

EEE 203L - CIRCUIT THEORY

MODULE-III

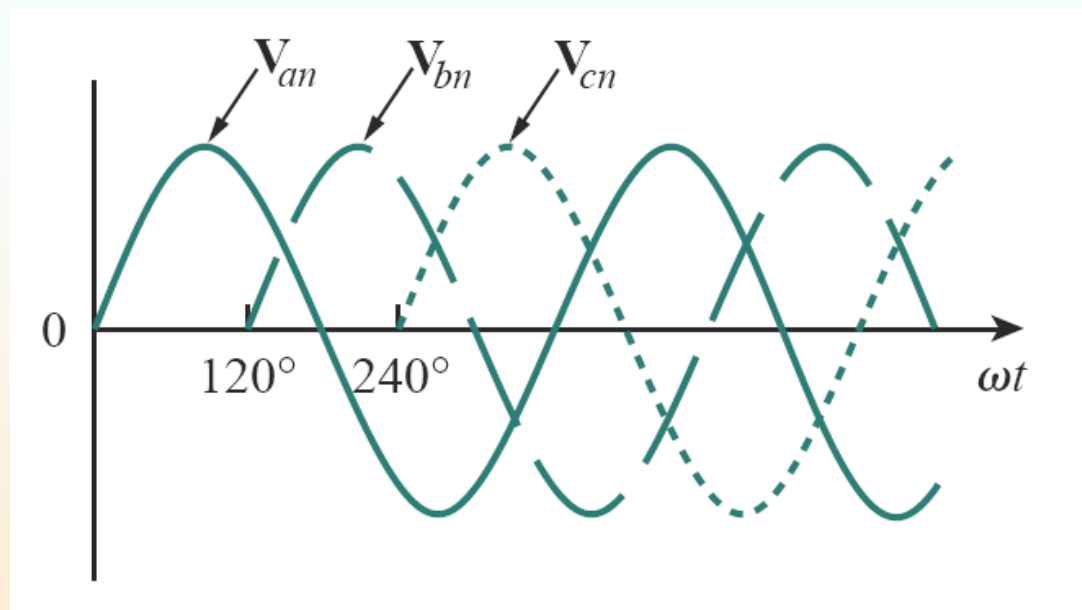
Review of balanced three-phase systems

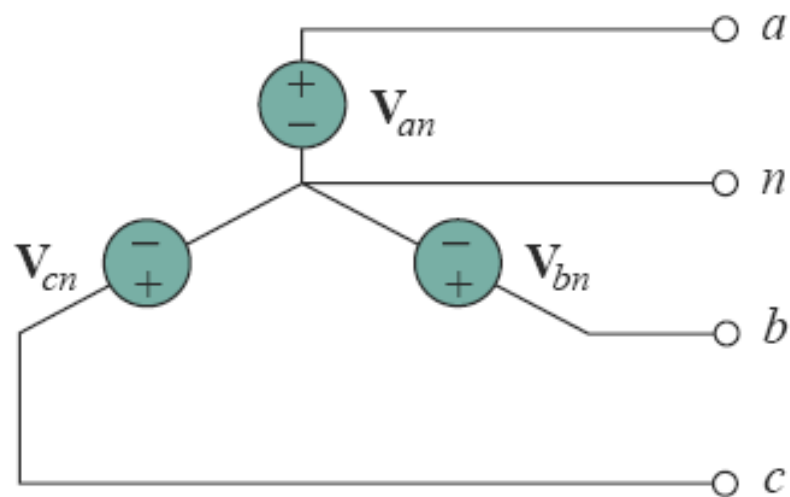
Lecture - 1

Three-phase systems

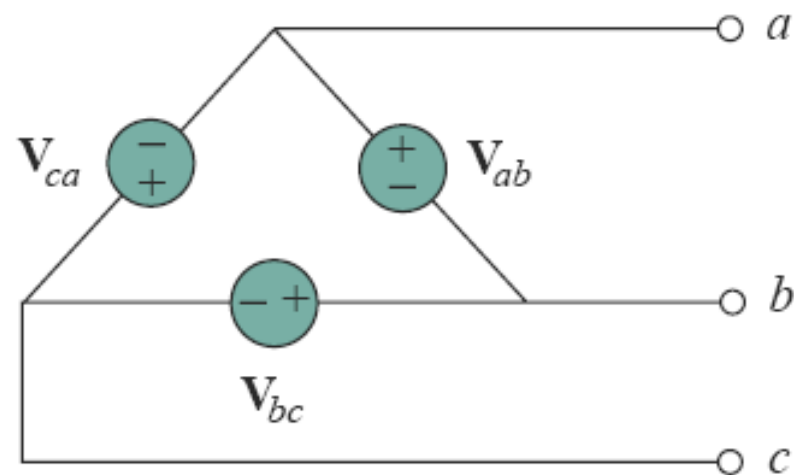
Advantages of three-phase systems over single-phase supplies include:

- (i) For a given amount of power transmitted through a system, the three-phase system requires conductors with a smaller cross sectional area. This means a saving of copper (or aluminium) and thus the original installation costs are less.
- (ii) The instantaneous power in a three-phase system can be constant (not pulsating).
- (iii) It is also used to drive large motors and other heavy loads.



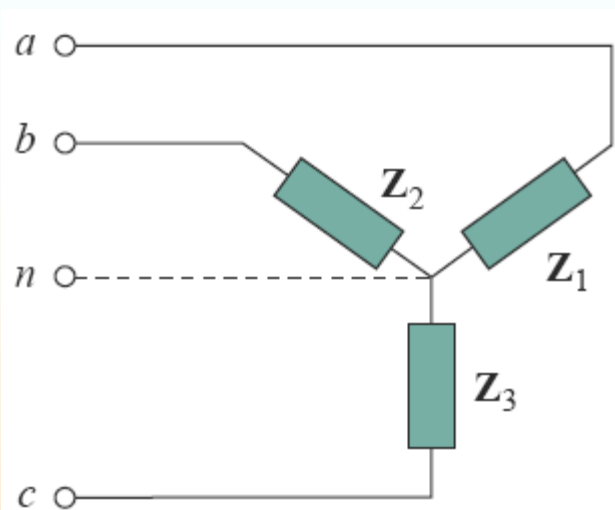


(a)

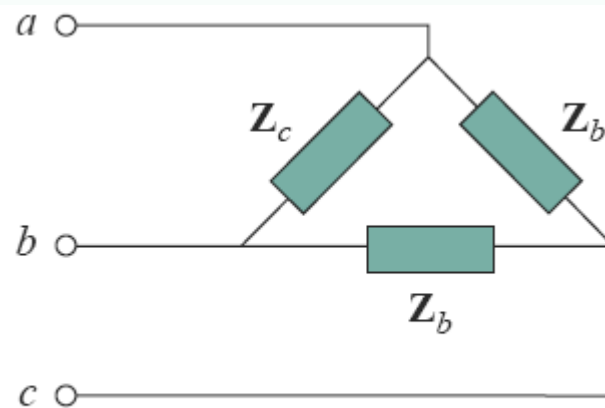


(b)

Three phase source

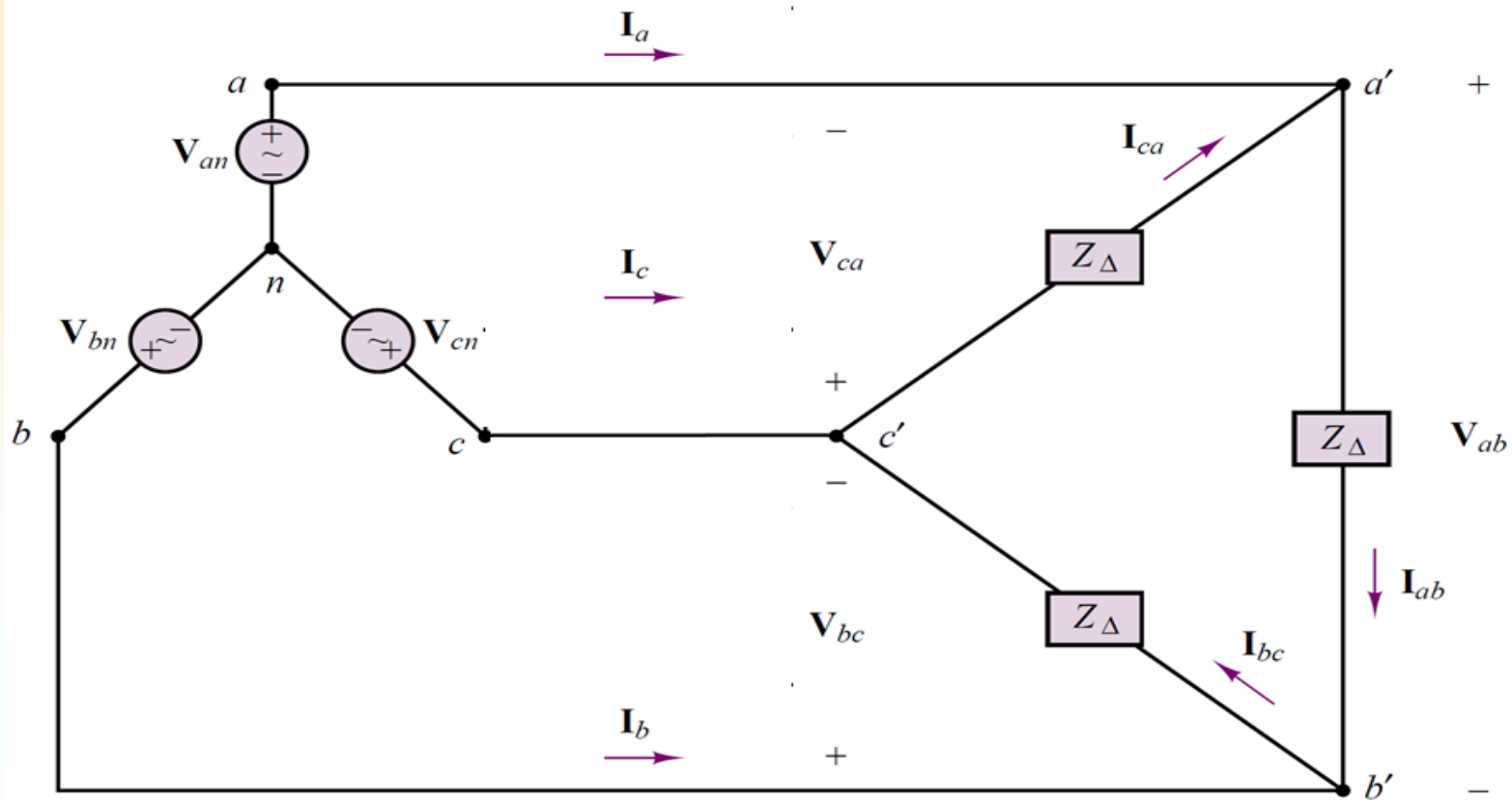


(a)



(b)

Three phase load



Source:

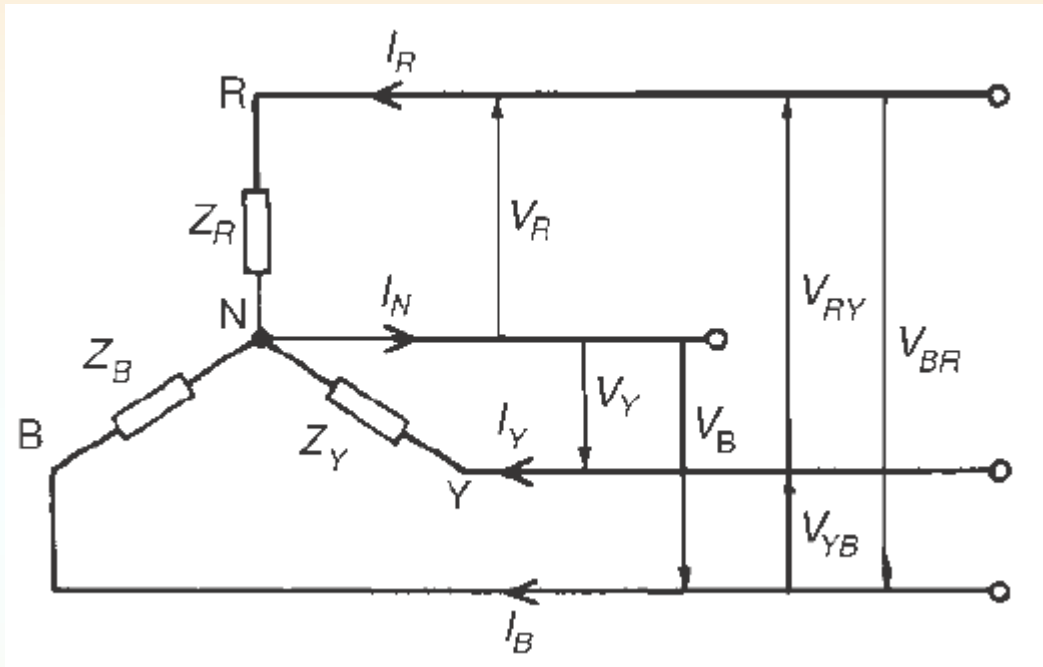
Phase current (I_a , I_b , I_c),
 Phase voltage (V_{an} , V_{bn} , V_{cn}),

Load:

Phase current (I_{ab} , I_{bc} , I_{ca}),
 Phase voltage ($V_{a'b'}$, $V_{b'c'}$, $V_{c'a'}$),

line current (I_a , I_b , I_c)
 line voltage (V_{ab} , V_{bc} , V_{ca})

Star connection



$$V_R = V_P \angle 0^\circ$$

$$V_Y = V_P \angle -120^\circ$$

$$V_B = V_P \angle -240^\circ$$

- (i) The three line conductors are connected to a load and the outlets from the loads are joined together at N to form the **neutral point** or the **star point**.
- (ii) The voltages, V_R , V_Y and V_B are called **phase voltages** or line to neutral voltages. Phase voltages are generally denoted by V_p .
- (iii) The voltages, V_{RY} , V_{YB} and V_{BR} are called **line voltages**.

(iv) The phase currents (generally denoted by I_p) are equal to their respective line currents I_R , I_Y and I_B , i.e. for a star connection:

$$I_L = I_p$$

(v) For a balanced system: $I_R = I_Y = I_B$, $V_R = V_Y = V_B$

$$V_{RY} = V_{YB} = V_{BR}, \quad Z_R = Z_Y = Z_B$$

and the current in the neutral conductor, $I_N = 0$

When a star connected system is balanced, then the neutral conductor is unnecessary and is often omitted.

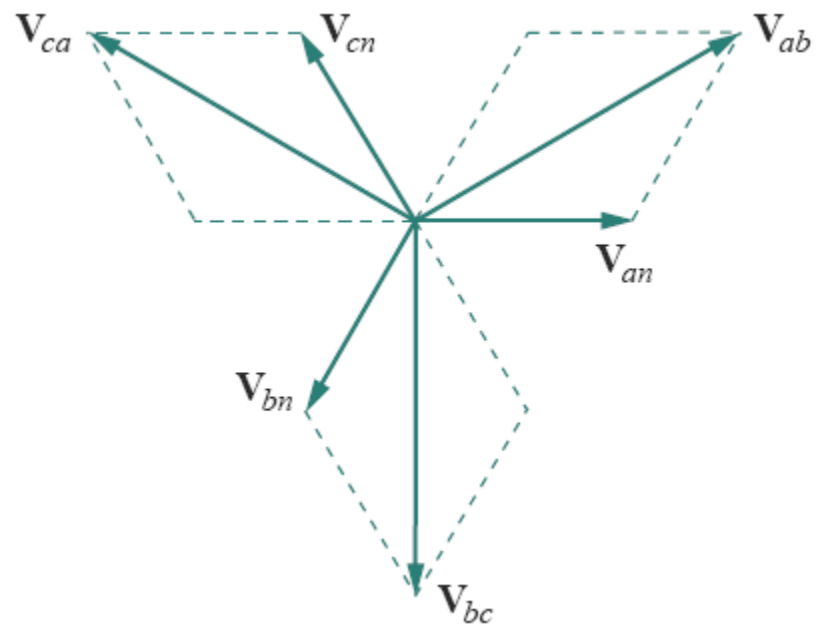
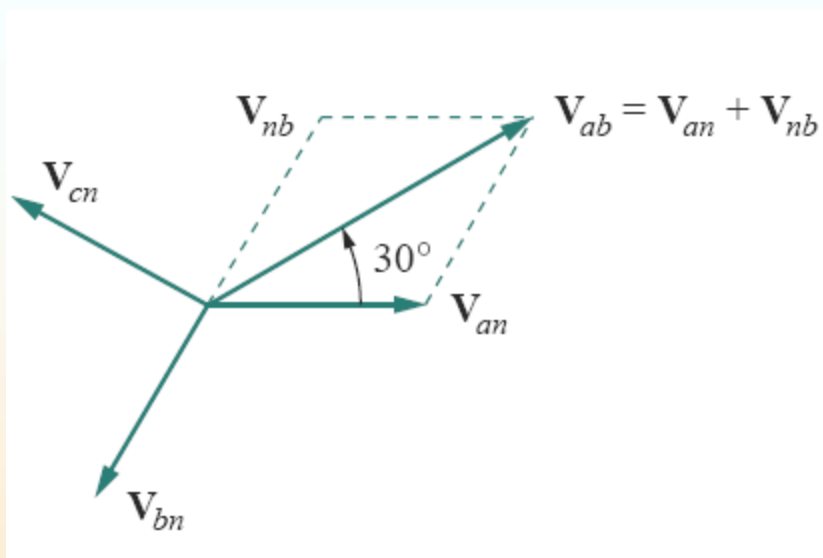
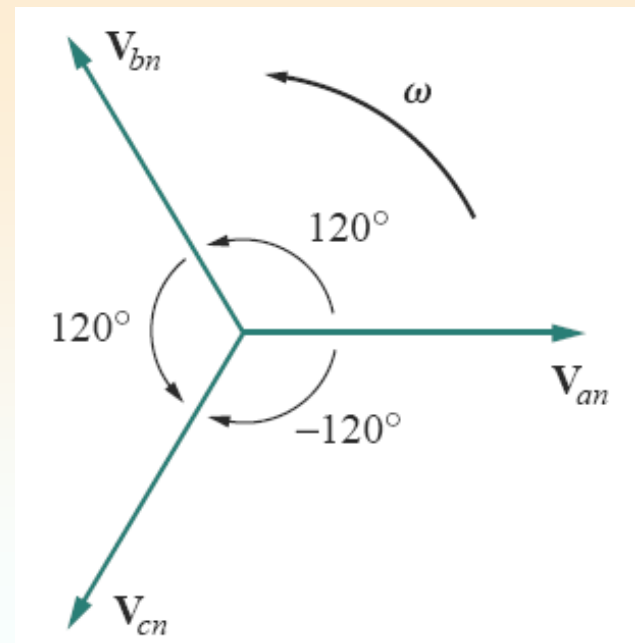
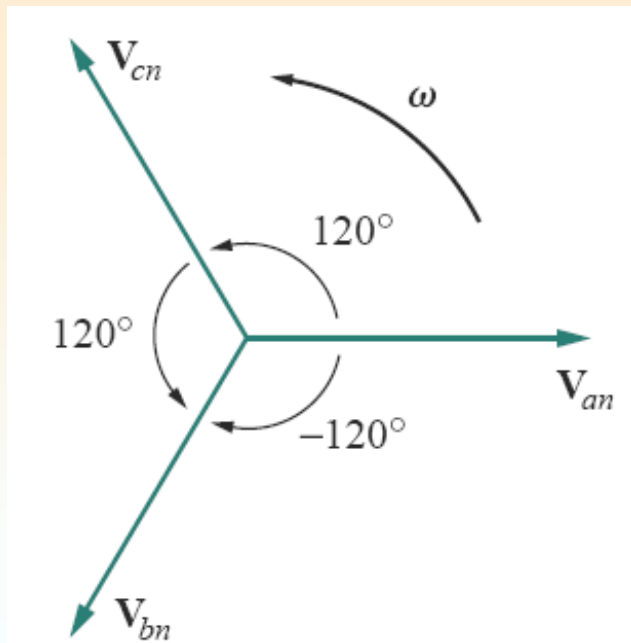
(vi) The line voltage, V_{RY} ,

$\mathbf{V}_{RY} = \mathbf{V}_R - \mathbf{V}_Y$ (V_Y is negative since it is in the opposite direction to V_{RY}).

$$= V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$= V_p \left(1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right) = \sqrt{3} V_p \angle 30^\circ$$

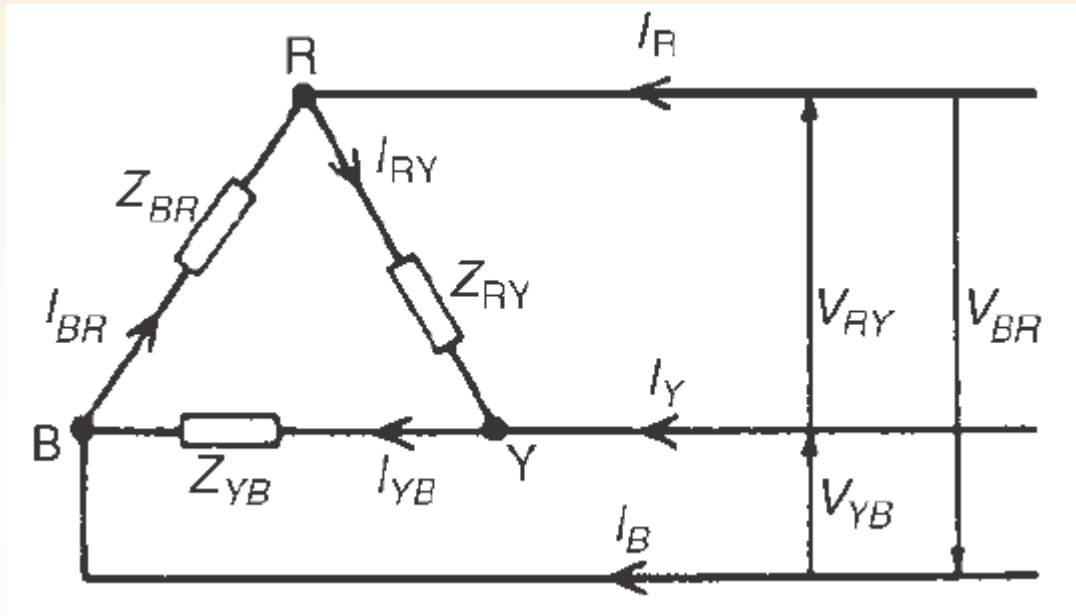
$$V_L = \sqrt{3} V_p$$



Phasor diagram for star connection

Review of balanced three-phase systems

Delta connection



$$I_{RY} = I_P \angle 0^\circ$$

$$I_{YB} = I_P \angle -120^\circ$$

$$I_{BR} = I_P \angle -240^\circ$$

(i) It can be seen that the line voltages V_{RY} , V_{YB} and V_{BR} are the respective phase voltages, i.e. for a delta connection:

$$V_L = V_p$$

(i) $I_R = I_{RY} - I_{BR} = I_{RY} + (-I_{BR})$

$$= I_P \angle 0^\circ - I_P \angle -240^\circ = \sqrt{3} I_P \angle -30^\circ$$

$$I_L = \sqrt{3} I_p$$

POWER IN A BALANCED SYSTEM

For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos \omega t,$$

$$v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ)$$

V_p has been defined as the rms value of the phase voltage.

If $\underline{Z}_Y = Z \angle \theta$, the phase currents lag behind their corresponding phase voltages by θ . Thus,

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta),$$

$$i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ)$$

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c$$

$$\begin{aligned} = 2V_p I_p [& \cos \omega t \cos(\omega t - \theta) \\ & + \cos(\omega t - 120^\circ) \cos(\omega t - \theta - 120^\circ) \\ & + \cos(\omega t + 120^\circ) \cos(\omega t - \theta + 120^\circ)] \end{aligned}$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$\begin{aligned} p &= V_p I_p [3 \cos \theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ &\quad + \cos(2\omega t - \theta + 240^\circ)] \\ &= V_p I_p [3 \cos \theta + \cos \alpha + \cos \alpha \cos 240^\circ + \sin \alpha \sin 240^\circ \\ &\quad + \cos \alpha \cos 240^\circ - \sin \alpha \sin 240^\circ] \\ &\quad \text{where } \alpha = 2\omega t - \theta \\ &= V_p I_p \left[3 \cos \theta + \cos \alpha + 2 \left(-\frac{1}{2} \right) \cos \alpha \right] = 3 V_p I_p \cos \theta \end{aligned}$$

The average power per phase P_p for either the Δ -connected load or the Y-connected load is $p/3$, or

$$P_p = V_p I_p \cos \theta$$

and the reactive power per phase is

$$Q_p = V_p I_p \sin \theta$$

The apparent power per phase is

$$S_p = V_p I_p$$

The complex power per phase is

$$\mathbf{S}_p = P_p + jQ_p = \mathbf{V}_p \mathbf{I}_p^*$$

The total average power is the sum of the average powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos \theta = \sqrt{3}V_L I_L \cos \theta$$

Similarly, the total reactive power is

$$Q = 3V_p I_p \sin \theta = 3Q_p = \sqrt{3}V_L I_L \sin \theta$$

and the total complex power is

$$S = 3S_p = 3\mathbf{V}_p \mathbf{I}_p^* = 3I_p^2 \mathbf{Z}_p = \frac{3V_p^2}{\mathbf{Z}_p^*}$$

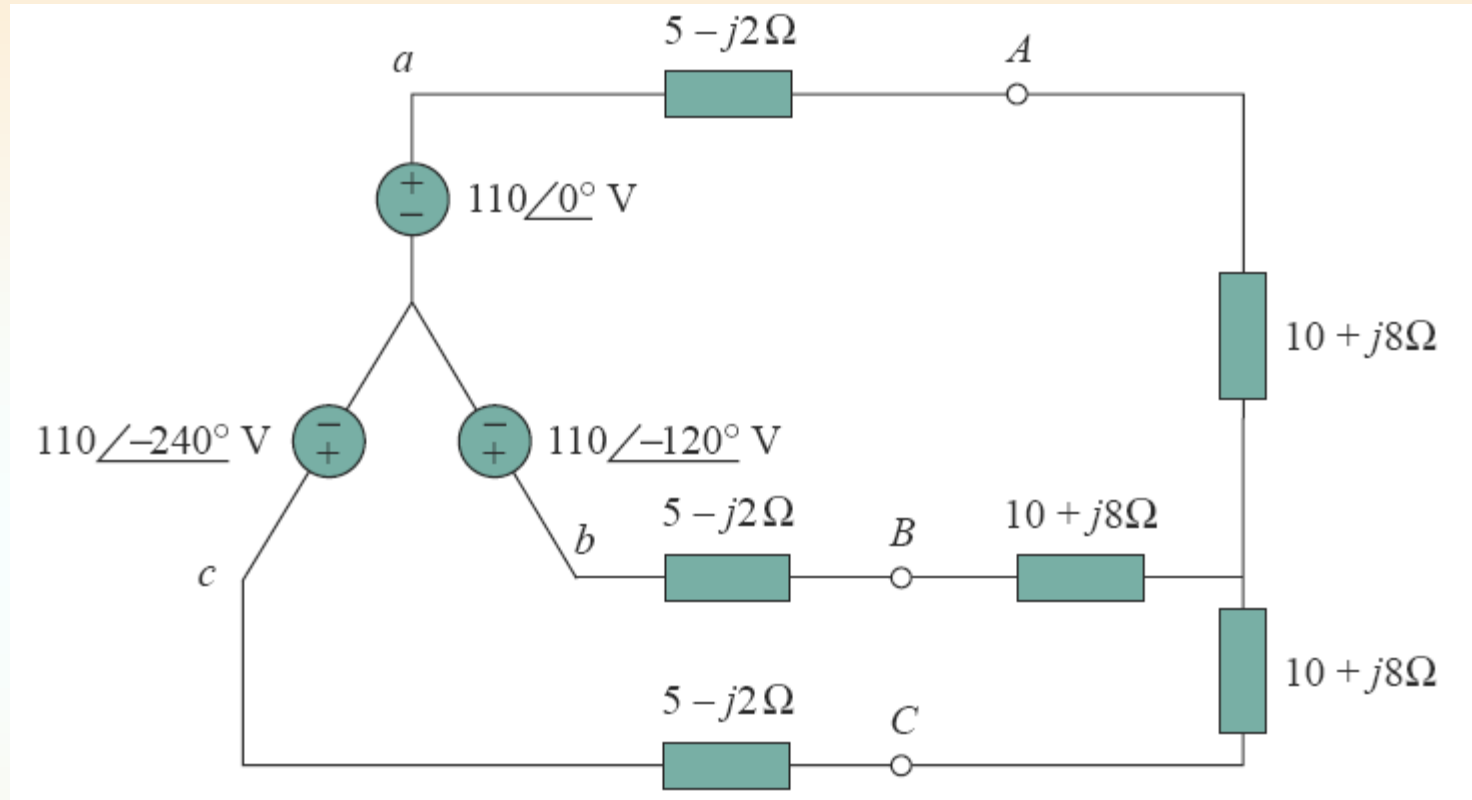
or

$$S = P + jQ = \sqrt{3}V_L I_L \angle \theta$$

Review of balanced three-phase systems
Numericals
Lecture - 2

P1.

Calculate the line currents in the three-wire Y-Y system of Fig.



$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8^\circ.$

$$\mathbf{I}_a = \frac{110 \angle 0^\circ}{16.155 \angle 21.8^\circ} = 6.81 \angle -21.8^\circ \text{ A}$$

Since the source voltages in Fig. are in positive sequence and the line currents are also in positive sequence,

$$\mathbf{I}_b = \mathbf{I}_a \angle -120^\circ = 6.81 \angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a \angle -240^\circ = 6.81 \angle -261.8^\circ \text{ A} = 6.81 \angle 98.2^\circ \text{ A}$$

P2.

A balanced *abc*-sequence Y-connected source with $\mathbf{V}_{an} = 100 \angle 10^\circ \text{ V}$ is connected to a Δ -connected balanced load $(8 + j4) \Omega$ per phase. Calculate the phase and line currents.

If the phase voltage $V_{an} = 100 \angle 10^\circ$, then the line voltage is

$$V_{ab} = V_{an} \sqrt{3} \angle 30^\circ = 100 \sqrt{3} \angle 10^\circ + 30^\circ = V_{AB}$$

The phase currents are

$$I_{AB} = \frac{V_{AB}}{Z_{\Delta}} = \frac{173.2 \angle 40^\circ}{8.944 \angle 26.57^\circ} = 19.36 \angle 13.43^\circ \text{ A}$$

$$I_{BC} = I_{AB} \angle -120^\circ = 19.36 \angle -106.57^\circ \text{ A}$$

$$I_{CA} = I_{AB} \angle +120^\circ = 19.36 \angle 133.43^\circ \text{ A}$$

The line currents are

$$\begin{aligned} I_a &= I_{AB} \sqrt{3} \angle -30^\circ = \sqrt{3}(19.36) \angle 13.43^\circ - 30^\circ \\ &= 33.53 \angle -16.57^\circ \text{ A} \end{aligned}$$

$$I_b = I_a \angle -120^\circ = 33.53 \angle -136.57^\circ \text{ A}$$

$$I_c = I_a \angle +120^\circ = 33.53 \angle 103.43^\circ \text{ A}$$

P3.

A positive-sequence, balanced Δ -connected source supplies a balanced Δ -connected load. If the impedance per phase of the load is $18 + j12 \Omega$ and $\mathbf{I}_a = 22.5 \angle 35^\circ \text{ A}$, find $\mathbf{I}_{AB}$ and $\mathbf{V}_{AB}$.

Answer: $13 \angle 65^\circ \text{ A}$, $281.2 \angle 98.69^\circ \text{ V}$.

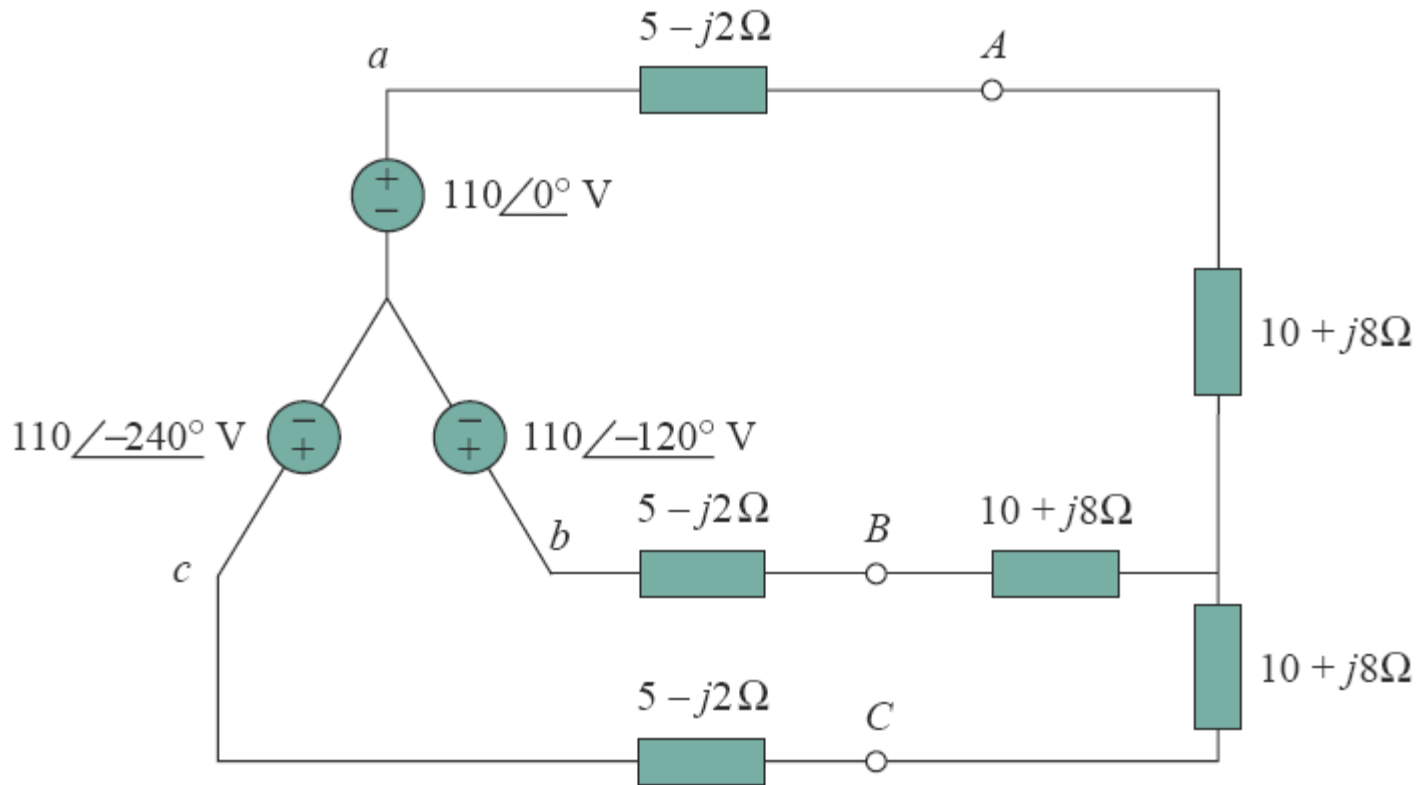
P4.

In a balanced Δ -Y circuit, $\mathbf{V}_{ab} = 240 \angle 15^\circ$ and $\mathbf{Z}_Y = (12 + j15) \Omega$. Calculate the line currents.

Answer: $7.21 \angle -66.34^\circ$, $7.21 \angle -186.34^\circ$, $7.21 \angle 53.66^\circ \text{ A}$.

P5.

Determine the total average power, reactive power, and complex power at the source and at the load.



$$\mathbf{V}_p = 110\angle 0^\circ \text{ V} \quad \text{and} \quad \mathbf{I}_p = 6.81\angle -21.8^\circ \text{ A}$$

At the load, the complex power absorbed is

$$\mathbf{S}_L = 3|\mathbf{I}_p|^2 \mathbf{Z}_p$$

where $\mathbf{Z}_p = 10 + j8 = 12.81 \angle 38.66^\circ$ and $\mathbf{I}_p = \mathbf{I}_a = 6.81 \angle -21.8^\circ$.
Hence

$$\begin{aligned}\mathbf{S}_L &= 3(6.81)^2 12.81 \angle 38.66^\circ = 1782 \angle 38.66^\circ \\ &= (1392 + j1113) \text{ VA}\end{aligned}$$

P6.

Three identical coils, each of resistance $15\ \Omega$ and inductance 42 mH are connected (a) in star and (b) in delta to a 415 V , 50 Hz , 3-phase supply. Determine the total power dissipated in each case.

(a) Star connection

$$\begin{aligned}\text{Inductive reactance } X_L &= 2\pi fL = 2\pi(50)(42 \times 10^{-3}) \\ &= 13.19\ \Omega\end{aligned}$$

$$\begin{aligned}\text{Phase impedance } Z_p &= \sqrt{(R^2 + X_L^2)} = \sqrt{(10^2 + 13.19^2)} \\ &= 16.55\ \Omega\end{aligned}$$

Line voltage $V_L = 415\text{ V}$ and

$$\text{phase voltage, } V_p = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 240\text{ V}$$

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{240}{16.55} = 14.50\text{ A}$$

$$\text{Line current, } I_L = I_p = 14.50\text{ A}$$

$$\text{Power factor} = \cos \phi = \frac{R_p}{Z_p} = \frac{10}{16.55} = 0.6042\text{ lagging}$$

$$\begin{aligned}\text{Power dissipated, } P &= \sqrt{3}V_L I_L \cos \phi = \sqrt{3}(415)(14.50)(0.6042) \\ &= \mathbf{6.3\text{ kW}}\end{aligned}$$

$$(\text{Alternatively, } P = 3I_p^2 R_p = 3(14.50)^2(10) = \mathbf{6.3\text{ kW}})$$

(b) **Delta connection**

$$V_L = V_p = 415 \text{ V}, Z_p = 16.55 \text{ } \Omega,$$

$\cos \phi = 0.6042$ lagging (from above).

$$\text{Phase current, } I_p = \frac{V_p}{Z_p} = \frac{415}{16.55} = 25.08 \text{ A}$$

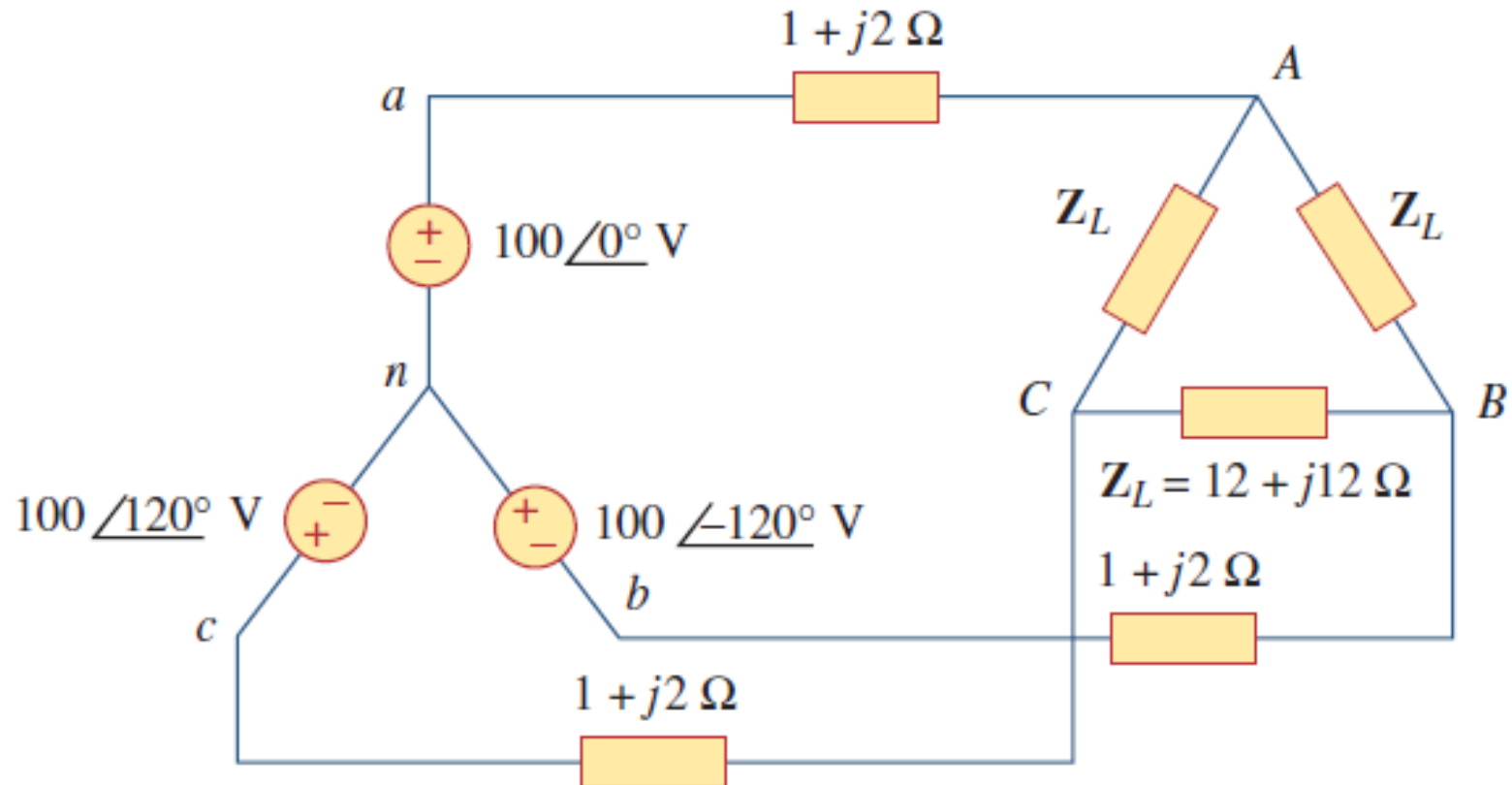
$$\text{Line current, } I_L = \sqrt{3}I_p = \sqrt{3}(25.08) = 43.44 \text{ A}$$

$$\begin{aligned} \text{Power dissipated, } P &= \sqrt{3}V_L I_L \cos \phi = \sqrt{3}(415)(43.44)(0.6042) \\ &= \mathbf{18.87 \text{ kW}} \end{aligned}$$

$$(\text{Alternatively, } P = 3I_p^2 R_p = 3(25.08)^2(10) = \mathbf{18.87 \text{ kW}})$$

P7

Calculate the line currents and complex power consumed by the load.



Unbalanced three-phase systems

Star connected

Lecture – 3

Lecture - 4

Unbalanced Three-phase systems

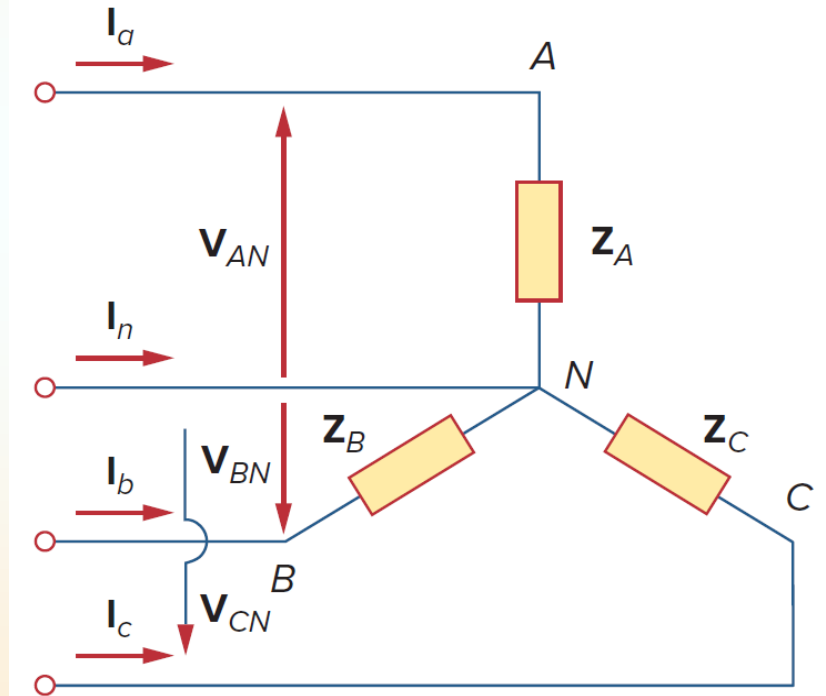
Unbalanced three-phase systems are solved by direct application of **mesh and nodal analysis**.

- a) **4-wire star connected** - apply **Ohm's law**
- b) **3-wire star connected** – apply **mesh/nodal analysis**
- c) **Delta connected**
 - i) apply **Ohm's law** – if line impedance is not given
if line voltage is given
 - ii) apply **mesh/nodal analysis** – if line impedance is given
- if phase voltage is given

Figure shows an example of an unbalanced three-phase system that consists of balanced source voltages (not shown in the figure) and an **unbalanced four-wire Y-connected load** (shown in the figure). Since the load is unbalanced, Z_A , Z_B , and Z_C are not equal. The line currents are determined by Ohm's law as

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A}, \quad \mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B}, \quad \mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C}$$

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c)$$



P8.

The unbalanced Y-load of Fig. has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $\mathbf{Z}_A = 15 \, \Omega$, $\mathbf{Z}_B = 10 + j5 \, \Omega$, $\mathbf{Z}_C = 6 - j8 \, \Omega$.

$$\begin{aligned}\mathbf{I}_a &= \frac{100\angle 0^\circ}{15} = 6.67\angle 0^\circ \text{ A} \\ \mathbf{I}_b &= \frac{100\angle 120^\circ}{10 + j5} = \frac{100\angle 120^\circ}{11.18\angle 26.56^\circ} = 8.94\angle 93.44^\circ \text{ A} \\ \mathbf{I}_c &= \frac{100\angle -120^\circ}{6 - j8} = \frac{100\angle -120^\circ}{10\angle -53.13^\circ} = 10\angle -66.87^\circ \text{ A}\end{aligned}$$

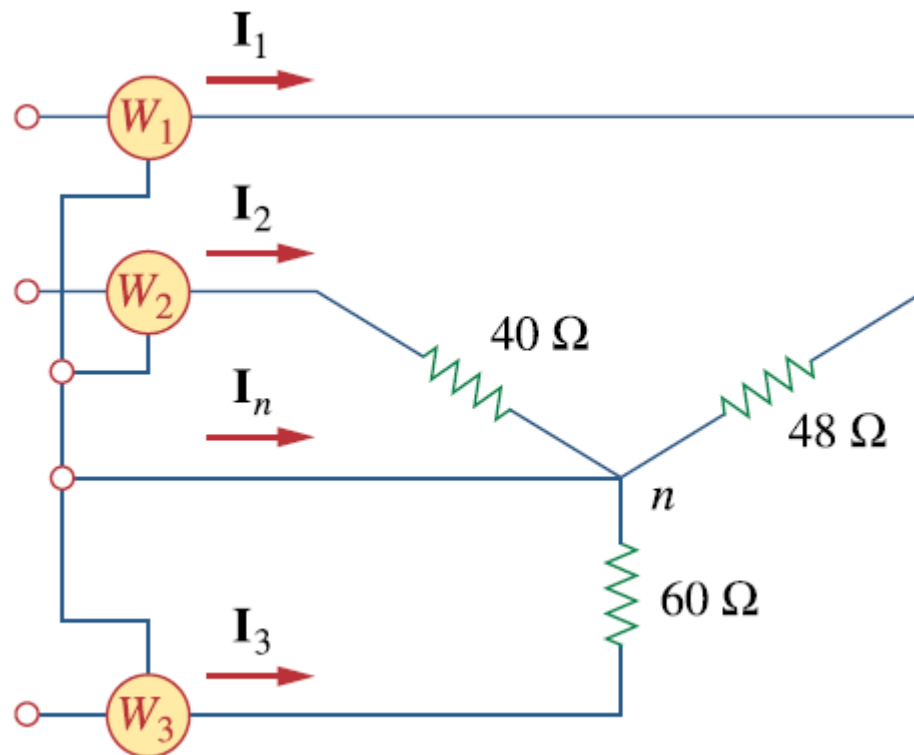
$$\begin{aligned}\mathbf{I}_n &= -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2) \\ &= -10.06 + j0.28 = 10.06\angle 178.4^\circ \text{ A}\end{aligned}$$

P9.

A three-phase, four-wire system operating with a 208-V line voltage is shown in Fig.

The source voltages are balanced. The power absorbed by the resistive wye-connected load is measured by the three-wattmeter method. Calculate:

- (a) the voltage to neutral
- (b) the currents I_1 , I_2 , I_3 , and I_n
- (c) the readings of the wattmeters
- (d) the total power absorbed by the load



$$(a) \quad V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = \underline{\underline{120 \text{ V}}}$$

(b) Because the load is unbalanced, we have an unbalanced three-phase system. Assuming an abc sequence,

$$\mathbf{I}_1 = \frac{120 \angle 0^\circ}{48} = 2.5 \angle 0^\circ \text{ A}$$

$$\mathbf{I}_2 = \frac{120 \angle -120^\circ}{40} = 3 \angle -120^\circ \text{ A}$$

$$\mathbf{I}_3 = \frac{120 \angle 120^\circ}{60} = 2 \angle 120^\circ \text{ A}$$

$$-\mathbf{I}_N = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 2.5 + (3) \left(-0.5 - j\frac{\sqrt{3}}{2} \right) + (2) \left(-0.5 + j\frac{\sqrt{3}}{2} \right)$$

$$\mathbf{I}_N = j\frac{\sqrt{3}}{2} = j0.866 = 0.866 \angle 90^\circ \text{ A}$$

Hence,

$$\mathbf{I}_1 = \underline{\underline{2.5 \text{ A}}}, \quad \mathbf{I}_2 = \underline{\underline{3 \text{ A}}}, \quad \mathbf{I}_3 = \underline{\underline{2 \text{ A}}}, \quad \mathbf{I}_N = \underline{\underline{0.866 \text{ A}}}$$

$$(c) \quad P_1 = I_1^2 R_1 = (2.5)^2 (48) = \underline{\underline{300 \text{ W}}}$$

$$P_2 = I_2^2 R_2 = (3)^2 (40) = \underline{\underline{360 \text{ W}}}$$

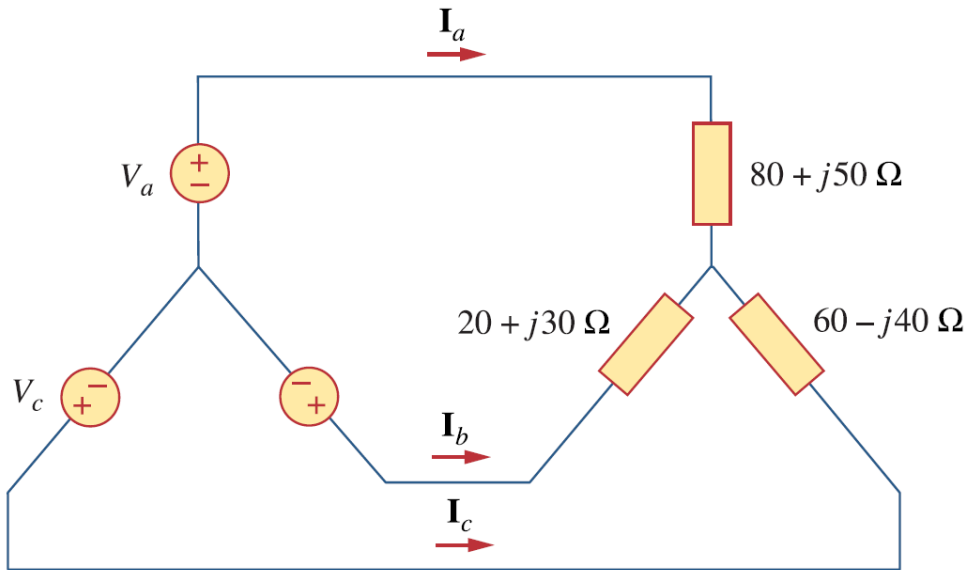
$$P_3 = I_3^2 R_3 = (2)^2 (60) = \underline{\underline{240 \text{ W}}}$$

$$(d) \quad P_T = P_1 + P_2 + P_3 = \underline{\underline{900 \text{ W}}}$$

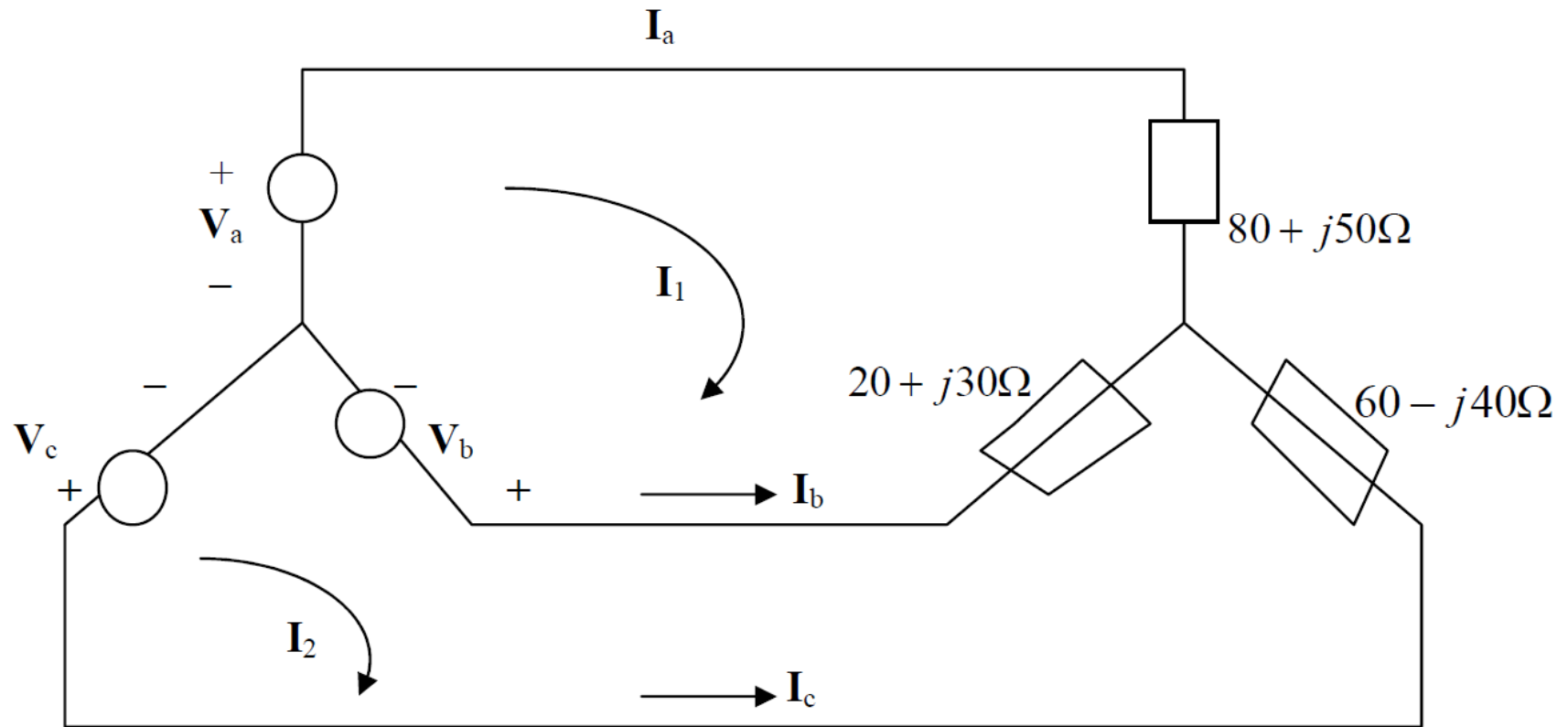
P10.

Determine the line currents for the three-phase circuit of Fig.
 $\mathbf{V}_b = 110 \angle -120^\circ$, $\mathbf{V}_c = 110 \angle 120^\circ$ V.

Let $\mathbf{V}_a = 110 \angle 0^\circ$,



We apply mesh analysis to the circuit shown below.



$$(100 + j80)I_1 - (20 + j30)I_2 = V_a - V_b = 165 + j95.263 \quad (1)$$

$$-(20 + j30)I_1 + (80 - j10)I_2 = V_b - V_c = -j190.53 \quad (2)$$

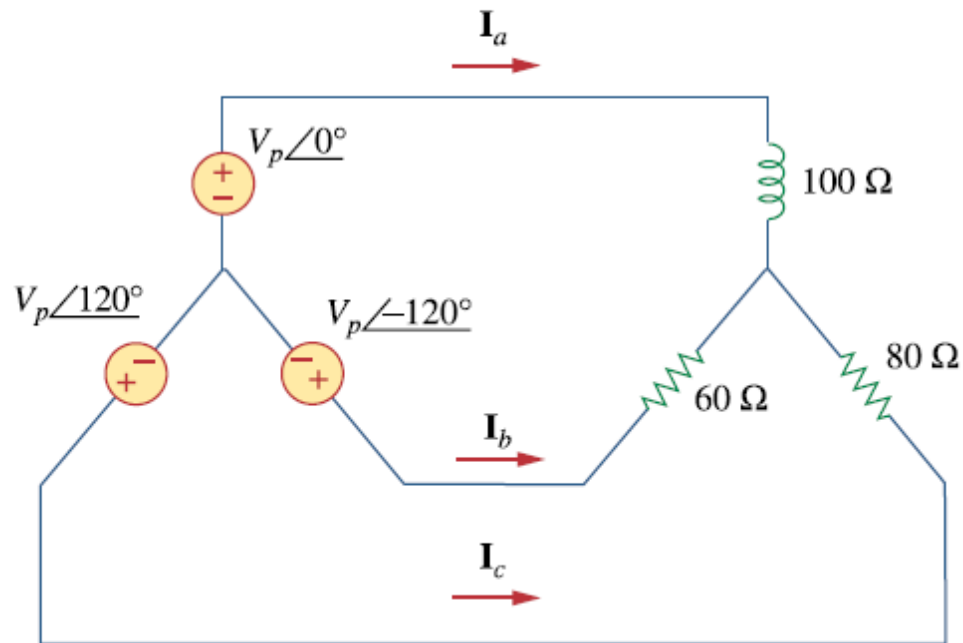
Solving (1) and (2) gives $I_1 = 1.8616 - j0.6084$, $I_2 = 0.9088 - j1.722$.

$$I_a = I_1 = \underline{1.9585 \angle -18.1^\circ \text{ A}}, \quad I_b = I_2 - I_1 = -0.528 - j1.1136 = \underline{1.4656 \angle -130.55^\circ \text{ A}}$$

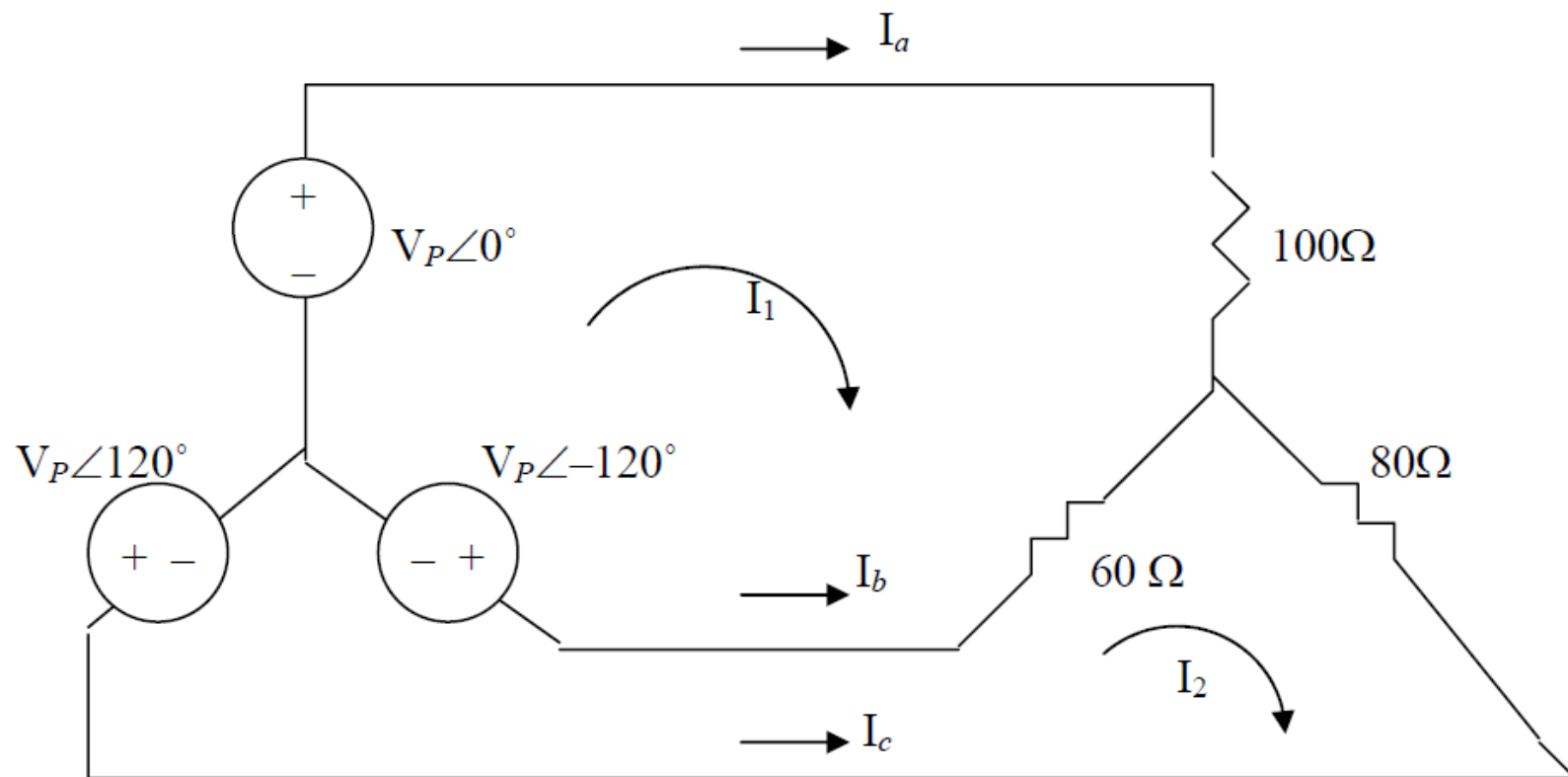
$$I_c = -I_2 = \underline{1.947 \angle 117.8^\circ \text{ A}}$$

P11.

In the Y-Y system shown in Fig , loads connected to the source are unbalanced. (a) Calculate $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$. (b) Find the total power delivered to the load. Take $V_p = 240 \text{ V rms}$.



Applying mesh analysis as shown below, we get.



$$240 < -120^\circ - 240 < 0^\circ + I_1 \times 160 - 60I_2 = 0 \quad \longrightarrow \quad 160I_1 - 60I_2 = 360 + j207.84 \quad (1)$$

$$240 < 120^\circ - 240 < -120^\circ + 140I_2 - 60I_1 = 0 \quad \longrightarrow \quad 140I_2 - 60I_1 = -j415.7 \quad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} 160 & -60 \\ -60 & 140 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 360 + j207.84 \\ -j415.7 \end{bmatrix}$$

Using MATLAB, we get,

```
>> Z=[160,-60;-60,140]
Z =
    160    -60
    -60    140
>> V=[(360+207.8i);-415.7i]
V =
  1.0e+002 *
    3.6000 + 2.0780i
         0 - 4.1570i
>> I=inv(Z)*V
I =
    2.6809 + 0.2207i
    1.1489 - 2.8747i
```

$$I_1 = 2.681 + j0.2207 \text{ and } I_2 = 1.1489 - j2.875$$

$$I_a = I_1 = \underline{\underline{2.69 \angle 4.71^\circ \text{ A}}}$$

$$I_b = I_2 - I_1 = -1.5321 - j3.096 = \underline{\underline{3.454 \angle -116.33^\circ \text{ A}}}$$

$$I_c = -I_2 = \underline{\underline{3.096 \angle 111.78^\circ \text{ A}}}$$

$$S_a = |I_a|^2 Z_a = (2.69)^2 \times 100 = 723.61 \text{ W}$$

$$S_b = |I_b|^2 Z_b = (3.454)^2 \times 60 = 715.81 \text{ W}$$

$$S_c = |I_c|^2 Z_c = (3.0957)^2 \times 80 = 766.67 \text{ W}$$

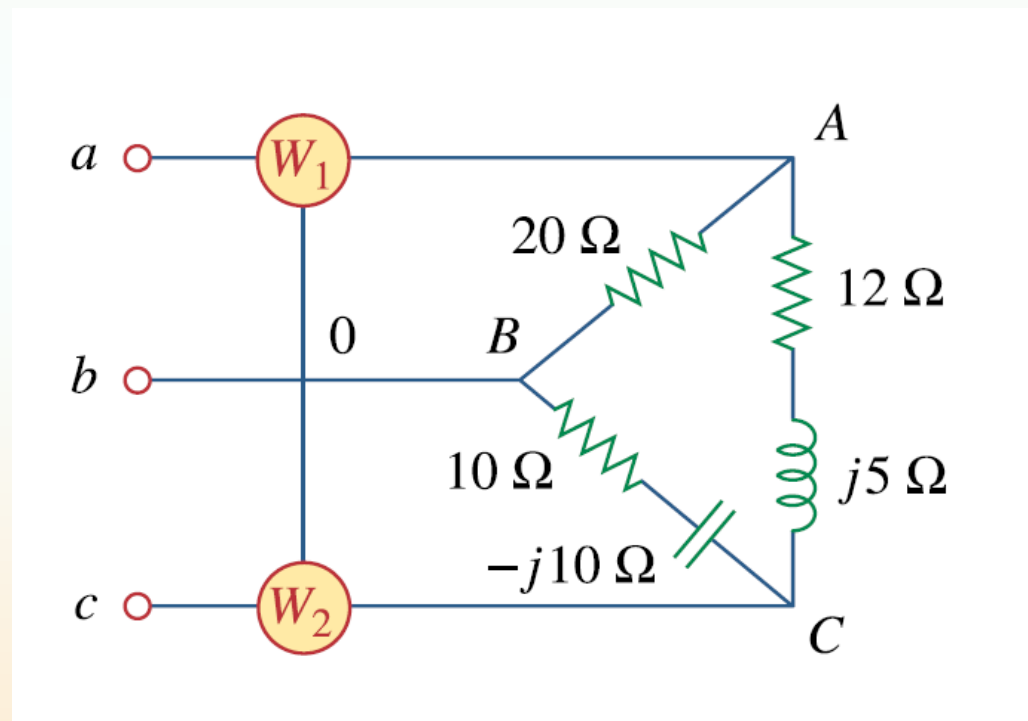
$$P = P_a + P_b + P_c = \underline{\underline{2.205 \text{ kW}}}$$

Unbalanced three-phase systems
Delta connected
Lecture - 5

P12.

In Fig., two wattmeters are properly connected to the unbalanced load supplied by a balanced source such that $\mathbf{V}_{ab} = 208 \angle 0^\circ \text{ V}$ with positive phase sequence.

- (a) Determine the reading of each wattmeter.
- (b) Calculate the total apparent power absorbed by the load.



(a) If $V_{ab} = 208\angle 0^\circ$, $V_{bc} = 208\angle -120^\circ$, $V_{ca} = 208\angle 120^\circ$,

$$I_{AB} = \frac{V_{ab}}{Z_{Ab}} = \frac{208\angle 0^\circ}{20} = 10.4\angle 0^\circ$$

$$I_{BC} = \frac{V_{bc}}{Z_{BC}} = \frac{208\angle -120^\circ}{10\sqrt{2}\angle -45^\circ} = 14.708\angle -75^\circ$$

$$I_{CA} = \frac{V_{ca}}{Z_{CA}} = \frac{208\angle 120^\circ}{13\angle 22.62^\circ} = 16\angle 97.38^\circ$$

$$I_{aA} = I_{AB} - I_{CA} = 10.4\angle 0^\circ - 16\angle 97.38^\circ$$

$$I_{aA} = 10.4 + 2.055 - j15.867$$

$$I_{aA} = 20.171\angle -51.87^\circ$$

$$I_{cC} = I_{CA} - I_{BC} = 16\angle 97.83^\circ - 14.708\angle -75^\circ$$

$$I_{cC} = 30.64\angle 101.03^\circ$$

$$P_1 = |\mathbf{V}_{ab}| |\mathbf{I}_{aA}| \cos(\theta_{V_{ab}} - \theta_{I_{aA}})$$

$$P_1 = (208)(20.171) \cos(0^\circ + 51.87^\circ) = \underline{\underline{2590 \text{ W}}}$$

$$P_2 = |\mathbf{V}_{cb}| |\mathbf{I}_{cC}| \cos(\theta_{V_{cb}} - \theta_{I_{cC}})$$

$$\text{But } \mathbf{V}_{cb} = -\mathbf{V}_{bc} = 208 \angle 60^\circ$$

$$P_2 = (208)(30.64) \cos(60^\circ - 101.03^\circ) = \underline{\underline{4808 \text{ W}}}$$

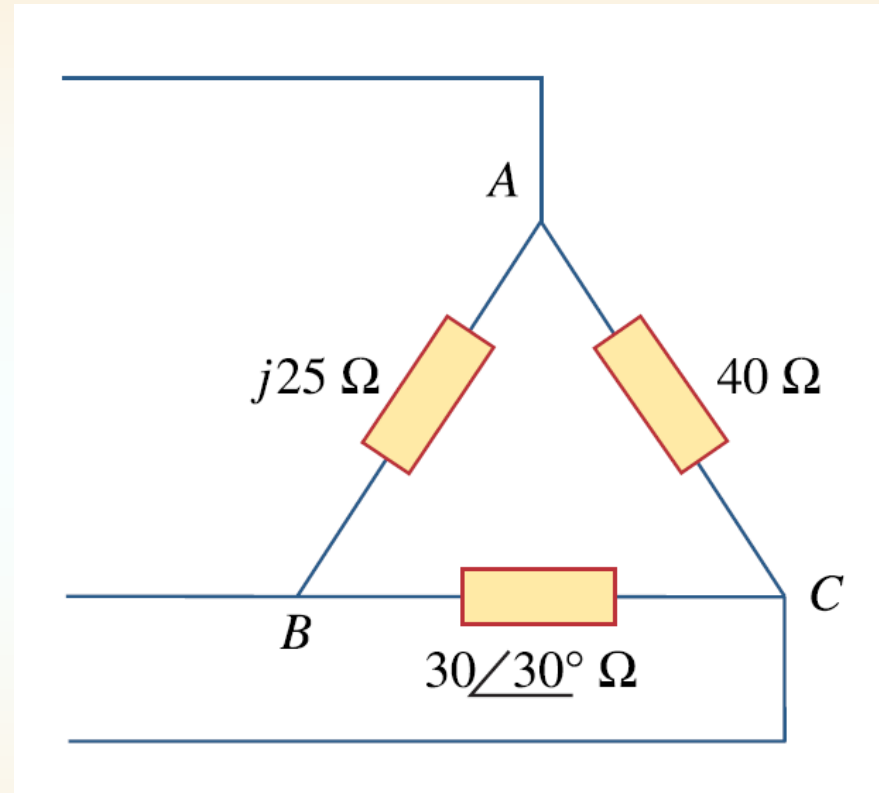
$$(b) \quad P_T = P_1 + P_2 = 7398.17 \text{ W}$$

$$Q_T = \sqrt{3} (P_2 - P_1) = 3840.25 \text{ VAR}$$

$$\mathbf{S}_T = P_T + jQ_T = 7398.17 + j3840.25 \text{ VA}$$

$$S_T = |\mathbf{S}_T| = \underline{\underline{8335 \text{ VA}}}$$

P13. A three-phase supply, with the line voltage 240 V rms positively phased, has an unbalanced delta-connected load as shown in Fig. Find the phase currents and the total complex power.



The phase currents are:

$$I_{AB} = 240/j25 = \underline{\underline{9.6\angle-90^\circ \text{ A}}}$$

$$I_{CA} = 240\angle120^\circ/40 = \underline{\underline{6\angle120^\circ \text{ A}}}$$

$$I_{BC} = 240\angle-120^\circ/30\angle30^\circ = \underline{\underline{8\angle-150^\circ \text{ A}}}$$

The complex power in each phase is:

$$S_{AB} = |I_{AB}|^2 Z_{AB} = (9.6)^2 j25 = j2304$$

$$S_{AC} = |I_{AC}|^2 Z_{AC} = (6)^2 40 \angle 0^\circ = 1440$$

$$S_{BC} = |I_{BC}|^2 Z_{BC} = (8)^2 30 \angle 30^\circ = 1662.77 + j960$$

The total complex power is,

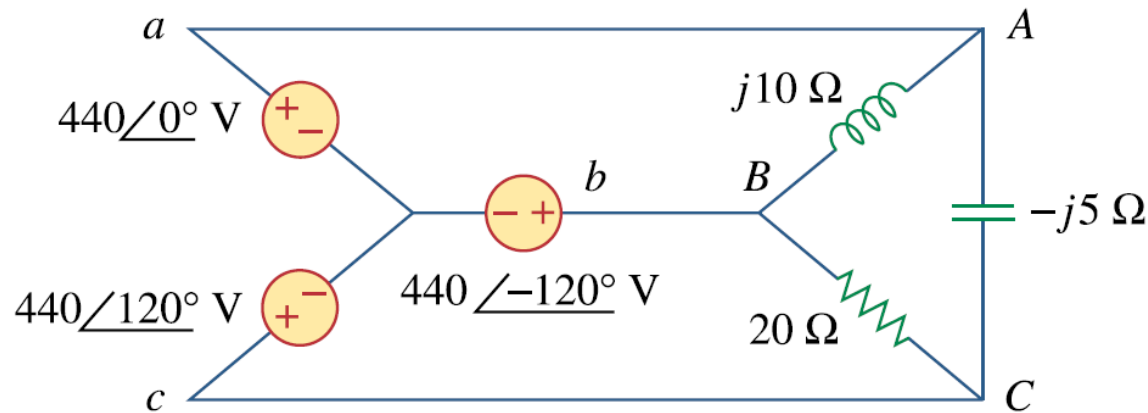
$$S = S_{AB} + S_{AC} + S_{BC} = \underline{\underline{3102.77 + j3264 \text{ VA}}} = \underline{\underline{3.103 + j3.264 \text{ kVA}}}$$

P14.

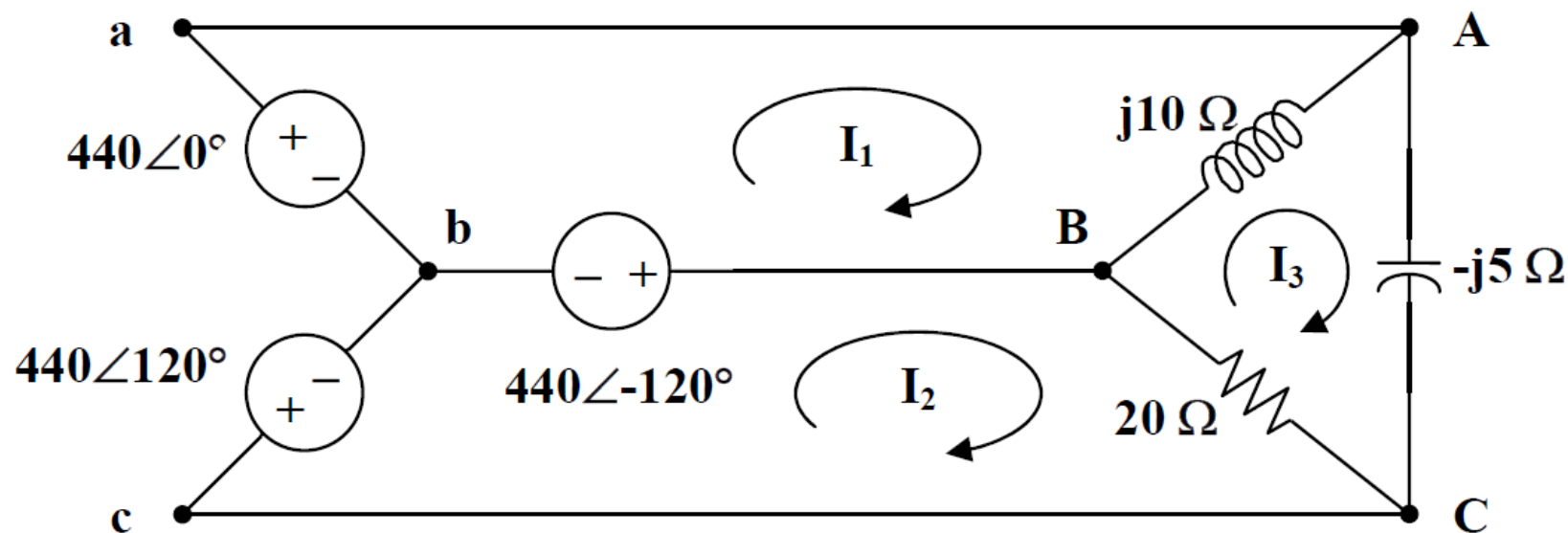
Refer to the unbalanced circuit of Fig.

Calculate:

- (a) the line currents
- (b) the real power absorbed by the load
- (c) the total complex power supplied by the source



(a) Consider the circuit below.



For mesh 1,

$$440 \angle -120^\circ - 440 \angle 0^\circ + j10(\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_1 - \mathbf{I}_3 = \frac{(440)(1.5 + j0.866)}{j10} = 76.21 \angle -60^\circ \quad (1)$$

For mesh 2,

$$440 \angle 120^\circ - 440 \angle -120^\circ + 20(\mathbf{I}_2 - \mathbf{I}_3) = 0$$

$$\mathbf{I}_3 - \mathbf{I}_2 = \frac{(440)(j1.732)}{20} = j38.1 \quad (2)$$

For mesh 3,

$$j10(\mathbf{I}_3 - \mathbf{I}_1) + 20(\mathbf{I}_3 - \mathbf{I}_2) - j5\mathbf{I}_3 = 0$$

Substituting (1) and (2) into the equation for mesh 3 gives,

$$\mathbf{I}_3 = \frac{(440)(-1.5 + j0.866)}{j5} = 152.42 \angle 60^\circ \quad (3)$$

From (1),

$$\mathbf{I}_1 = \mathbf{I}_3 + 76.21 \angle -60^\circ = 114.315 + j66 = 132 \angle 30^\circ$$

From (2),

$$\mathbf{I}_2 = \mathbf{I}_3 - j38.1 = 76.21 + j93.9 = 120.93 \angle 50.94^\circ$$

$$\mathbf{I}_a = \mathbf{I}_1 = \underline{\underline{132 \angle 30^\circ \text{ A}}}$$

$$\mathbf{I}_b = \mathbf{I}_2 - \mathbf{I}_1 = -38.105 + j27.9 = \underline{\underline{47.23 \angle 143.8^\circ \text{ A}}}$$

$$\mathbf{I}_c = -\mathbf{I}_2 = \underline{\underline{120.9 \angle 230.9^\circ \text{ A}}}$$

$$(b) \quad \mathbf{S}_{AB} = |\mathbf{I}_1 - \mathbf{I}_3|^2 (j10) = j58.08 \text{ kVA}$$

$$\mathbf{S}_{BC} = |\mathbf{I}_2 - \mathbf{I}_3|^2 (20) = 29.04 \text{ kVA}$$

$$\mathbf{S}_{CA} = |\mathbf{I}_3|^2 (-j5) = (152.42)^2 (-j5) = -j116.16 \text{ kVA}$$

$$\mathbf{S} = \mathbf{S}_{AB} + \mathbf{S}_{BC} + \mathbf{S}_{CA} = 29.04 - j58.08 \text{ kVA}$$

$$\text{Real power absorbed} = \underline{\underline{29.04 \text{ kW}}}$$

(c) Total complex supplied by the source is

$$\mathbf{S} = \underline{\underline{29.04 - j58.08 \text{ kVA}}}$$

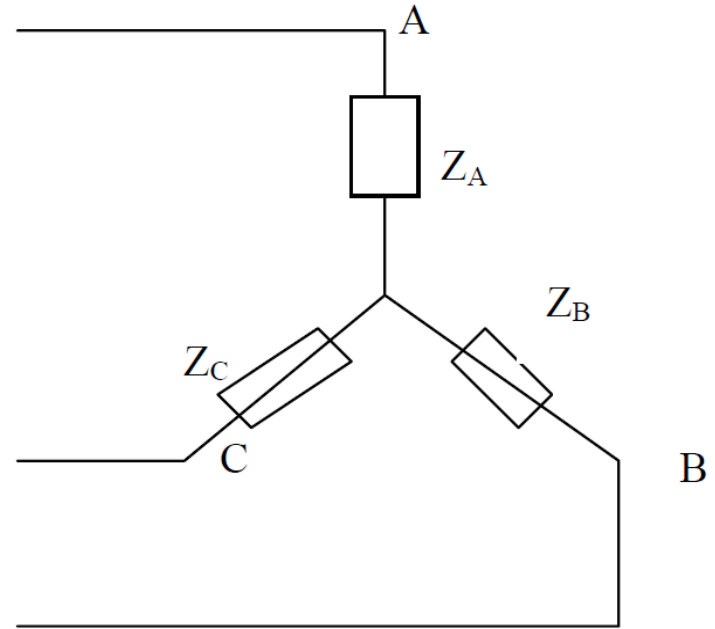
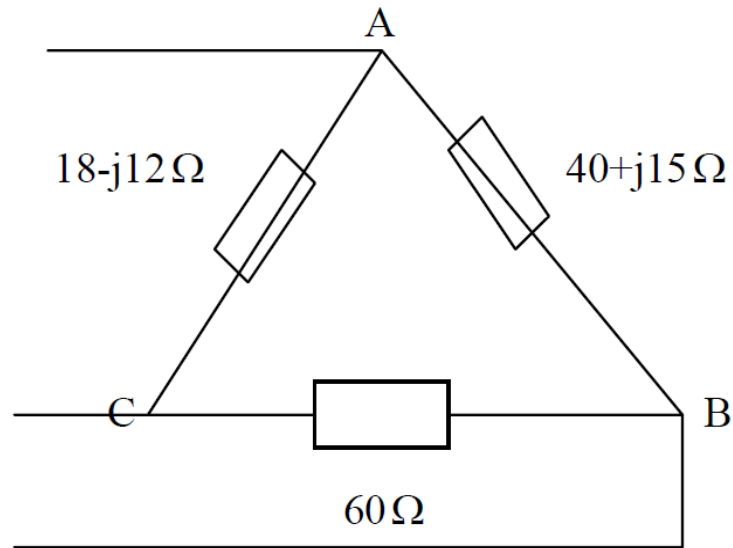
P15.

A balanced, positive-sequence wye-connected source has $\mathbf{V}_{an} = 240 \angle 0^\circ$ V rms and supplies an unbalanced delta-connected load via a transmission line with impedance $2 + j3 \Omega$ per phase.

- (a) Calculate the line currents if $\mathbf{Z}_{AB} = 40 + j15 \Omega$, $\mathbf{Z}_{BC} = 60 \Omega$, $\mathbf{Z}_{CA} = 18 - j12 \Omega$.
- (b) Find the complex power supplied by the source.

Unbalanced three-phase systems
Star/Delta conversion
Lecture - 6

(a) We first convert the delta load to its equivalent wye load, as shown below.

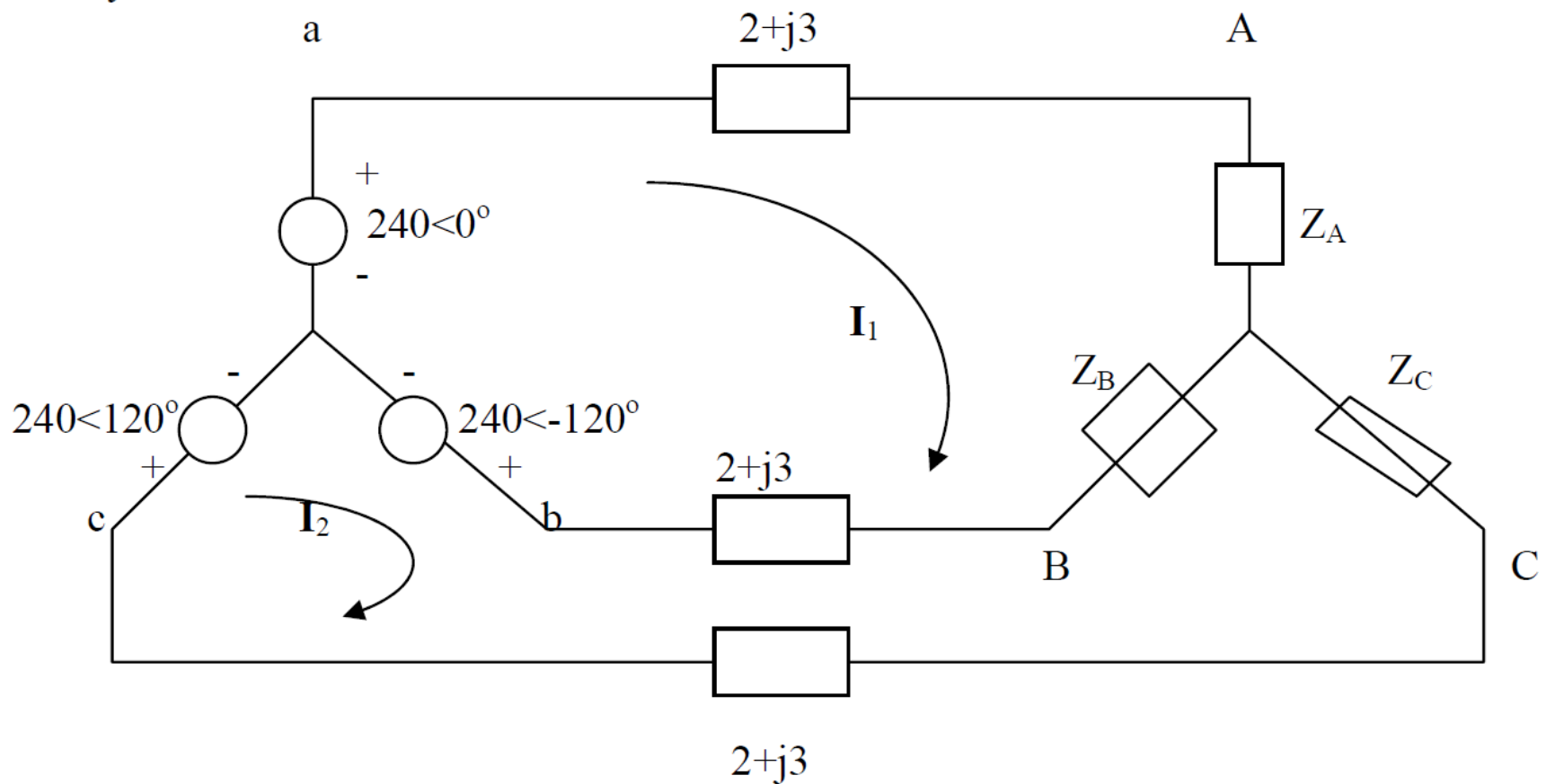


$$Z_A = \frac{(40 + j15)(18 - j12)}{118 + j3} = 7.577 - j1.923$$

$$Z_B = \frac{60(40 + j15)}{118 + j3} = 20.52 + j7.105$$

$$Z_C = \frac{60(18 - j12)}{118 + j3} = 8.992 - j6.3303$$

The system becomes that shown below.



We apply KVL to the loops. For mesh 1,

$$-240 + 240\angle -120^\circ + I_1(2Z_l + Z_A + Z_B) - I_2(Z_B + Z_l) = 0$$

or

$$(32.097 + j11.13)I_1 - (22.52 + j10.105)I_2 = 360 + j207.85 \quad (1)$$

For mesh 2,

$$240\angle 120^\circ - 240\angle -120^\circ - I_1(Z_B + Z_l) + I_2(2Z_l + Z_B + Z_C) = 0$$

or

$$-(22.52 + j10.105)I_1 + (33.51 + j6.775)I_2 = -j415.69 \quad (2)$$

Solving (1) and (2) gives

$$I_1 = 23.75 - j5.328, \quad I_2 = 15.165 - j11.89$$

$$I_{aA} = I_1 = \underline{24.34\angle -12.64^\circ \text{ A}}, \quad I_{bB} = I_2 - I_1 = \underline{10.81\angle -142.6^\circ \text{ A}}$$

$$I_{cC} = -I_2 = \underline{19.27\angle 141.9^\circ \text{ A}}$$

$$(b) \quad \bar{S}_a = (240\angle 0^\circ)(24.34\angle 12.64^\circ) = 5841.6\angle 12.64^\circ$$

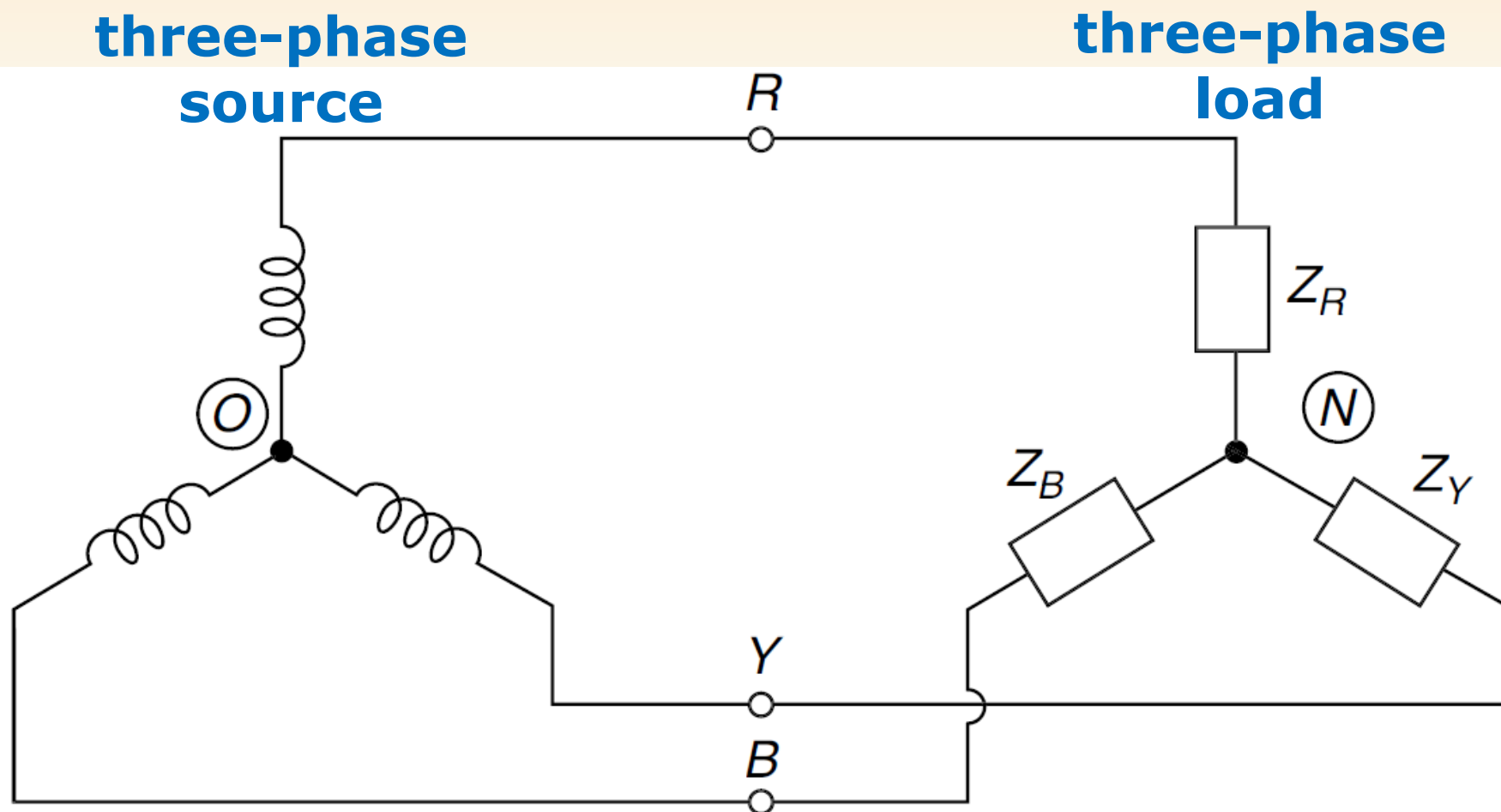
$$\bar{S}_b = (240\angle -120^\circ)(10.81\angle 142.6^\circ) = 2594.4\angle 22.6^\circ$$

$$\bar{S}_c = (240\angle 120^\circ)(19.27\angle -141.9^\circ) = 4624.8\angle -21.9^\circ$$

$$\bar{S} = \bar{S}_a + \bar{S}_b + \bar{S}_c = 12.386 + j0.55 \text{ kVA} = \underline{12.4\angle 2.54^\circ \text{ kVA}}$$

Unbalanced three-phase systems
Star/Delta using Millman's theorem
Lecture – 7 & 8

An unbalanced three-wire star-connected load can also be solved by **Millman's theorem**.

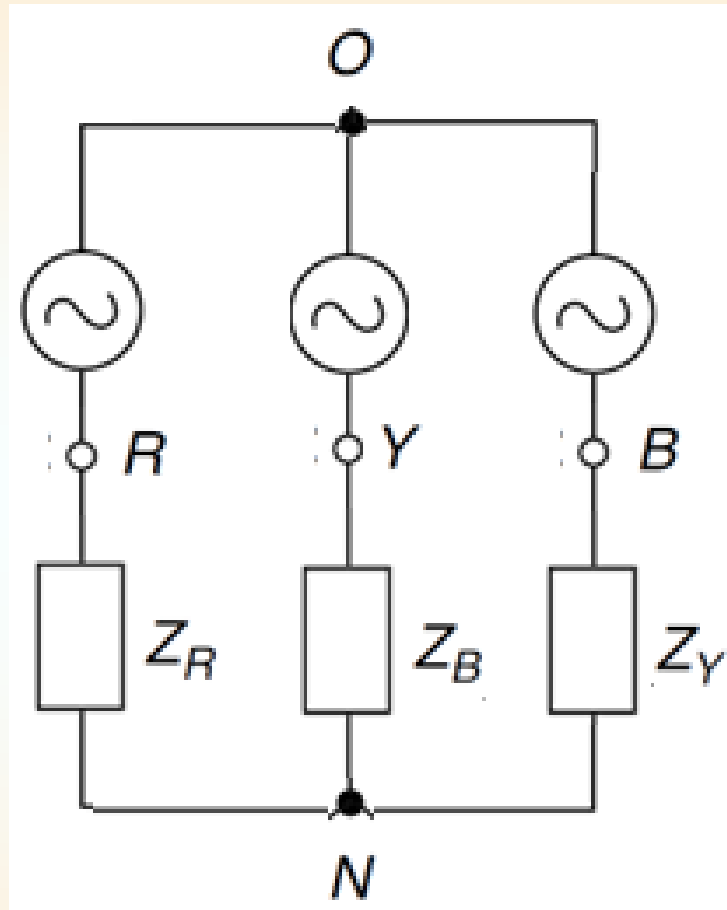


Unbalanced three-wire star-connected load

An unbalanced three-wire star-connected load is redrawn in parallel form.

**three-phase
source**

**three-phase
load**



Let V_{RO} , V_{YO} and V_{BO} be the phase voltages of the supply which are equal in magnitude but differ in phase from one another by 120° . Let V_{RN} , V_{YN} and V_{BN} be the load phase voltages which are unequal in magnitude as well as differ in phase by unequal angles. The voltage between the neutral points, N and O, i.e, V_{NO} , is calculated using Millman's theorem.

$$V_{NO} = \frac{V_{RO} Y_R + V_{YO} Y_Y + V_{BO} Y_B}{Y_R + Y_Y + Y_B}$$

where Y_R , Y_Y and Y_B are the admittances of the branches of the unbalanced star-connected load.

The voltage across phase impedance, i.e., load phase voltage can be calculated as,

$$\mathbf{V}_{RN} = \mathbf{V}_{RO} - \mathbf{V}_{NO}$$

$$\mathbf{V}_{YN} = \mathbf{V}_{YO} - \mathbf{V}_{NO}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{BO} - \mathbf{V}_{NO}$$

The phase currents in the load are given by

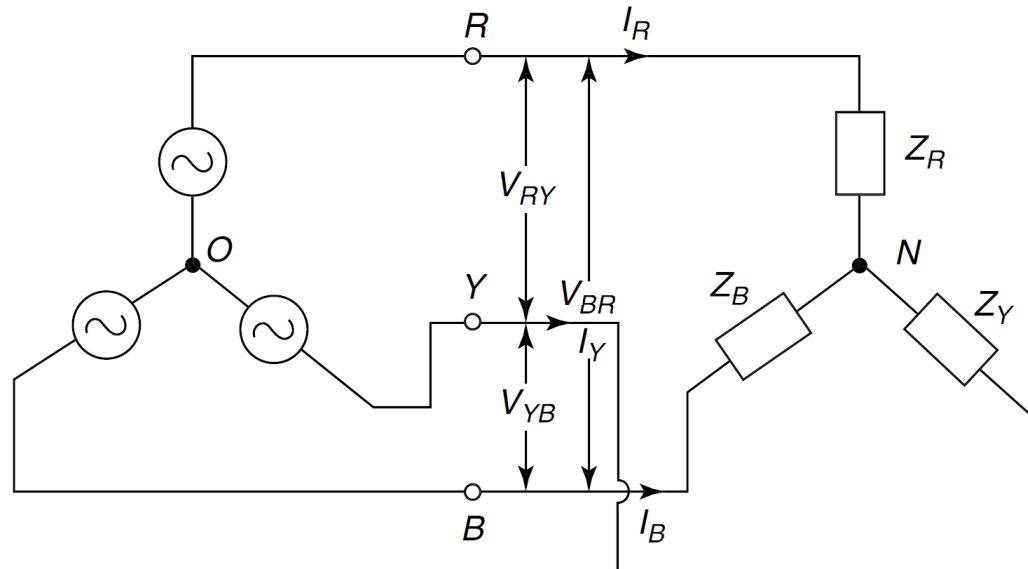
$$\mathbf{I}_R = \frac{\mathbf{V}_{RN}}{\mathbf{Z}_R}$$

$$\mathbf{I}_Y = \frac{\mathbf{V}_{YN}}{\mathbf{Z}_Y}$$

$$\mathbf{I}_B = \frac{\mathbf{V}_{BR}}{\mathbf{Z}_B}$$

The line currents are equal to the phase currents for a star-connected load.

A symmetrical 440 V, 3-phase system supplied a star-connected load with the branch impedances $Z_R = 10 \Omega$, $Z_Y = j5 \Omega$, $Z_B = -j5 \Omega$ as shown in Fig. Calculate line currents and voltage across each phase impedance by Millman's theorem. The phase sequence is RYB.



Solution $V_L = 440 \text{ V}, \quad \mathbf{Z}_R = 10 \, \Omega, \quad \mathbf{Z}_Y = j5 \, \Omega, \quad \mathbf{Z}_B = -j5 \, \Omega$

For a star-connected load,

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

Let O be the neutral point of the supply system.

Let $V_{RO} = 254.03 \angle 0^\circ \text{ V}$

$$V_{YO} = 254.03 \angle -120^\circ \text{ V}$$

$$V_{BO} = 254.03 \angle -240^\circ \text{ V}$$

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{10} = \frac{1}{10 \angle 0^\circ} = 0.1 \angle 0^\circ \text{ S}$$

$$\mathbf{Y}_Y = \frac{1}{\mathbf{Z}_Y} = \frac{1}{j5} = \frac{1}{5 \angle 90^\circ} = 0.2 \angle -90^\circ \text{ S}$$

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{1}{-j5} = \frac{1}{5 \angle -90^\circ} = 0.2 \angle 90^\circ \text{ S}$$

By Millman's theorem,

$$\begin{aligned}V_{NO} &= \frac{V_{RO} Y_R + V_{YO} Y_Y + V_{BO} Y_B}{Y_R + Y_Y + Y_B} \\&= \frac{(254.03 \angle 0^\circ)(0.1 \angle 0^\circ) + (254.03 \angle -120^\circ)(0.2 \angle -90^\circ) + (254.03 \angle -240^\circ)(0.2 \angle 90^\circ)}{0.1 \angle 0^\circ + 0.2 \angle -90^\circ + 0.2 \angle 90^\circ} \\&= 625.96 \angle 180^\circ \text{ V}\end{aligned}$$

Voltage across phase impedance

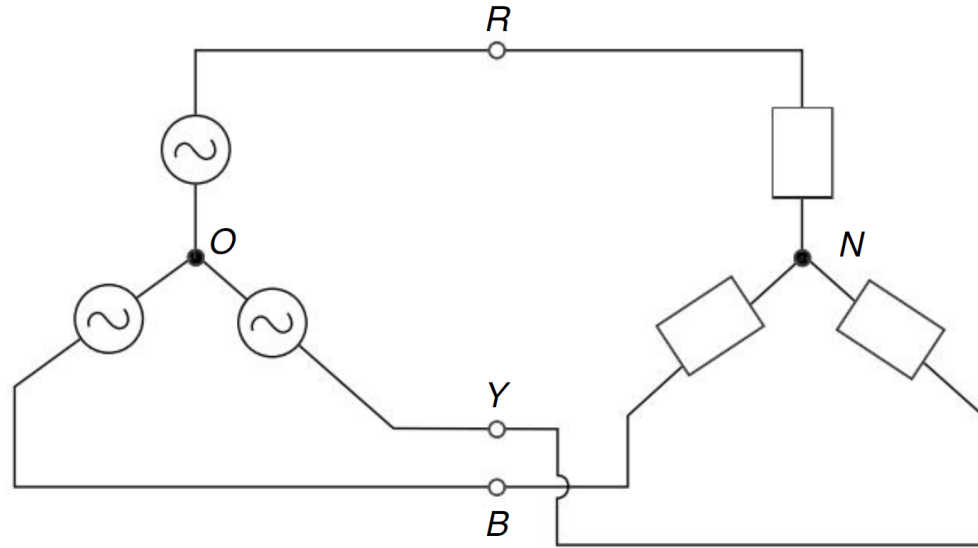
$$\begin{aligned}V_{RN} &= V_{RO} - V_{NO} = 254.03 \angle 0^\circ - 625.96 \angle 180^\circ = 880 \angle 0^\circ \text{ V} \\V_{YN} &= V_{YO} - V_{NO} = 254.03 \angle -120^\circ - 625.96 \angle 180^\circ = 545.29 \angle -23.79^\circ \text{ V} \\V_{BN} &= V_{BO} - V_{NO} = 254.03 \angle -240^\circ - 625.96 \angle 180^\circ = 545.29 \angle 23.79^\circ \text{ V}\end{aligned}$$

Phase currents/ line currents

$$\begin{aligned}I_R &= \frac{V_{RN}}{Z_R} = \frac{880 \angle 0^\circ}{10 \angle 0^\circ} = 88 \angle 0^\circ \text{ A} \\I_Y &= \frac{V_{YN}}{Z_Y} = \frac{545.29 \angle -23.79^\circ}{5 \angle 90^\circ} = 109.06 \angle -113.79^\circ \text{ A} \\I_B &= \frac{V_{BN}}{Z_B} = \frac{545.29 \angle 23.79^\circ}{5 \angle -90^\circ} = 109.06 \angle 113.79^\circ \text{ A}\end{aligned}$$

A three-phase three-wire unbalanced load is star-connected as shown in Fig.

The phase voltage of two of the arms are $V_{RN} = 100 \angle -10^\circ \text{ V}$ and $V_{YN} = 150 \angle 100^\circ \text{ V}$. Calculate voltage between star point of the load and the supply neutral.



Solution

$$V_{RN} = 100 \angle -10^\circ \text{ V}, \quad V_{YN} = 150 \angle 100^\circ \text{ V}$$

Let

$$V_{RO} = V \angle 0^\circ$$

$$V_{YO} = V \angle -120^\circ$$

$$\mathbf{V}_{RO} = \mathbf{V}_{RN} + \mathbf{V}_{NO}$$

$$\mathbf{V}_{NO} = \mathbf{V}_{RO} - \mathbf{V}_{RN} = V\angle 0^\circ - 100\angle -10^\circ$$

Similarly,

$$\mathbf{V}_{NO} = \mathbf{V}_{YO} - \mathbf{V}_{YN} = V\angle -120^\circ - 150\angle 100^\circ$$

From Eqs. (i) and (ii),

$$V\angle 0^\circ - 100\angle -10^\circ = V\angle -120^\circ - 150\angle 100^\circ$$

$$V\angle 0^\circ - V\angle -120^\circ = 100\angle -10^\circ - 150\angle 100^\circ$$

$$\sqrt{3} V\angle 30^\circ = 206.79\angle -52.97^\circ$$

$$V = 119.39\angle -82.97^\circ$$

$$\mathbf{V}_{NO} = 119.39\angle -82.97^\circ - 100\angle -10^\circ = 131.88\angle -129.67^\circ \text{ V}$$