



KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any **TEN** Questions

(10 X 10 = 100 Marks)

1. Consider the function $f(x) = x(6 - 2x)^2$
 - (i) Identify where the extrema of f occur.
 - (ii) Find the intervals on which f is increasing and decreasing.
 - (iii) Find the intervals on which f is concave up and concave down.
2. (i) Find the area of the region enclosed by $y = x^4$ and $y = 8x$. [5]
 (ii) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ about the y -axis. [5]
3. If z is a function of x and y , where $x = e^u \cos v$, $y = e^u \sin v$ prove that $y \frac{\partial z}{\partial u} + x \frac{\partial z}{\partial v} = e^{2u} \frac{\partial z}{\partial y}$.
4. Using Taylor's series expansion expand $e^x \sin y$ in powers of x and y up to third degree terms.
5. The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = a^2$.
6. By changing the order of integration, evaluate $\int_0^1 \int_y^{2-y} xy dx dy$.
7. By transforming into spherical polar coordinates, evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} x dx dy dz$.
8. Using Gamma function, evaluate $(\int_0^\infty xe^{-x^2} dx) (\int_0^\infty x^2 e^{-x^4} dx)$.
9. Show that $\vec{F} = (2x + yz)\vec{i} + (4y + zx)\vec{j} - (6z - xy)\vec{k}$ is irrotational. Hence find its scalar potential ϕ .
10. If $r = |\vec{r}|$, where \vec{r} is the position vector of the point (x, y, z) , then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$.
11. Verify Green's theorem in the plane for $\int_C \{(2x - y)dx + (x + y)dy\}$ where C is the boundary of the circle $x^2 + y^2 = a^2$.
12. Verify Stokes theorem for the function $\vec{F} = x^2\vec{i} + xy\vec{j}$, integrated round the square in the plane $z = 0$ whose sides are along the lines $x = 0, y = 0, x = a, y = a$.