VIT

Reg. No. :

Final Assessment Test (FAT) – January/February 2023

Programme	**B.Tech.**	Semester	**Fall Semester 2022-23**
Course Title	**CALCULUS**	Course Code	**BMAT101L**
Faculty Name	**Prof. Nathiya N**	Slot	**A1+TA1**
		Class Nbr	**CH2022231700422**
Time	**3 Hours**	Max. Marks	**100**

Part A (10 X 10 Marks)
Answer any 10 questions

1. Determine the volume of the solid obtained by rotating the region bounded by $y = x^{\frac{1}{2}}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y axis. [10]

2. a) Determine by integration the area bounded by the three lines $y = 4 - x$, $y = 3x$ and $3y = x$. **[5 marks]**
   b) If $u + v = e^x \cos y$ and $u - v = e^x \sin y$ then find the Jacobian of the function u and v with respect to x and y. **[5 marks]** [10]

3. Consider the function $f(x, y)$ of two variables x and y defined as $f(x, y) = -\frac{xy}{x^2+y^2}$. Find the limit along the following curves as $(x, y) \to (0, 0)$. [10]
   a) the x -axis. b) the y - axis. c) the line $y = x$. d) the line $y = -x$. e) the parabola $y = x^2$.

4. A toy manufactures estimates a production function to be $f(x, y) = 100x^{\frac{3}{4}}y^{\frac{1}{4}}$, where x represents the units of labour (at Rs. 150 per unit) and y represents the units of capital (at Rs. 250 per unit). The total cost of labour and capital to Rs. 50,000. Find the maximum production level for this manufacturer. [10]

5. a) Obtain the local maximum and local minimum values of the function [10]
   $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$. [5 marks]
   b) Evaluate the integral $\int_0^1 x^{q-1}\left(\log \frac{1}{x}\right)^{p-1} dx$ by using Beta, Gamma functions. [5 marks]

6. Sketch the region of integration and evaluate by changing to polar coordinates [10]
   $\int_0^1 \int_{x^2}^{x} (x^2 + y^2) dy dx$.

7. Changing to cylindrical coordinates, find the volume of the portion of the sphere [10]
   $x^2 + y^2 + z^2 = a^2$ lying inside the cylinder $x^2 + y^2 = ay$.

8. Evaluate the integral $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx \times \int_0^1 \frac{1}{\sqrt{1+x^4}} dx$ by using Beta, Gamma functions. [10]

9. Find the points on the surface defined by $x^2 + 2y^2 + 3z^2 = 1$, where the tangent plane is parallel to the plane defined by $3x - y + 3z = 1$. [10]

10. a) Find ∇r^n, where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$. [5 marks] [10]
    b) Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $v = 3\vec{i} - 4\vec{j}$. [5 marks]

11. Verify Stoke's theorem for $F = (x - 2y)\vec{i} - 3yz^2\vec{j} - 3y^2z\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the circular boundary on $z = 0$ plane. [10]

12. Show that $F = (2xy - z^3)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative force field. Find the scalar potential and the work done by F in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$ [10]