Final Assessment Test – April 2025

VIT Vellore Institute of Technology

Course: BMAT101L - Calculus

Class NBR(s): 1212 / 1213 / 6584

Slot: E1+TE1

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1. (i) Find the critical points of $f(x)=x^3-12x-5$ and identify the open intervals on which f is increasing and on which f is decreasing. [5]

   (ii) Verify Lagrange's Mean value theorem for the function $f(x)=x^2+2x-1,\ [0,1]$ and find the value of c. [5]

2. The region bounded by the curve $y=x^2+1$ and the line $y=-x+3$ is revolved about the x-axis to generate a solid. Find the volume of the solid. [10]

3. (i) Check the continuity of the function $f(x,y)=\begin{cases}\dfrac{x-y}{x+y}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ at the point (0, 0). [5]

   (ii) If $u=\sin^{-1}(x-y), x=3t, y=4t^3$, Find $\dfrac{du}{dt}$. [5]

4. Find the Taylor series polynomial upto the degree three for the function $f(x,y)=2x^3+3y^3-4x^2y$ about the point $(1,2)$. [10]

5. The temperature T at any point (x,y,z) in space is given by $T=400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$. [10]

6. Sketch the region and evaluate the integral $\int_0^2\int_{x^2}^{2x}(4x+2)\,dy\,dx$ by changing the order of integral. [10]

7. Find the volume of the region cut from the cylinder $x^2+y^2=4$ by the plane $z=0$ and the plane $x+z=3$. [10]

8. (i) Evaluate $\int_0^{\frac{\pi}{2}}\sqrt{\tan x}\,dx$. [10]

   (ii) Express $\int_0^1 x^m(1-x^p)^n\,dx$ in terms of Beta function and hence evaluate the integral $\int_0^1 x^{\frac{3}{2}}\left(1-\sqrt{x}\right)^{\frac{1}{2}}dx$.

9.a) Find the directional derivative of the scalar function $g(x,y,z)=x^2y-y^2z-xyz$, $(1,-1,0)$, in the direction of $\vec{i}-\vec{j}+2\vec{k}$. [10]

OR

9.b) Show that the vector field $\mathbf{v}=e^{xy}(y\,\mathbf{i}+x\,\mathbf{j})+2e^{z}\mathbf{k}$ is irrotational and find a scalar potential function $f(x,y,z)$ such that $\mathbf{v}=grad\,f$. [10]

10.a) Find the work done by the force $\mathbf{F}=(x^2-y^3)\mathbf{i}+(x+y)\mathbf{j}$ in moving a particle along the closed path C containing the curves $x+y=0$, $x^2+y^2=16$ and $y=x$ in the first and fourth quadrants. [10]

OR

10.b) Verify Green's theorem for $f(x,y)=e^{-x}\sin y$, $g(x,y)=e^{-x}\cos y$ and C is the square with vertices at $(0,0),\left(\frac{\pi}{2},0\right),\left(\frac{\pi}{2},\frac{\pi}{2}\right),\left(0,\frac{\pi}{2}\right)$. [10]

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