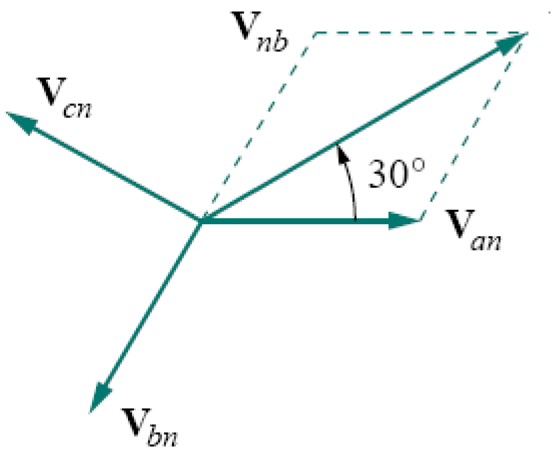
- Y-Y connection (i.e., Y-connected source with a Y-connected load).
- Y-Δ connection.
- Δ-Δ connection.
- Δ-Y connection.

A balanced Y-Y system, showing the source, line, and load impedances.

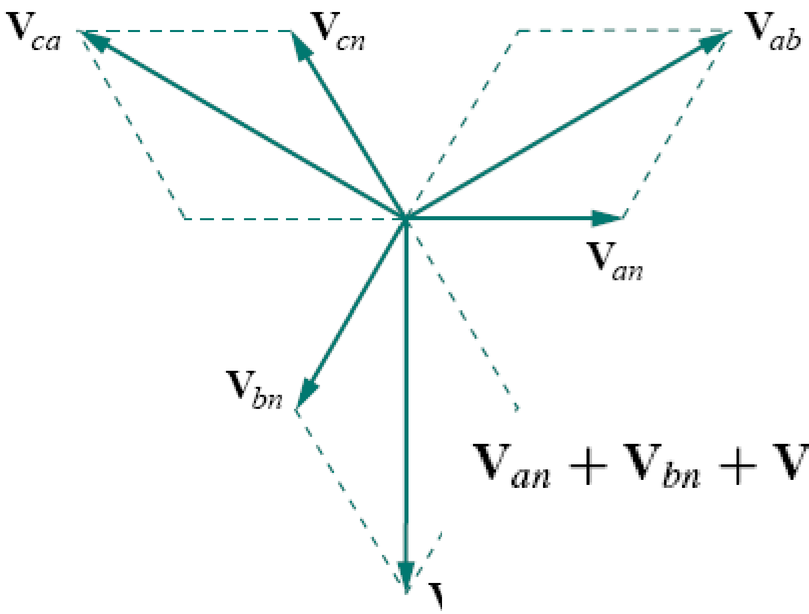


Balanced Y-Y connection.





$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\underline{/0^\circ} - V_p\underline{/-120^\circ}$$
$$= V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p\underline{/30^\circ}$$
$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\underline{/-90^\circ}$$
$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\underline{/-210^\circ}$$



$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = V_p\underline{/0^\circ} + V_p\underline{/-120^\circ} + V_p\underline{/+120^\circ}$$
$$= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866)$$
$$= 0$$

Power in a balanced three-phase system

For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos \omega t, \qquad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ)$$

$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120^\circ)$$

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \qquad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ)$$

$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ)$$

$$\begin{aligned} p &= p_a + p_b + p_c = v_{AN} i_a + v_{BN} i_b + v_{CN} i_c \\ &= 2V_p I_p [\cos \omega t \cos(\omega t - \theta) \\ &\qquad + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) \\ &\qquad + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)] \end{aligned}$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

gives

$$p = V_p I_p[3\cos\theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ)$$
$$+ \cos(2\omega t - \theta + 240^\circ)]$$
$$= V_p I_p[3\cos\theta + \cos\alpha + \cos\alpha\cos 240^\circ + \sin\alpha\sin 240^\circ$$
$$+ \cos\alpha\cos 240^\circ - \sin\alpha\sin 240^\circ]$$

where $\alpha = 2\omega t - \theta$

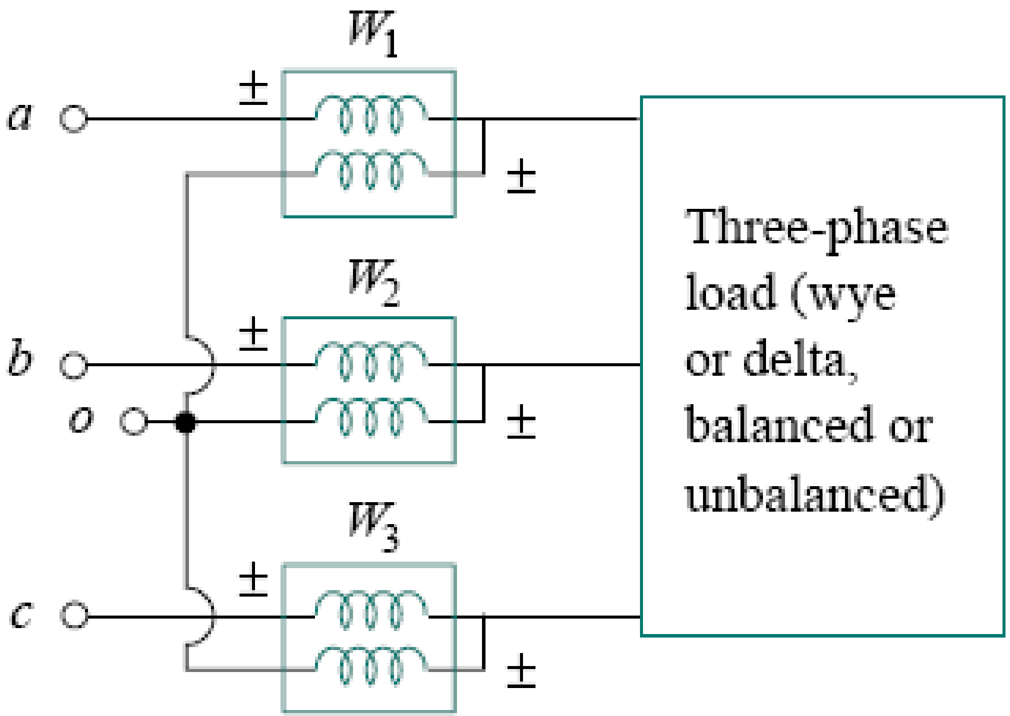
$$= V_p I_p\left[3\cos\theta + \cos\alpha + 2\left(-\frac{1}{2}\right)\cos\alpha\right] = 3V_p I_p\cos\theta$$

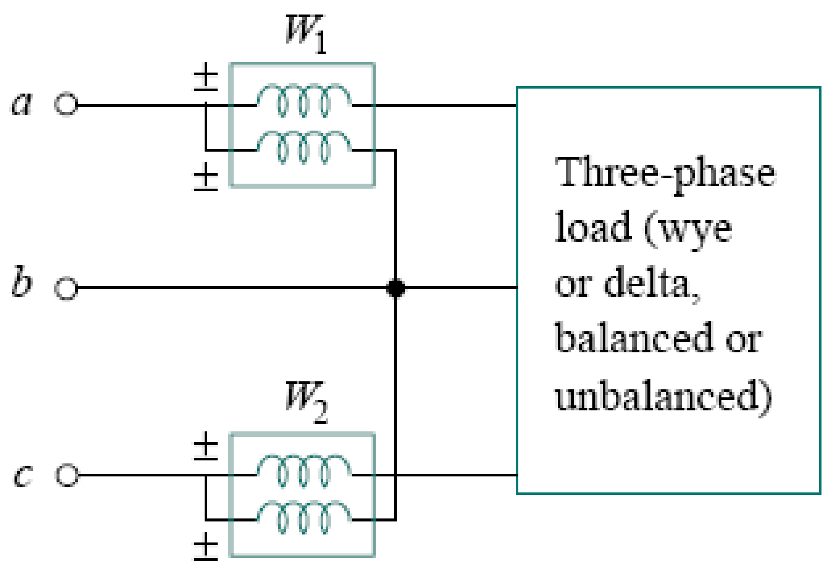
$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p\cos\theta = \sqrt{3}V_L I_L\cos\theta$$

$$Q = 3V_p I_p \sin\theta = 3Q_p = \sqrt{3}V_L I_L \sin\theta$$

and the total complex power is

$$\mathbf{S} = 3\mathbf{S}_p = 3\mathbf{V}_p\mathbf{I}_p^* = 3I_p^2\mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*}$$





$$P_1 = \mathrm{Re}[\mathbf{V}_{ab}\mathbf{I}_a^*] = V_{ab}I_a\cos(\theta + 30^\circ) = V_L I_L \cos(\theta + 30^\circ)$$

$$P_2 = \mathrm{Re}[\mathbf{V}_{cb}\mathbf{I}_c^*] = V_{cb}I_c\cos(\theta - 30^\circ) = V_L I_L \cos(\theta - 30^\circ)$$

$$\begin{aligned}
P_1 + P_2 &= V_L I_L[\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ)] \\
&= V_l I_L(\cos\theta\cos 30^\circ - \sin\theta\sin 30^\circ \\
&\qquad + \cos\theta\cos 30^\circ + \sin\theta\sin 30^\circ) \\
&= V_L I_L 2\cos 30^\circ\cos\theta = \sqrt{3}V_L I_L\cos\theta
\end{aligned}$$

$$\begin{aligned}
P_1 - P_2 &= V_L I_L[\cos(\theta + 30^\circ) - \cos(\theta - 30^\circ)] \\
&= V_l I_L(\cos\theta\cos 30^\circ - \sin\theta\sin 30^\circ \\
&\qquad - \cos\theta\cos 30^\circ - \sin\theta\sin 30^\circ) \\
&= -V_L I_L 2\sin 30^\circ\sin\theta
\end{aligned}$$

$$P_2 - P_1 = V_L I_L\sin\theta$$

$$Q_T = \sqrt{3}(P_2 - P_1)$$

$$\tan\theta = \frac{Q_T}{P_T} = \sqrt{3}\frac{P_2 - P_1}{P_2 + P_1}$$

1. If $P_2 = P_1$, the load is resistive.
2. If $P_2 > P_1$, the load is inductive.
3. If $P_2 < P_1$, the load is capacitive.

The two-wattmeter method produces wattmeter readings $P_1 = 1560$ W and $P_2 = 2100$ W when connected to a delta-connected load. If the line voltage is 220 V, calculate: (a) the per-phase average power, (b) the per-phase reactive power, (c) the power factor, and (d) the phase impedance.

Solution:

We can apply the given results to the delta-connected load.
(a) The total real or average power is

$$P_T = P_1 + P_2 = 1560 + 2100 = 3660 \text{ W}$$

The per-phase average power is then

$$P_p = \frac{1}{3} P_T = 1220 \text{ W}$$

(b) The total reactive power is

$$Q_T = \sqrt{3}(P_2 - P_1) = \sqrt{3}(2100 - 1560) = 935.3 \text{ VAR}$$

so that the per-phase reactive power is

$$Q_p = \frac{1}{3} Q_T = 311.77 \text{ VAR}$$

(c) The power angle is

$$\theta = \tan^{-1} \frac{Q_T}{P_T} = \tan^{-1} \frac{935.3}{3660} = 14.33^\circ$$

Hence, the power factor is

$$\cos\theta = 0.9689 \text{ (leading)}$$

It is a leading pf because Q_T is positive or $P_2 > P_1$.

(c) The phase impedance is $\mathbf{Z}_p = Z_p \underline{/\theta}$. We know that θ is the same as the pf angle; that is, $\theta = 14.57°$.

$$Z_p = \frac{V_p}{I_p}$$

We recall that for a delta-connected load, $V_p = V_L = 220$ V. From Eq. (12.46),

$$P_p = V_p I_p \cos\theta \qquad \Longrightarrow \qquad I_p = \frac{1220}{220 \times 0.9689} = 5.723 \text{ A}$$

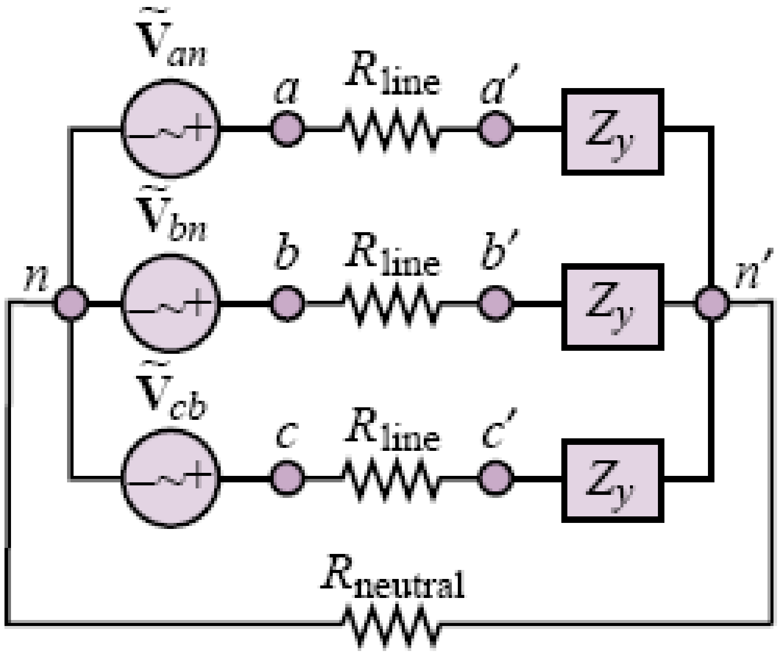
Hence,

$$Z_p = \frac{V_p}{I_p} = \frac{220}{5.723} = 38.44\ \Omega$$

and

$$\mathbf{Z}_p = 38.44 \underline{/14.33°}\ \Omega$$

Compute the power delivered to the load by the three-phase generator in the circuit shown in Figure



Schematics, Diagrams, Circuits, and Given Data: $\tilde{\mathbf{V}}_{an} = 480\angle(0)$ V; $\tilde{\mathbf{V}}_{bn} = 480\angle(-2\pi/3)$ V; $\tilde{\mathbf{V}}_{cn} = 480\angle(2\pi/3)$ V; $Z_y = 2 + j4 = 4.47\angle(1.107)\ \Omega$; $R_{line} = 2\ \Omega$; $R_{neutral} = 10\ \Omega$.

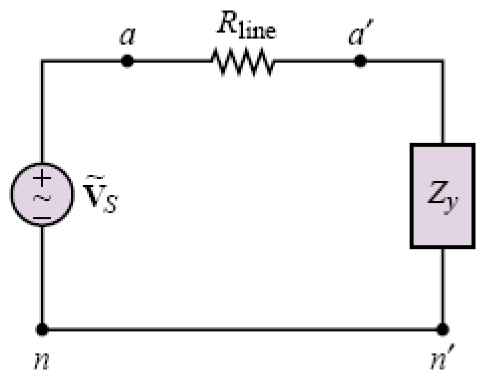
$$P_a = |\tilde{\mathbf{I}}|^2 R_L$$

where

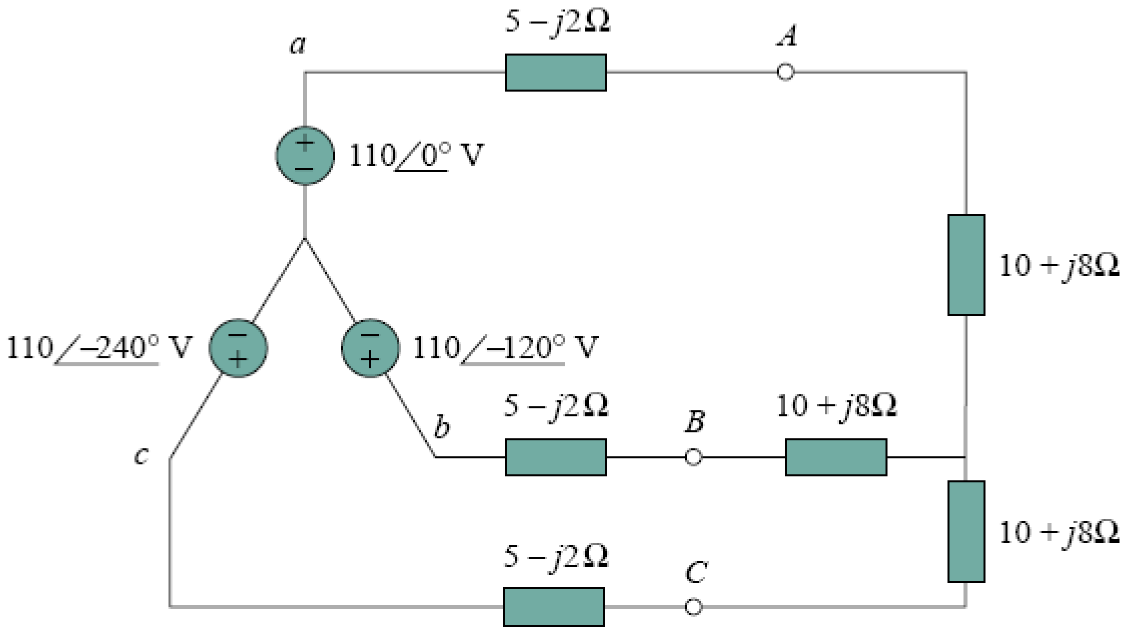
$$|\tilde{\mathbf{I}}| = \left|\frac{\tilde{\mathbf{V}}_a}{Z_y + R_{\text{line}}}\right| = \left|\frac{480\angle 0}{2 + j4 + 2}\right| = \left|\frac{480\angle 0}{5.66\angle\left(\frac{\pi}{4}\right)}\right| = 84.85 \text{ A}$$

and $P_a = (84.85)^2 \times 2 = 14.4$ kW. Since the circuit is balanced, the results for phases b and c are identical, and we have:

$$P_L = 3P_a = 43.2 \text{ kW}$$



Determine the total average power, reactive power, and complex power at the source and at the load.



Solution:

It is sufficient to consider one phase, as the system is balanced. For phase a,

$$\mathbf{V}_p = 110\underline{/0^\circ} \text{ V} \qquad \text{and} \qquad \mathbf{I}_p = 6.81\underline{/-21.8^\circ} \text{ A}$$

Thus, at the source, the complex power supplied is

$$\mathbf{S}_s = -3\mathbf{V}_p\mathbf{I}_p^* = 3(110\underline{/0^\circ})(6.81\underline{/21.8^\circ})$$
$$= -2247\underline{/21.8^\circ} = -(2087 + j834.6) \text{ VA}$$

The real or average power supplied is -2087 W and the reactive power is -834.6 VAR.

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2\mathbf{Z}_p$$

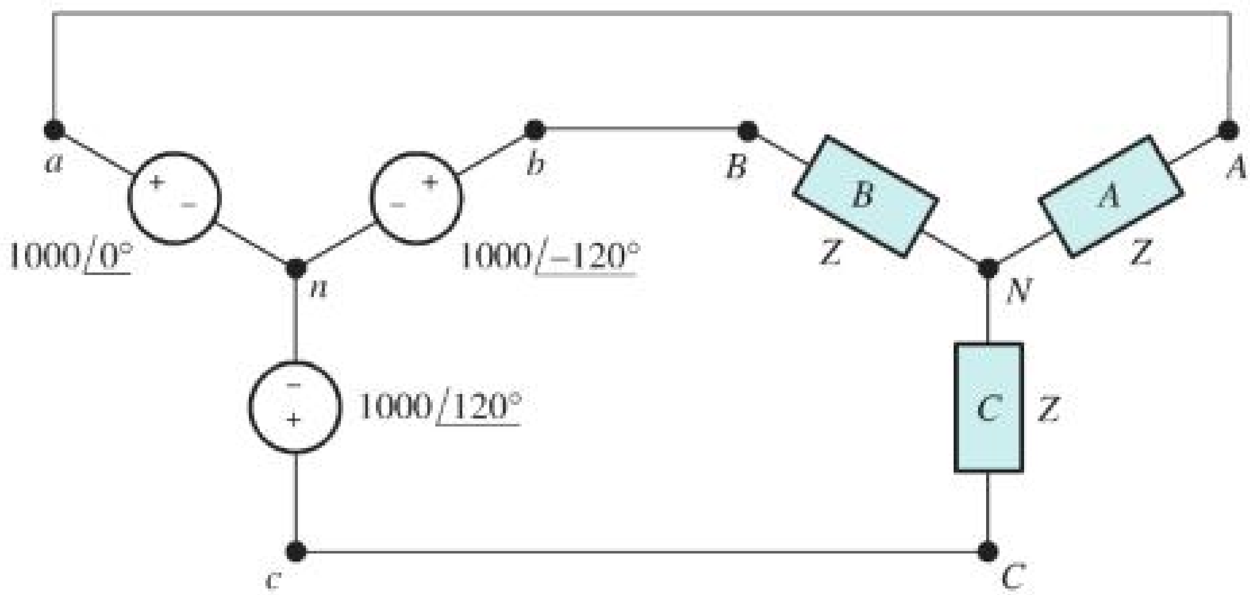
where $\mathbf{Z}_p = 10 + j8 = 12.81\underline{/38.66^\circ}$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81\underline{/-21.8^\circ}$. Hence

$$\mathbf{S}_L = 3(6.81)^2 12.81\underline{/38.66^\circ} = 1782\underline{/38.66}$$
$$= (1392 + j1113) \text{ VA}$$

The real power absorbed is 1391.7 W and the reactive power absorbed is 1113.3 VAR. The difference between the two complex powers is absorbed by the line impedance $(5 - j2)$ Ω. To show that this is the case, we find the complex power absorbed by the line as

$$\mathbf{S}_\ell = 3|\mathbf{I}_p|^2\mathbf{Z}_\ell = 3(6.81)^2(5 - j2) = 695.6 - j278.3 \text{ VA}$$

A balanced positive-sequence wye-connected 60-Hz three-phase source has line-to-neutral voltages of $V_Y = 1000$ V. This source is connected to a balanced wye-connected load. Each phase of the load consists of a 0.1-H inductance in series with a 50-Ω resistance. Find the line currents, the line-to-line voltages, the power, and the reactive power delivered to the load. Draw a phasor diagram showing the line-to-neutral voltages, the line-to-line voltages, and the line currents. Assume that the phase angle of $\mathbf{V}_{an}$ is zero.



Solution First, by computing the complex impedance of each phase of the load, we find that

$$Z = R + j\omega L = 50 + j2\pi(60)(0.1) = 50 + j37.70$$
$$= 62.62\ \angle 37.02^\circ$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{Z} = \frac{1000\ \angle 0^\circ}{62.62\ \angle 37.02^\circ} = 15.97\ \angle -37.02^\circ$$

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{bn}}{Z} = \frac{1000\ \angle -120^\circ}{62.62\ \angle 37.02^\circ} = 15.97\ \angle -157.02^\circ$$

$$\mathbf{I}_{cC} = \frac{\mathbf{V}_{cn}}{Z} = \frac{1000\ \angle 120^\circ}{62.62\ \angle 37.02^\circ} = 15.97\ \angle 82.98^\circ$$

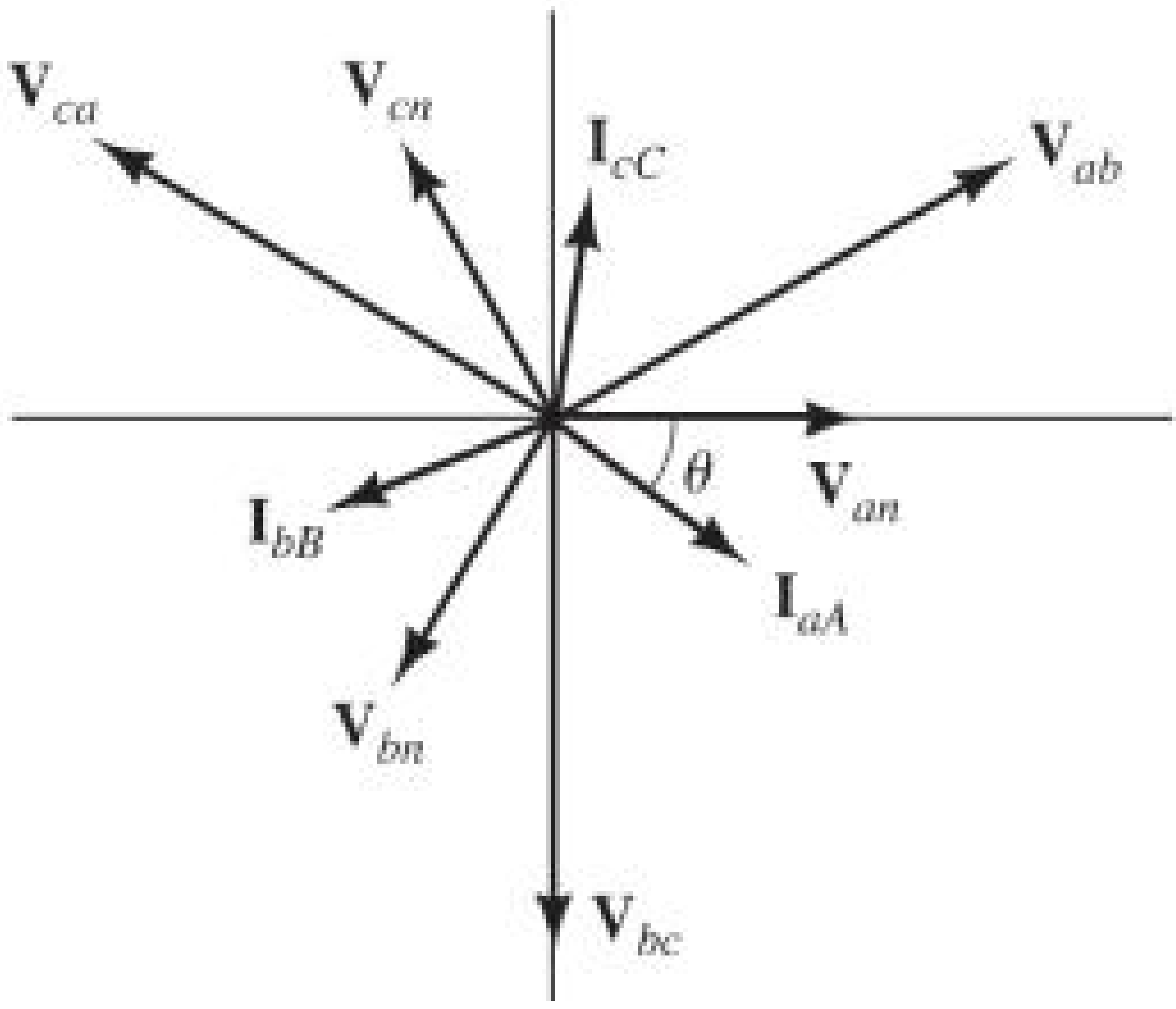
$$\mathbf{V}_{ab} = \mathbf{V}_{an} \times \sqrt{3}\ \angle 30^\circ = 1732\ \angle 30^\circ$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} \times \sqrt{3}\ \angle 30^\circ = 1732\ \angle -90^\circ$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} \times \sqrt{3}\ \angle 30^\circ = 1732\ \angle 150^\circ$$

$$P = 3\frac{V_Y I_L}{2}\cos(\theta) = 3\left(\frac{1000 \times 15.97}{2}\right)\cos(37.02^\circ) = 19.13 \text{ kW}$$

$$Q = 3\frac{V_Y I_L}{2}\sin(\theta) = 3\left(\frac{1000 \times 15.97}{2}\right)\sin(37.02^\circ) = 14.42 \text{ kVAR}$$



$\mathbf{V}_{ca}$
$\mathbf{V}_{cn}$
$\mathbf{I}_{cC}$
$\mathbf{V}_{ab}$
θ
$\mathbf{V}_{an}$
$\mathbf{I}_{bB}$
$\mathbf{I}_{aA}$
$\mathbf{V}_{bn}$
$\mathbf{V}_{bc}$

Three identical coils, each of resistance 10 and inductance 42 mH are connected (a) in star and (b) in delta to a 415 V, 50 Hz, 3-phase supply. Determine the total power dissipated in each case.

(a) **Star connection**

$$\text{Inductive reactance } X_L = 2\pi f L = 2\pi(50)(42 \times 10^{-3})$$

$$= 13.19\ \Omega$$

$$\text{Phase impedance } Z_p = \sqrt{(R^2 + X_L^2)} = \sqrt{(10^2 + 13.19^2)}$$

$$= 16.55\ \Omega$$

Line voltage $V_L = 415$ V and

$$\text{phase voltage, } V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240 \text{ V}$$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{240}{16.55} = 14.50 \text{ A}$$

$$\text{Line current, } I_L = I_p = 14.50 \text{ A}$$

$$\text{Power factor} = \cos\phi = \frac{R_p}{Z_p} = \frac{10}{16.55} = 0.6042 \text{ lagging}$$

Power dissipated, $P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3}(415)(14.50)(0.6042)$

$$= \mathbf{6.3\ kW}$$

(Alternatively, $P = 3{I_p}^2 R_p = 3(14.50)^2(10) = \mathbf{6.3\ kW}$)

(b) **Delta connection**

$V_L = V_p = 415$ V, $Z_p = 16.55\ \Omega$,

$\cos\phi = 0.6042$ lagging (from above).

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{415}{16.55} = 25.08 \text{ A}$$

$$\text{Line current, } I_L = \sqrt{3} I_p = \sqrt{3}(25.08) = 43.44 \text{ A}$$

$$\textbf{Power dissipated, } P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3}(415)(43.44)(0.6042)$$

$$= \mathbf{18.87\ kW}$$

$$\text{(Alternatively, } P = 3{I_p}^2 R_p = 3(25.08)^2(10) = \mathbf{18.87\ kW}\text{)}$$

Hence loads connected in delta dissipate three times the power than when connected in star, and also take a line current three times greater.