



# VIT

Vellore Institute of Technology

## Final Assessment Test - November 2024

Course: **BMAT101L - Calculus**

Time: **Three Hours**

Max. Marks: **100**

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1. For the function  $f(x) = 3x^4 - 4x^3 - 12x^2 + 4$  [10]
  - i) find the local maxima and local minima.
  - ii) find the intervals where the function is increasing and the intervals where it is decreasing.
  - iii) identify the intervals where the function is concave up and concave down, hence find the points of inflection.
2. i) Find the area of the region enclosed by  $y = x^2$  and  $y = \sqrt{x}$ . [10]  
 ii) Find the volume of the solid generated by revolving the region bounded by the line  $x + y = 2$  and the curve  $x^2 = 4 - y$  about the  $x$ -axis.
3. i) Examine whether  $u = y + z, v = x + 2z^2, w = x - 4yz - 2y^2$  are functionally dependent. If so find the relation between them. [10]  
 ii) If  $u = x^2y^2 + x^3y$ , where  $x = 2t^2$  and  $y = 4t$ , then find  $\frac{du}{dt}$ .
4. Expand  $e^x \log(1 + y)$  in a Taylor's series about  $(0,0)$  up to terms of third degree. [10]
5. A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction. [10]

6. Change the order of the integration and hence evaluate it. [10]

$$\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy dx.$$

7. Evaluate  $\iiint (x^2 + y^2 + z^2) dx dy dz$  taken over the volume enclosed by the sphere  $x^2 + y^2 + z^2 = 1$  by transforming into spherical polar coordinates. [10]
8. i) Evaluate  $\int_0^1 \frac{1}{\sqrt{\log \frac{1}{x}}} dx$ . [10]  
 ii) Find the value of the integral  $\int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin \theta}}$ .
- 9.a) Find the directional derivative of  $\phi = x^2yz + 4xz^2$  at  $(1,1,1)$  in the direction of  $\vec{i} + \vec{j} - \vec{k}$ . Also find  $\text{curl}(\text{grad } \phi)$ . [10]

OR

- 9.b) Show that  $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$  is irrotational and find its scalar potential. [10]

- 10.a) Evaluate  $\iint_S \vec{F} \cdot \hat{n} ds$  where  $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$  and  $S$  is the surface bounding the region  $x^2 + y^2 = 4, z = 0$  and  $z = 3$ . [10]

OR

- 10.b) Verify Green's theorem in the  $xy$  plane for  $\int_C \{(xy + y^2)dx + x^2dy\}$ , where  $C$  is the closed curve of the region bounded by  $y = x$  and  $y = x^2$ . [10]

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