Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

Vellore – 632014, Tamil Nadu, India
DEPARTMENT OF MATHEMATICS
SCHOOL OF ADVANCED SCIENCES
FALL SEMESTER 2022-2023

CONTINUOUS ASSESSMENT TEST – II

Programme Name & Branch : B. Tech
Course Code : BMAT205L
Course Name : Discrete Mathematics and Graph Theory
Slot : A2 + TA2 +TAA2
Date of the Examination :
Duration : 90 minutes **Max. Marks : 50**

General instruction(s): Answer All The Questions

Q. No	Question	Marks	Course Outcome (CO)	Bloom's Taxonomy (BL)
1.	The Cayley's table for a group defined on the set S = { e, a, b, c, d, f } with respect to a binary operator * is given below. Determine | * | e | a | b | c | d | f | | e | e | a | b | c | d | f | | a | a | e | d | f | b | c | | b | b | f | e | d | c | a | | c | c | d | f | e | a | b | | d | d | c | a | b | f | e | | f | f | b | c | a | e | d | a. A set $H_1 \subseteq S$ such that $< H_1, * >$ is not a subgroup of $< S, * >$. b. A set $H_2 \subseteq S$ such that $< H_2, * >$ is a subgroup of $< S, * >$. c. The left and right cosets of H_2 in S.	10	2	BL5
2.	Let $H = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ be a parity check matrix. Determine the (2, 5) group code function $e_H : B^2 \rightarrow B^5$. Create the decoding table. What is the original message if the received message is 11101.	10	2	BL5

3.	How many integers are there between 1 and 1000 both inclusive, that are divisible by atleast one of the integers 2, 3, 5 and 7.	10	3	BL2
4.	Find the generating function of the recurrence relation $S(n) - 2S(n-1) - 3S(n-2) = 0$, $n \geq 2$, with $S(0) = 3$, $S(1) = 1$, and hence find its solution.	10	3	BL1
5.	Let n = 60, and let X be the set of all positive integers which are divisors of 60. Let '$\leq$ 'be the relation ' divisor of ' on X. a. Draw the Hasse diagram of $< X, \leq >$. b. Find the least upper bound and greatest lower bound of (2, 5) and (12, 30) . c. What is the greatest and least element of $< X, \leq >$.	10	4	BL3

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Fall Semester 2022-2023

Continuous Assessment Test -II (October 2022)

Slot: A1+TA1+TAA1 **Programme:** B.Tech(CSE/IT)

Course : BMAT205L – Discrete Mathematics and Graph Theory

Max. Time: 90 minutes **Max. Marks:** 50

Answer all the questions (5x10=50)

1. Verify Lagrange's theorem for the subgroup generated by 4 of the group $< Z_{17}^{*} , \times_{17} >$

2. An encoding function $e: B^3 \rightarrow B^6$ is given by the generator matrix $\begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$.

   a) Determine all code words generated by the matrix. b) Find the associated parity check matrix H c) Use H to decode the following received words 100100, 010100

3. a) How many positive integers between 100 and 999 inclusive
   (i) are not divisible by either 3 or 4? (ii) are divisible by 3 but not by 4?

   b)How many different ways are there to choose 3 donuts from the 5 varieties at a donut shop? Assume that there are at least 3 donuts of each variety.

4. Solve the recurrence relation
   $a_{n+2} + 3a_{n+1} + 2a_n = 3^n, \ n \geq 0, a_0 = 0, a_1 = 1$

5. Draw the Hasse diagram for the poset $\{S_{30}, D\}$, divisors of 30 under the relation divides. Hence or otherwise prove that it is a lattice, also find $3 * (10 \oplus 3)$ and $3 \oplus (10 * 3)$

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Fall Semester – 2023 ∼ 2024

Department of Mathematics

School of Advanced Sciences

Continuous Assessment Test – II

Answer Key

Course Code & Name : BMAT205L – Discrete Mathematics and Graph Theory **Slot : A1+TA1+TAA1**

Common Question Paper for A1+TA1+TAA1 slot **Programme Name & Branch : B.Tech.**

Exam Duration : 90 Minutes **Maximum Marks : 50**

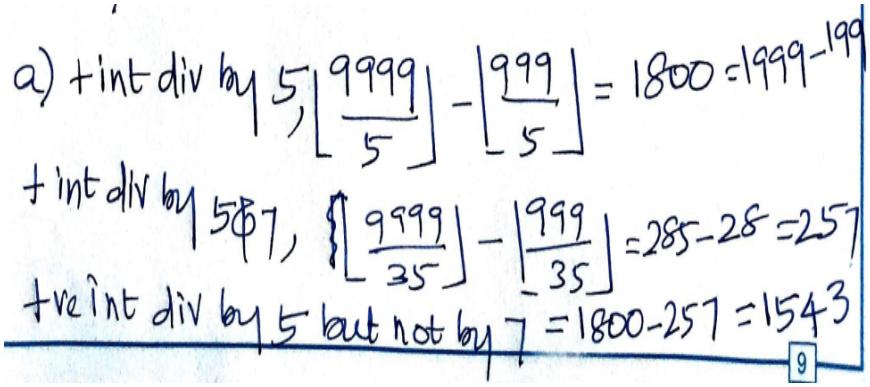
Answer ALL the Questions

Students are permitted to bring any number of text books and hand written note books (class notes)

Each question carries equal marks (5 × 10 = 50 Marks)

1. (*a*). How many positive integers between 1000 and 9999 both inclusive that are divisible by 5 but not by 7?

Solution:



(*b*). Find the least number of ways of choosing three different numbers from 1 to 10 so that all choices have the same sum.

Solution:

Find the least number of ways of choosing three different numbers from 1 to 10 so that all choices have the same sum.

Solution.
From the numbers 1 to 10, we can choose three different numbers in $10C_3 = 120$ ways.

The smallest possible sum that we get from a choice is $1+2+3=6$, and the largest sum is $8+9+10=27$. Thus, the sums vary from 6 to 27 (both inclusive), and these sums are 22 in number.

Accordingly, there are 120 choices (pigeons) and 22 sums (pigeonholes).

Therefore, the least number of choices assigned to the same sum is, by the generalized pigeonhole principle, $\left[\frac{120-1}{22}\right]+1=6$.

[10 M] [CO: 3][BL: L2]

2. Solve the relation $a_n - 3a_{n-1} = n$ where $n \geq 1$ and $a_0 = 1$ using the method of generating functions.

Solution:

To bring in generating function, we multiply n=1 with x, n=2 with x^2, and so on. We have

$$n=1:\ a_1x^1 - 3a_0x^1 = 1x^1$$
$$n=2:\ a_2x^2 - 3a_1x^1 = 2x^2$$
$$\cdots\cdots$$

Then we have $\sum_{n=1}^{\infty} a_n x^n - 3\sum_{n=1}^{\infty} a_{n-1}x^n = \sum_{n=1}^{\infty} nx^n$

Let f(x) be the ordinary generating function of a_0, a_1, a_2,.., then we have $(f(x)-a_0) - 3x\sum_{n=1}^{\infty} a_{n-1}x^{n-1} = \sum_{n=0}^{\infty} nx^n$

And then $(f(x)-1) - 3xf(x) = \sum_{n=0}^{\infty} nx^n$

Recall the generating function of 0, 1, 2, 3, … is $\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + \cdots$

Therefore $(f(x)-1) - 3xf(x) = \frac{1}{(1-x)^2}$

We write $\frac{x}{(1-x)^2(1-3x)} = \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{1-3x}$

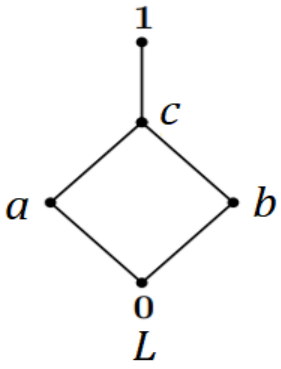
Solving it we get A=-1/4, B=-1/2, and C=3/4

That is: $f(x) = \frac{7/4}{1-3x} + \frac{-1/4}{1-x} + \frac{-1/2}{(1-x)^2}$

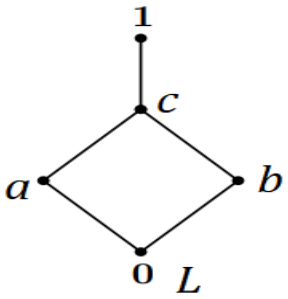
Using the formulas learned in the generating functions, we have $a_n = \frac{7}{4}3^n - \frac{1}{2}n - \frac{3}{4}$

[10 M] [CO: 3][BL: L5]

3. (*i*). Verify the Complemented and Distributive properties for the given lattice L.



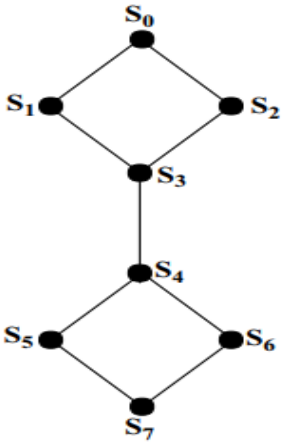
Solution:



Distributive but NOT complemented

(*ii*). Let the sets $S_0, S_1, \cdots S_7$ be given by $S_0 = \{a, b, c, d, e, f\}$, $S_1 = \{a, b, c, d, e\}$, $S_2 = \{a, b, c, e, f\}$, $S_3 = \{a, b, c, e\}$, $S_4 = \{a, b, c\}$, $S_5 = \{a, b\}$, $S_6 = \{a, c\}$, $S_7 = \{a\}$. Draw the diagram of $\langle L, \subseteq \rangle$ where $L = \{S_0, S_1, \cdots S_7\}$.

Solution:



Hasse diagram of $\langle L, \subseteq \rangle$

[10 M] [CO: 4][BL: L3]

4. (*i*). Simplify the boolean expression $c * (b \oplus c) * (a \oplus b \oplus c)$

Solution:

$$\begin{aligned}
\mathbf{c * (b \oplus c) * (a \oplus b \oplus c)} &= \mathbf{(c * b \oplus c * c) * (a \oplus b \oplus c)} \\
&= \mathbf{(c * b \oplus c) * (a \oplus b \oplus c)} \\
&= \mathbf{c * (b \oplus 1) * (a \oplus b \oplus c)} \\
&= \mathbf{c * (a \oplus b \oplus c)} \\
&= \mathbf{(c * a) \oplus (c * b) \oplus (c * c)} \\
&= \mathbf{(c * a) \oplus (c * b) \oplus c} \\
&= \mathbf{c * (a \oplus b \oplus 1)} \\
&= \mathbf{c * (a \oplus 1)} \\
&= \mathbf{c * 1} \\
&= \mathbf{c}
\end{aligned}$$

(*ii*). If $x \oplus y = x \oplus z$ and $x' \oplus y = x' \oplus z$, then prove that $y = z$.

Solution:

Given that $(x + y) = (x + z)$ *i.e.* $(x \vee y) = (x \vee z)$

$(x' + y) = (x' + z)$ *i.e.* $(x' \vee y) = (x' \vee z)$

$$\begin{aligned}
y &= y \vee 0 \\
&= y \vee (x \wedge x') \\
&= (y \vee x) \wedge (y \vee x') \\
&= (x \vee y) \wedge (x' \vee y) \\
&= (x \vee z) \wedge (x' \vee z) \\
&= (z \vee x) \wedge (z \vee x') \\
&= z \vee (x \wedge x') \\
&= z \vee 0 \\
&= z
\end{aligned}$$

, $y = z$.

[10 M] [CO: 4][BL: L2]

5. Obtain the product-of-sums canonical forms and sum-of-products canonical forms for the expression $(x \oplus z) * y$

Solution:

x	y	z	$x + z$	$f = (x + z)y$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

Product-of-sums canonical forms: $(x \oplus y \oplus z) * (x \oplus y \oplus z') * (x \oplus y' \oplus z) * (x' \oplus y \oplus z) * (x' \oplus y \oplus z')$
Similarly, sum-of-products canonical forms: $(x * y * z') \oplus (x' * y * z) \oplus (x * y * z)$

[10 M] [CO: 4][BL: L4]

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