Final Assessment Test– Jan/Feb 2023

VIT Vellore Institute of Technology

Course: **BMAT101L - Calculus**

Class NBR(s):**6215 / 6291** Slot: **C2+TC2**

Time: **Three Hours** Max. Marks: **100**

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Use Intermediate Value Theorem and Rolle's Theorem to show that the equation $x^3 + 7x + 5$ has exactly one real root.

2. The region bounded between the curves $y = x^2 + 1$ and $y = -x + 3$ is rotated about the X-axis to generate a solid. Find the volume of this solid.

3. Does the limit for the following function exist as (x, y) approaches $(0, 0)$? Justify.

$$f(x,y) = \frac{4x^2y}{x^4 + y^2}$$

4. Find the critical points of the given function $f(x,y)$. Further, use the second derivative test to find the points of local maxima, local minima and the saddle points.

$$f(x,y) = 10xye^{-(x^2+y^2)}$$

5. Find the point on the plane $2x + y - z - 5 = 0$ that is closest to origin.

6. Find the value of $\int_0^2 \int_{x^2}^{2x} (4x + 2)\, dy\, dx$ by reversing the order of integration.

7. Find the volume of the ice cream cone D cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$ by using spherical co-ordinates.

8. The cylinder $x^2 + z^2 = 1$ is cut by the planes $x = 0, y = 0, y = x$. Find the volume of the region in the first octant by using Gamma function.

(x+y+z)

(1,1,1)

(0)

9. Let $H(x,y,z) = \frac{1}{\sqrt{(x^2+y^2+z^2)}}$. Then find $div\ grad\ H$.

10. Find the directional derivative of $f(x,y) = xe^y + \cos(xy)$ at the point $(2, 0)$ in the direction of $\overline{v} = 3\hat{\imath} - 4\hat{\jmath}$.

(0,5)

(1,13)

(2,27)

(-1,-3)

(-1,-17)

11. Find the line integral of $f(x,y,z) = x - 3y^2 + z$ over $C_1 \cup C_2$ where C_1 is the line segment joining $(0,0,0)$ to $(1,1,0)$ and C_2 is the line segment joining $(1,1,0)$ to $(1,1,1)$.

12. Use Stoke's Theorem to find the circulation of the field $\overline{F} = x^2\hat{\imath} + 2x\hat{\jmath} + z^2\hat{k}$ around $C: 4x^2 + y^2 = 4$ in the XY-plane in the anti-clockwise when viewed from above.

⇔⇔⇔

$y = x^3+7x+5$

$\frac{f(b)-f(a)}{b-a}$