N/E/TX

Reg. No: 23BBS0060

# Final Assessment Test – May 2024

VIT
Vellore Institute of Technology

Course: MAT2004 - Linear Algebra
Class NBR(s): 3811 / 3814
Slot: C2+TC2+TCC2
Time: Three Hours
Max. Marks: 100

- KEEPING MOBILE PHONE/ELECTRONIC DEVICES EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

**Answer any TEN Questions**
**(10 X 10 = 100 Marks)**

1. (a) Write down any five properties of determinants of a matrix. **[10]**

   (b) Check whether the matrix is $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ is singular? Find the rank of $A$.

2. Let the system of linear equations be given by **[10]**
   $x + 3y - 2z + t = 0;\ 2x + 6y - 5z - 2t = -1; z + 2t = 1, x + 3y + 4t = 2.$
   Test for consistency of the linear system and hence solve it completely.

3. Find the inverse of $A = \begin{bmatrix} 2 & 1 & 2 \\ -3 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ using Gauss-Jordan elimination and hence find the solution vector for the linear system $AX = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$. **[10]**

4. Find the $LU$ factorization of the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 4 \\ 2 & 2 & -3 \end{bmatrix}$ using elementary matrices and then solve the system $AX = [1 \ \ 2 \ \ 2]^T$ with the help of $L$ and $U$. **[10]**

5. (a) Let $P = \{p_1, p_2, p_3, p_4\}$ where $p_1 = 2 + x + 4x^2$; $p_2 = 1 - x + 3x^2$; $p_3 = 3 + 2x + 5x^2$ and $p_4 = 7 + 8x + 9x^2$ spans $P_2$. Determine maximal linearly independent subset of the set $P$. **[10]**

   (b) Define the subspace. Let $W = \left\{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : 2a + b - c + 5d = 0\right\}$. Is $W$ a subspace of $M_{22}$? Explain.

6. Find the bases for row space ($R(A)$), column space ($C(A)$) and nullspace ($N(A)$) of the matrix $A_{m\times n} = \begin{bmatrix} 1 & 3 & -9 & 5 \\ 2 & 6 & 7 & 1 \\ 1 & 3 & -8 & 1 \end{bmatrix}$. Hence, verify the statement $\dim(R(A)) + \dim(N(A)) = n$. **[10]**

7. (a) Consider the space $P_2$ with inner product $\langle f, g\rangle = \frac{1}{2}\int_{-1}^{1} f(t)g(t)dt$. What is the angle between $f(t) = t^2 - 1$ and $g(t) = 2t + 3$ in $P_2$ space. **[10]**

   (b) Define an Isometry linear transformation. Check whether the linear transformation $T: R^2 \to R^2$ be defined by $T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$ is an isometry?

8. Let $S = \{(1,0,1,1), (1,2,1,3), (0,2,1,1), (0,1,0,0)\}$ be a basis for the space $R^4$. Find the orthonormal basis for $S$ in $R^4$ with respect to the standard inner product. **[10]**

9. Find the eigenvlaues and corresponding eigenvectors for the matrix **[10]**
$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

10. Check whether the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is positive definite? Hence, find the transformation matrix that transforms $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ into canonical form. **[10]**

11. Let the alphabets A to Z are encoded as $A \leftrightarrow 0, B \leftrightarrow 1, \dots, z \leftrightarrow 25$. Encode and decode the message $'FLIGHT\ TAKE\ OFF'$ with the use of encoded matrix $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$. **[10]**

12. Determine the least square solution to $Ax = b$ where $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 2 & 5 & 3 \\ 2 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$. **[10]**

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