BEEE201L- Electronic Materials

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Cabin: CBMR, 207-C

BEEE102L- Basic Electrical and Electronics Engineering

Module-4 : Dielectric materials and Insulation

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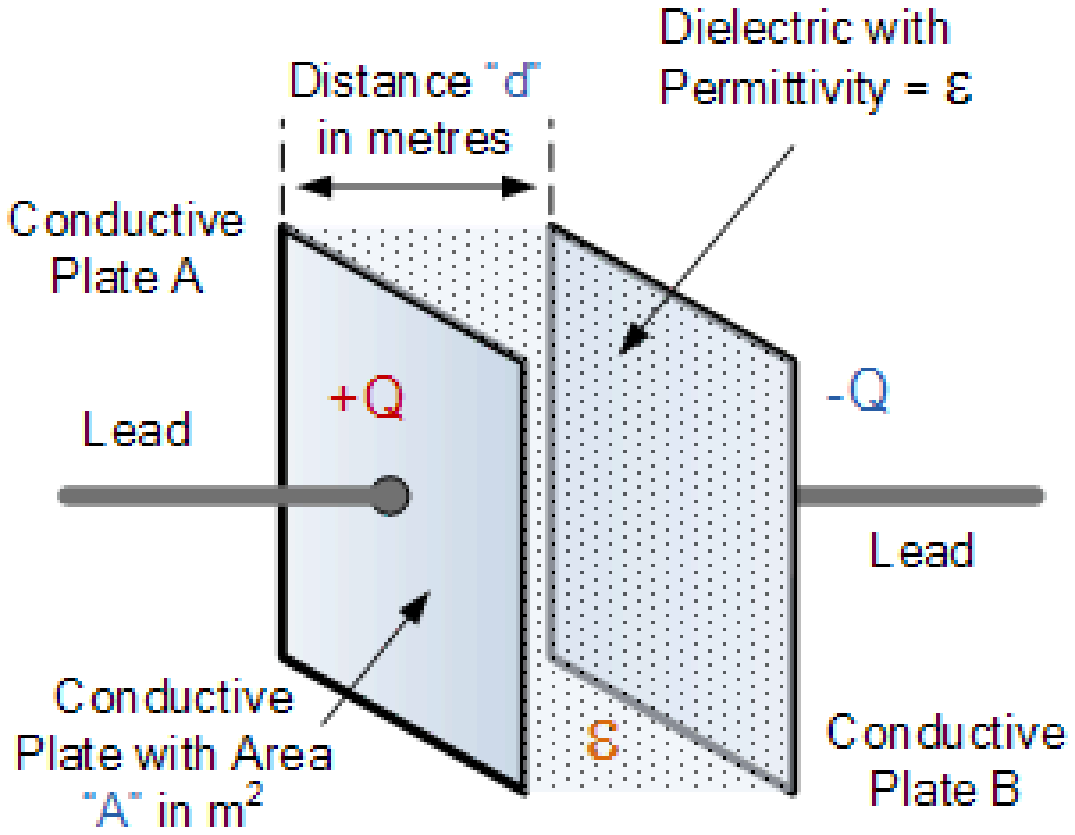
CONTENT

- Requirement of insulating materials,
- Electrical and molecular properties,
- Dependence of permittivity on temperature, pressure and humidity; 685, 697, 706
- Dipole movement and electronic polarization; 7.1.2
- Clausius-Mossotto equation, polarization mechanisms; 7.1.4, 7.3
- Behaviour of dielectrics under static and alternating fields
- Frequency dependence; 7.4
- Complex dielectric constants and dielectric loss, dipolar relaxation and characteristics. 7.4

Capacitance: When a two conducting plates are separated by a dielectric material then it will act as a capacitor and offer capacitance. Conductive plates are electrodes and dielectric materials are insulators. It is represented by (C).

Units: Farads (F).



$$C = \frac{Q}{V}$$

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

d = distance between conducting plates(m)

ϵ_0= Absolute Primitivity (F/m)

$= 8.854 \times 10^{-12}$

ϵ_r= Relative Primitivity

A= Area of plate (m^2)

https://phet.colorado.edu/sims/html/capacitor-lab-basics/latest/capacitor-lab-basics_en.html

Requirement of Good insulating materials

Inters of MΩ/cm

1. High insulation resistance to avoid leakage current.

Air 3kV/mm or 30kV/cm, paper 16kV/cm, Mica 118kV/mm

2. High dielectric strength to avoid electrical breakdown of the insulator.

Air 1, Mica 6.8, paper 3.6, Porcelain 6.5, Water 80

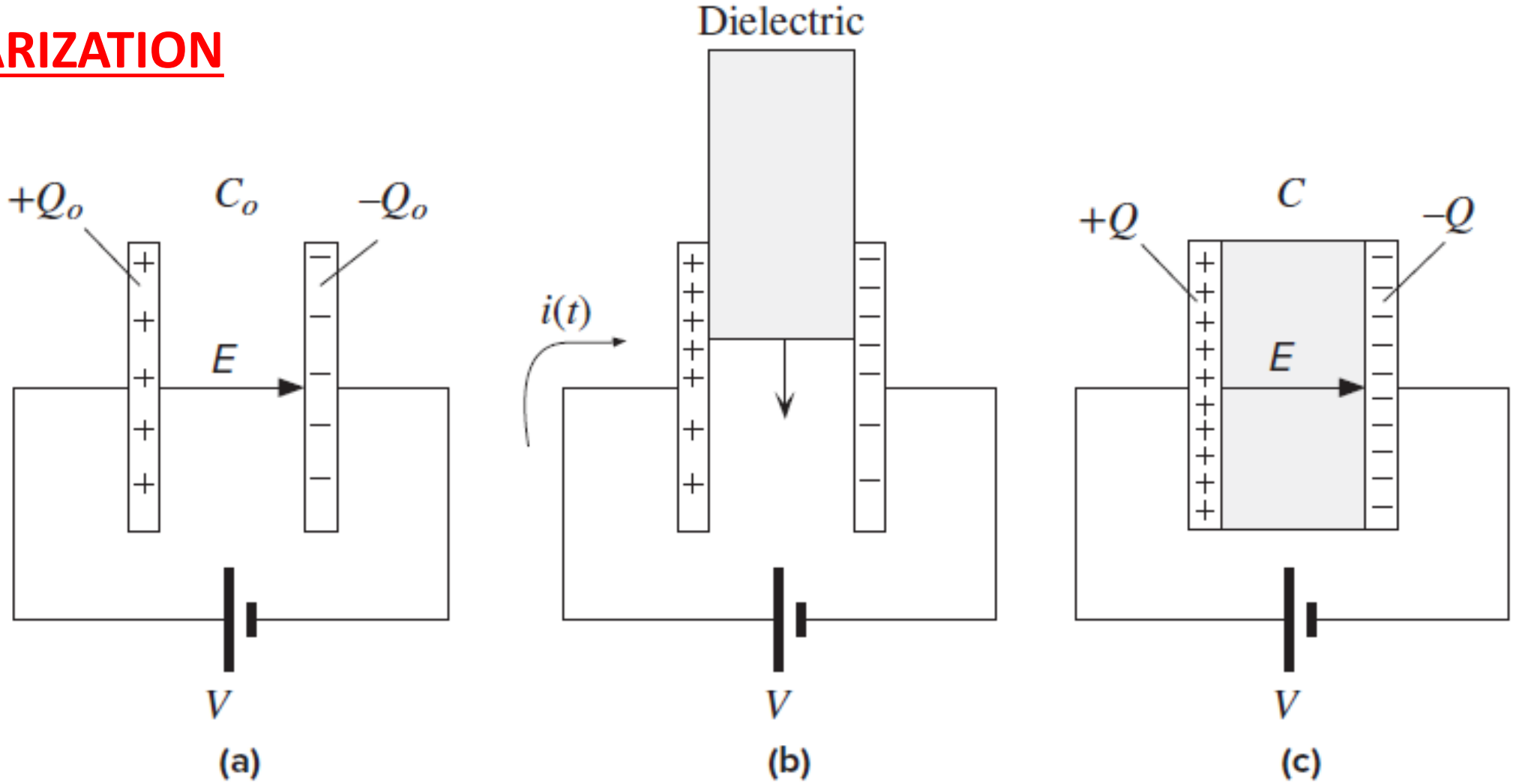
3. High relative permittivity of insulator material in order that dielectric strength is high.
4. High mechanical strength to withstand the mechanical handling of cables/conductor.
5. Non-hygroscopic *i.e.,* it should not absorb moisture from air or soil.
6. Non-inflammable.
7. Low cost so as to make the underground system a viable proposition.
8. Unaffected by acids and alkalies to avoid any chemical action.
9. The insulator material should be non-porous, free from impurities and cracks otherwise the permittivity will be lowered.
10. It is desirable to have high value of safety factor.
11. **No one insulating material possesses all the above mentioned properties**.

Some important dielectric materials for electrical applications

1. Hydrogen, nitrogen and helium (Inert gases).
2. Transformer oil and synthetic liquids.
3. Resins, nylon, varnishes.
4. Mica, Vinayl, PVC
5. Plastic, rubber.
6. Ceramic, Glass, Porcelain, Oil Impregnated Paper.
7. Cross-linked polyethylene material.

- The charge storage ability per unit voltage, increases by a factor of ε_r, where ε_r is called the **dielectric constant** of the medium or its **relative permittivity.**
- The increase in the capacitance is due to the **polarization** of the medium in which positive and negative charges are displaced with respect to their equilibrium positions.
- The opposite surfaces of the dielectric medium acquire opposite surface charge densities that are related to the amount of polarization in the material.
- **Electric dipole moment p**, which is a measure of the electrostatic effects of a pair of opposite charges +Q and −Q separated by a finite distance 'd', $\boldsymbol{p = Qd}$.
- **Dielectric strength** is the maximum field (E_{br}) that can be sustained in a dielectric beyond which dielectric breakdown ensues; that is, there is a large conduction current through the dielectric shorting the plates.

MATTER POLARIZATION



$$C = \frac{Q}{V}$$

Figure 7.1 (a) Parallel plate capacitor with free space between the plates. (b) As a slab of insulating material is inserted between the plates, there is an external current flow indicating that more charge is stored on the plates. (c) The capacitance has been increased due to the insertion of a medium between the plates.

The **electric field, directed from high to low potential**, is defined by the gradient of the potential $E = -dV/dx$. Thus, the electric field **E between the plates is just V/d** where d is the separation of the plates.

The **increase in the stored charge is due to the polarization of the dielectric by the applied field.**

It is important to remember that **when the dielectric medium is inserted, the electric field remains unchanged**, provided that the insulator fills the whole space between the plates as shown in Figure 7.1c.

The voltage V remains the same and therefore so does the gradient V/d, which means that E remains constant.

The flow of current during the insertion of the dielectric in Fig. 7.1b, is due to the additional free charges $Q - Q_O$ needed on the capacitor plates to neutralize the opposite polarity polarization charges Q_P appearing on the dielectric surfaces.

DIPOLE MOMENT AND ELECTRONIC POLARIZATION

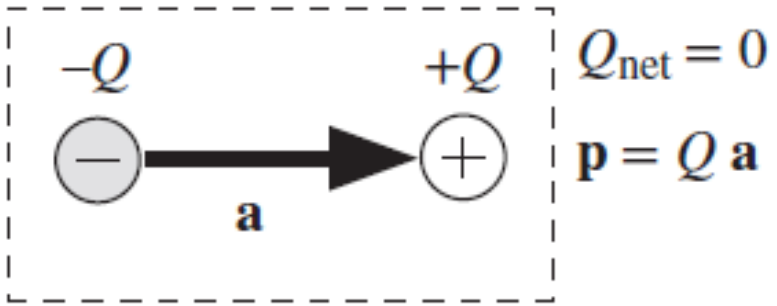


Figure 7.2 The definition of electric dipole moment.

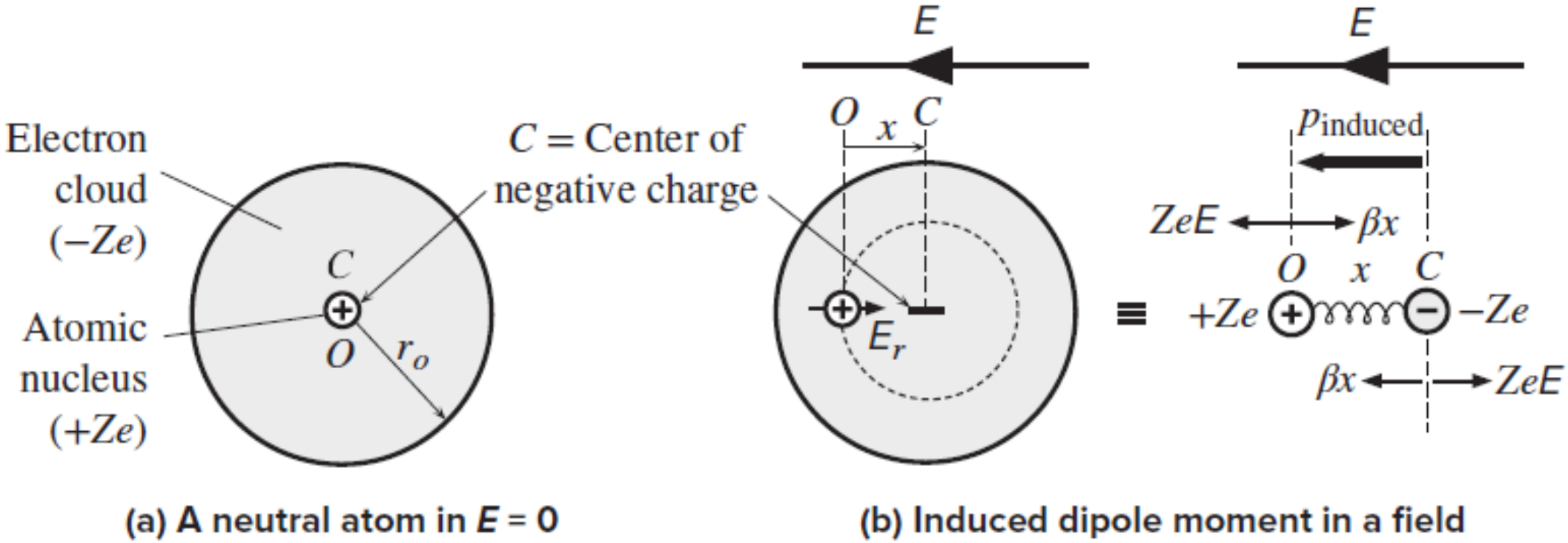


Figure 7.3 The origin of electronic polarization.

The induced dipole moment depends on the electric field causing it. We define a quantity called the **polarizability** α to relate the induced dipole moment $p_{induced}$ to the field E causing it,

$$p_{induced} = \alpha E$$

where α is a coefficient called the polarizability of the atom. It depends on the polarization mechanism. Since the polarization of a neutral atom involves the displacement of electrons, α is called **electronic polarization** and denoted as α_e. Inasmuch as the electrons in an atom are not rigidly fixed, all atoms possess a certain amount of electronic polarizability.

$$p_{induced} = \propto_e E$$

In the absence of an electric field, the center of mass C of the orbital motions of the electrons coincides with the positively charged nucleus O and the electronic dipole moment is zero as in Figure 7.3a. Ex: H_2, O_2, CO_2 etc

Suppose that the atom has Z number of electrons orbiting the nucleus and all the electrons are contained within a certain sphere region of radius r_o. When an electric field E is applied, the light electrons become displaced in the opposite direction to E so that their center of mass C is shifted by some distance x with respect to the nucleus at O, which we take to be the origin as shown in Figure 7.3b. Ex: Hcl, NaCl, H_20, NH_3.

As the electrons are "pushed" away by the applied field, the Coulombic attraction between the electrons and nuclear charge "pulls in" the electrons; tries to restore the electron cloud back to its original position.

In equilibrium, the net force on the negative charge center is zero $\boxed{ZeE = \beta x}$

Therefore, the *magnitude* of the induced electronic dipole moment p_e is given by

$$p_e = (Ze)x = (Z^2e^2/\beta)E$$

Restoring force constant $$\beta = \frac{(Ze)^2}{4\pi\varepsilon_o r_o^3}$$

Classical atomic polarizability $$\alpha_e \approx 4\pi\varepsilon_o r_o^3$$

The electronic dipole moment in above is **valid under static conditions**, that is, when the **electric field is a DC field**.

Go through Example 7.1 in Kasap Textbook

With sinusoidal displacement at any time is given by $x(t) = x_0 \cos(w_0 t)$

Electronic polarization resonance frequency $\omega_0 = \left(\frac{\beta}{Zm_e}\right)^{1/2}$

Static polarizability and resonance $\alpha_e = \frac{Ze^2}{m_e w_0^2}$

POLARIZATION VECTOR P

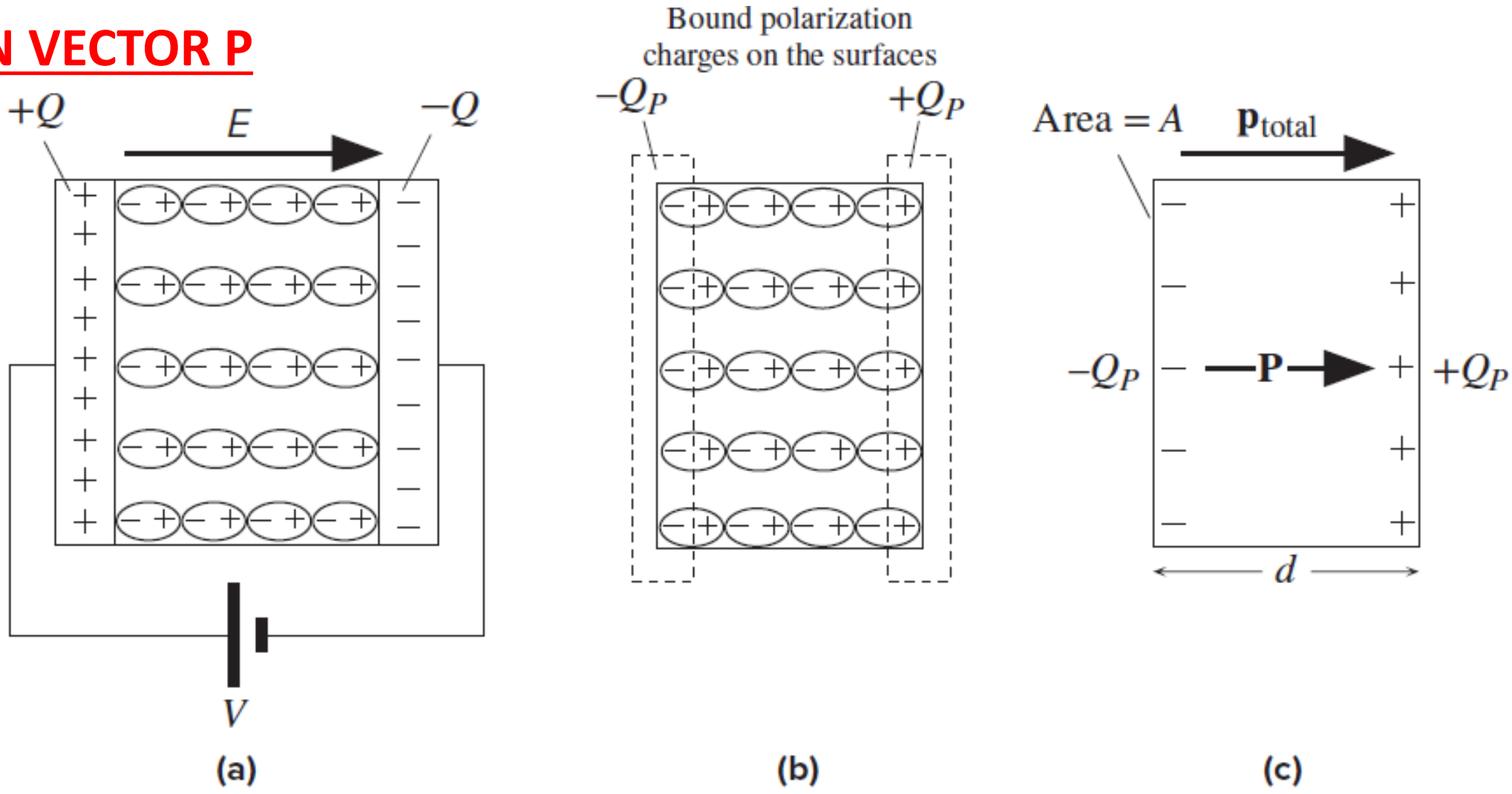


Figure 7.5 (a) When a dielectric is placed in an electric field, bound polarization charges appear on the opposite surfaces. (b) The origin of these polarization charges is the polarization of the molecules of the medium. (c) We can represent the whole dielectric in terms of its surface polarization charges $+Q_P$ and $-Q_P$.

- The induced dipole moments all point in the direction of the field as shown in Fig. 7.5a.
- Consider the polarized medium alone, as shown in Figure 7.5b. In the bulk of the material, the dipoles are aligned head to tail. There is therefore no net charge within the bulk.
- But the positive charges of the dipoles appearing at the right-hand face are not canceled by negative charges of any dipoles at this face. Therefore a surface charge $+Q_P$ on the right-hand face that results from the polarization of the medium.
- Similarly, there is a negative charge $-Q_P$ with the same magnitude appearing on the left-hand face due to the negative charges of the dipoles at this face.
- We see that charges $+Q_P$ and $-Q_P$ appear on the opposite surfaces of a material when it becomes polarized in an electric field, as shown in Figure 7.5c.
- These charges are **bound** and are a direct result of the polarization of the molecules. They are termed **surface polarization charges.**

Total dipole movement from $-Qp\ to\ +Qp$

$$p_{\text{total}} = Q_P d$$

Polarization (P)

$$P = \frac{p_{\text{total}}}{\text{Volume}} = \frac{Q_P d}{Ad} = \frac{Q_P}{A}$$

Surface polarization charge density,

$$\sigma_P = \frac{Q_P}{A}$$

So,

$$P_{normal} = \sigma_P$$

To express the dependence of P on the field E, we define a quantity called the **electric susceptibility** χ_e by

$$P = \chi_e \varepsilon_0 E$$

We know, $p_{induced} = \alpha_e E$

$$P = N p_{induced} = N\alpha_e E$$

where 'N' is the number of molecules per unit volume

From above two equations,
Electric susceptibility and polarization

$$\chi_e = \frac{N\alpha_e}{\varepsilon_0}$$

It is important to recognize the difference between *free* and *polarization* (or *bound*) charges.

The charges stored on the metal plates in Figure 7.5a are free because they result from the motion of free electrons in the metal.

For example both Q_o and Q, before and after the dielectric insertion in Figure 7.1, are free charges that arrive on the plates from the battery.

The polarization charges $+Q_P$ and $-Q_P$, on the other hand, are bound to the molecules. They cannot move within the dielectric or on its surface.

The free surface without any dielectric medium between the plates (Figure 7.1a) , the field E is given by

$$E = \frac{V}{d} = \frac{Q_o}{C_o d} = \frac{Q_o}{\varepsilon_o A} = \frac{\sigma_o}{\varepsilon_o}$$

The total charge $Q = Q_0 + Q_P$ and dividing by A

$$\frac{Q}{A} = \frac{Q_0}{A} + \frac{Q_P}{A}$$

$$\sigma = E \times \varepsilon_0 + \sigma_P \qquad \text{From, } \sigma_P = P = \chi_e \varepsilon_0 E$$

$$\sigma = E \times \varepsilon_0 + \chi_e \varepsilon_0 E$$

$$\sigma = \varepsilon_0 (1 + \chi_e) E$$

The relative permeability

$$\varepsilon_r = \frac{Q}{Q_0} = \frac{\sigma}{\sigma_0}$$

$$\varepsilon_r = (1 + \chi_e)$$

$$\varepsilon_r = 1 + \frac{N\alpha_e}{\varepsilon_0}$$

It relates the microscopic polarization mechanism that determines α_e to the macroscopic property ε_r.

LOCAL FIELD E_{loc} AND CLAUSIUS–MOSSOTTI EQUATION

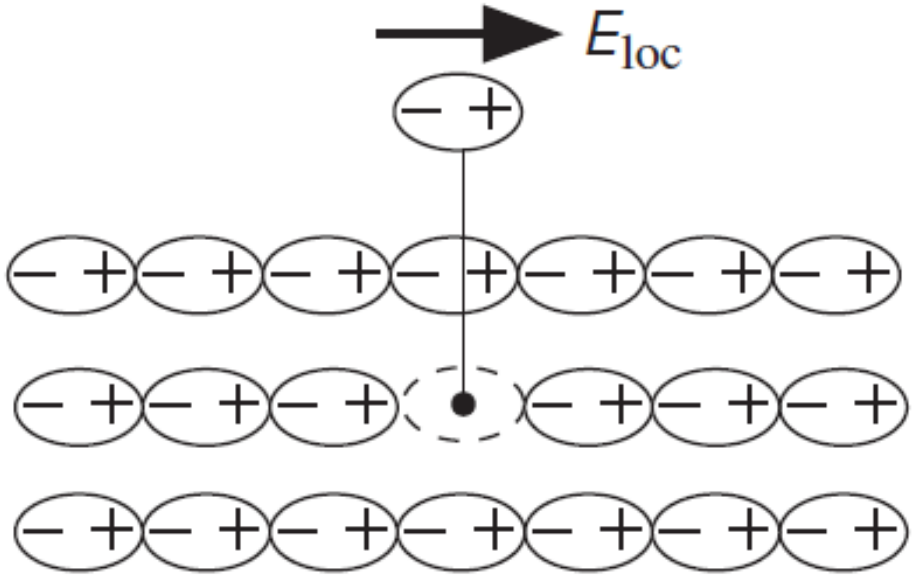
Previously, it assumes that the field acting on an individual atom or molecule is the field E, which is assumed to be uniform within the dielectric. In other words, the induced polarization, $p_{induced} \propto E$.

The field experienced by an individual molecule is actually different than E, which represents the average field in the dielectric. As soon as the dielectric becomes polarized, the field at some arbitrary point depends not only on the charges on the plates (Q) but also on the orientations of all the other dipoles around this point in the dielectric.

The actual field experienced by a molecule in a dielectric is defined as the **local field** and denoted by E_{loc}. It depends not only on the free charges on the plates but also on the arrangement of all the polarized molecules around this point.

In evaluating E_{loc} we simply remove the molecule from this point and calculate the field at this point coming from all sources, including neighboring polarized molecules, as visualized in Figure 7.7.

Lorentz field $$E_{\text{loc}} = E + \frac{1}{3\varepsilon_o}P$$



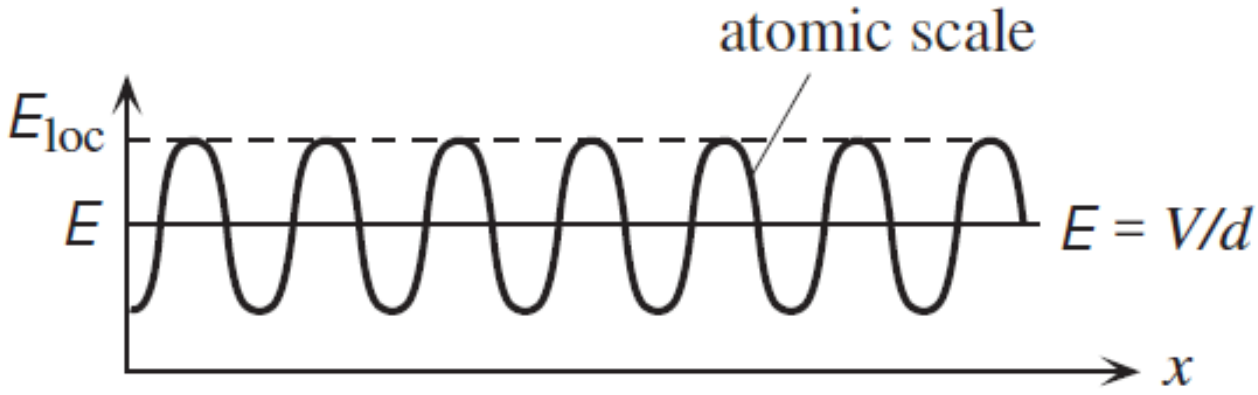


Figure 7.7 The electric field inside a polarized dielectric at the atomic scale is not uniform.

The local field is the actual field that acts on a molecule. It can be calculated by removing that molecule and evaluating the field at that point from the charges on the plates and the dipoles surrounding the point.

$$E_{loc} = E + \frac{1}{3\varepsilon_0} P \quad \ldots\ldots Eq.(1)$$

$$p_{induced} = \alpha_e E_{local}$$

$$P = N p_{induced} = N\alpha_e E_{loc} \quad \ldots Eq.(2)$$

$$P = \chi_e \varepsilon_0 E$$

$$P = (\varepsilon_r - 1)\varepsilon_0 E$$

$$E = \frac{P}{(\varepsilon_r - 1)\varepsilon_0} \quad \ldots Eq.(3)$$

Go through Example 7.2 in Kasap Textbook

Substitute (3) in (1)

$$E_{loc} = P\frac{2\varepsilon_0 + \varepsilon}{3\varepsilon_0(\varepsilon - \varepsilon_0)} \ldots\ldots Eq.(4)$$

Substitute (4) in (2)

$$P = N\alpha_e P \frac{2\varepsilon_0 + \varepsilon}{3\varepsilon_0(\varepsilon - \varepsilon_0)}$$

After simplification

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0}$$

$$\varepsilon_r = \frac{1 + \frac{2N\alpha_e}{3\varepsilon_o}}{1 - \frac{N\alpha_e}{3\varepsilon_o}}$$

This is the **Clausius–Mossotti equation**

ELECTRONIC POLARIZATION: COVALENT SOLIDS

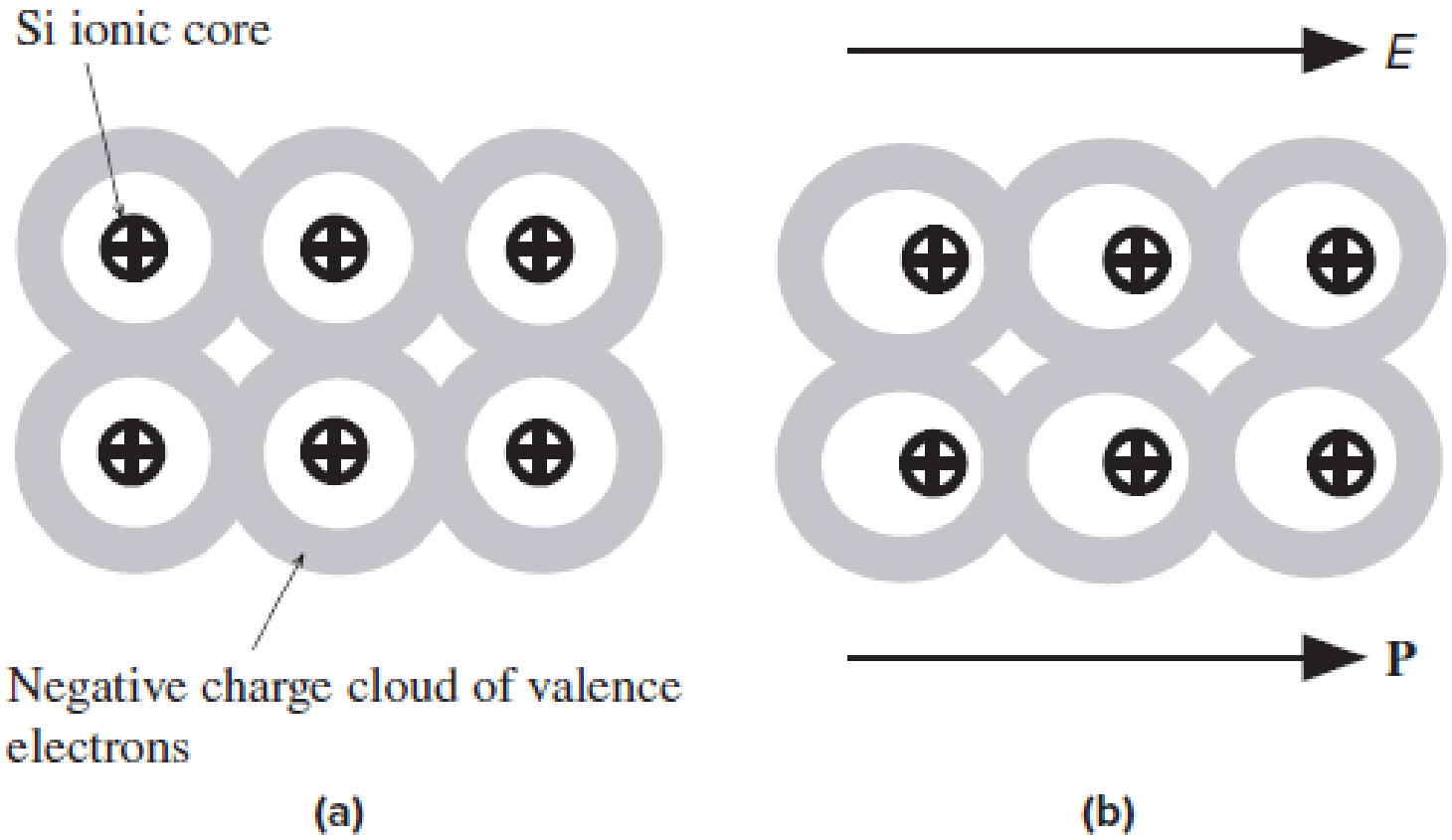


Figure 7.8 (a) Valence electrons in covalent bonds in the absence of an applied field. (b) When an electric field is applied to a covalent solid, the valence electrons in the covalent bonds are shifted very easily with respect to the positive ionic cores. The whole solid becomes polarized due to the collective shift in the negative charge distribution of the valence electrons.

The electron clouds within each atom become shifted by the field, and this gives rise to **electronic polarization.** This type of electronic polarization within an atom, however, is quite small compared with the polarization due to the valence electrons in the covalent bonds within the solid.

- In the covalent solid, the valence electrons are not rigidly tied to the ionic cores left in the Si atoms.
- When an electric field is applied, the negative charge distribution associated with these valence electrons becomes readily shifted with respect to the positive charges of the ionic Si cores, as depicted in Figure 7.8b and the crystal exhibits polarization, or develops a polarization vector.
- Thus, the valence electrons in the bonds readily respond to an applied field and become displaced.
- This type of electronic polarization, due to the displacement of electrons in covalent bonds, is responsible for the large dielectric constants of covalent crystals. **For example $\varepsilon_r = 11.9$ for the Si crystal and $\varepsilon_r = 16$ for the Ge crystal.**

Go through Example 7.3 in Kasap Textbook

POLARIZATION MECHANISMS

In addition to electronic polarization, we can identify a number of other polarization mechanisms that may also contribute to the relative permittivity.

1. IONIC POLARIZATION

- This type of polarization occurs in ionic crystals such as NaCl, KCl, and LiBr.
- The ionic crystal has distinctly identifiable ions, for example, Na+ and Cl-, located at well-defined lattice sites, so each pair of oppositely charged neighboring ions has a dipole moment.
- **In the absence of an applied field**, the solid has no net polarization because the dipole moments of equal magnitude are lined up head to head and tail to tail so that the net dipole moment is zero.
- The dipole moment p+ in the positive x direction has the same magnitude as p− in the negative x direction, so the net dipole moment is zero.

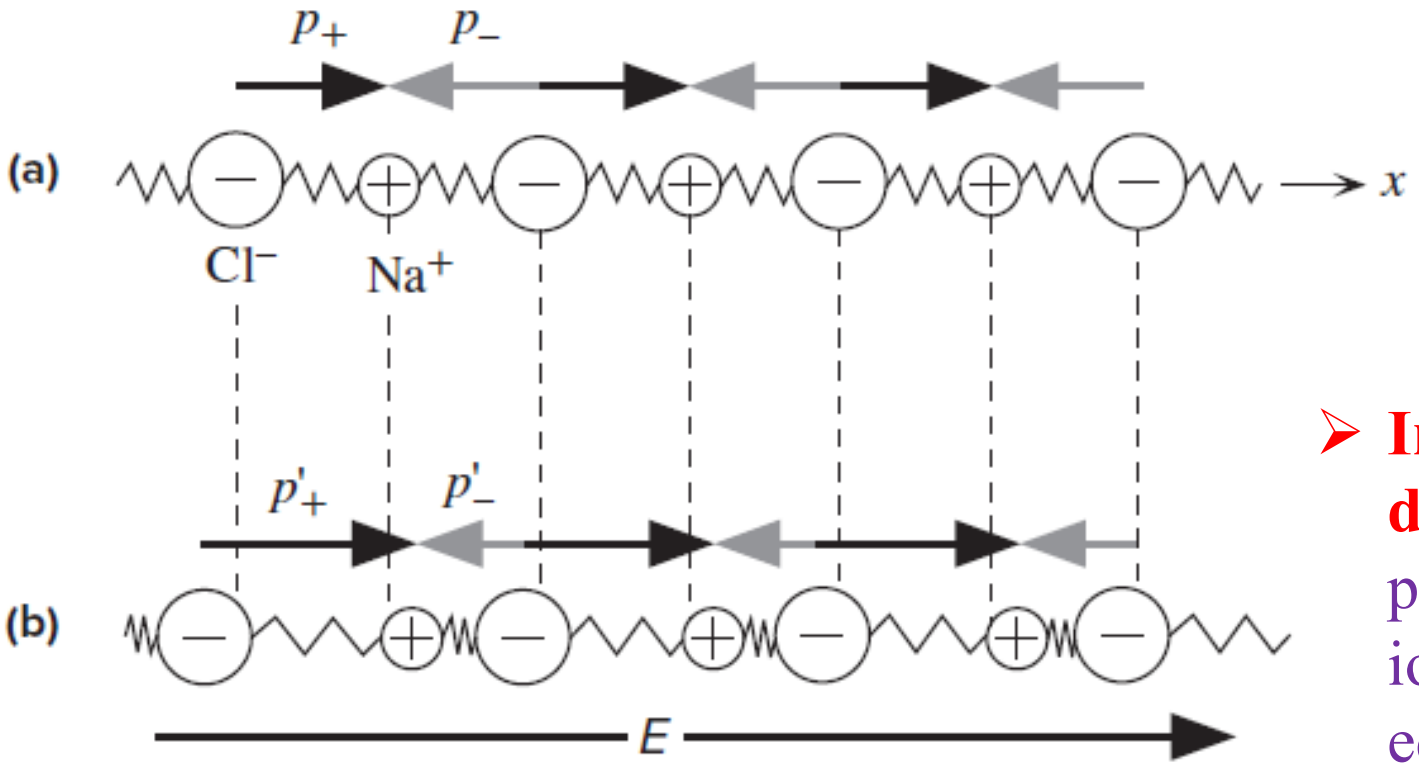


Figure 7.9 (a) A NaCl chain in the NaCl crystal without an applied field. Average or net dipole moment per ion is zero. (b) In the presence of an applied field, the ions become slightly displaced, which leads to a net average dipole moment per ion.

- **In the presence of a field E along the *x* direction**, however, the Cl- ions are pushed in the $-x$ direction and the Na+ ions in the $+x$ direction about their equilibrium positions.

- Consequently, the dipole moment p+ in the +x direction increases to p′+ and the dipole moment p− decreases to p′−, as shown in Figure 7.9b. The net dipole moment is now no longer zero. The **net dipole moment, or the average dipole moment, per ion pair is now (p′+)−(p′−), which depends on the electric field E**.

- Thus the induced average dipole moment per ion pair p_{av} depends on the field E. The ionic polarizability α_i is defined in terms of the local field experienced by the ions,

Ionic polarizability $$p_{av} = \alpha_i E_{loc}$$

- Generally, α_i is larger than the electronic polarizability α_e by a factor of 10 or more, which leads to ionic solids having large dielectric constants. The polarization P exhibited by the ionic solid is therefore given by

$$P = N p_{av} = N_i \alpha_i E_{loc}$$ By relating the local field to E and using $$P = (\varepsilon_r - 1)\varepsilon_0 E$$

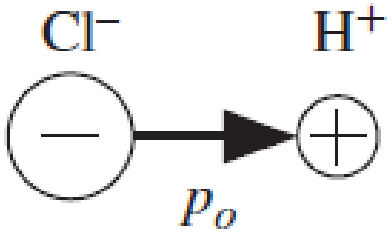
where N_i is the number of ion pairs per unit volume.

- Clausius–Mossotti equation, but now due to ionic polarization

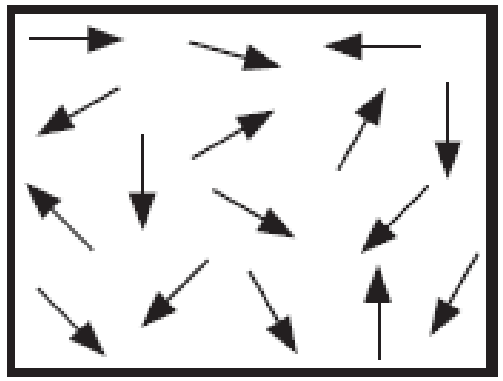
$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N_i \alpha_i}{3\varepsilon_0}$$

- Each ion also has a core of electrons that become displaced in the presence of an applied field with respect to their positive nuclei and therefore also contribute to the polarization of the solid. This electronic polarization simply adds to the ionic polarization. Its magnitude is invariably much smaller than the ionic contribution in these solids.

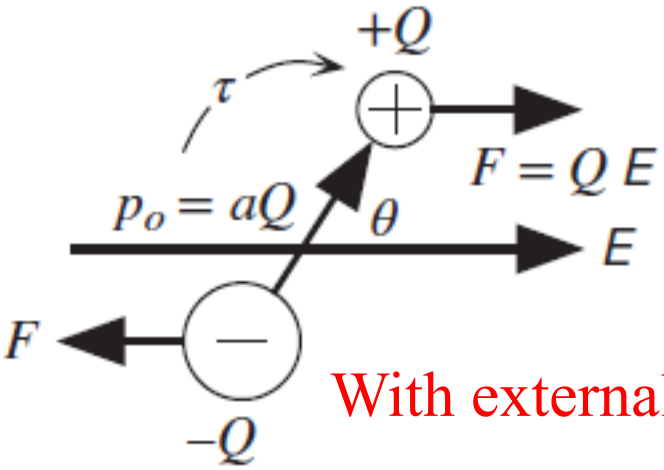
2. DIPOLAR (ORIENTATIONAL) POLARIZATION



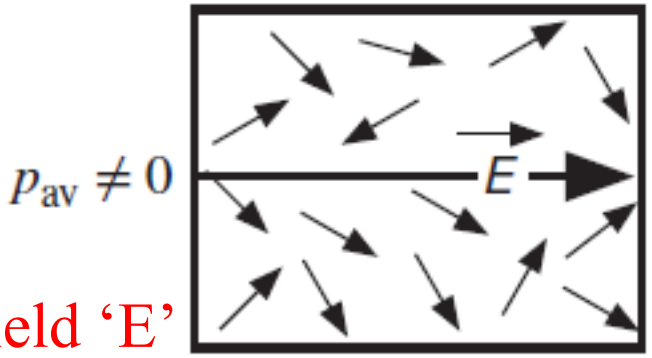
Without external field 'E'



(a) (b)



With external field 'E'



(c) (d)

Figure 7.10 (a) A HCl molecule possesses a permanent dipole moment p_o. (b) In the absence of a field, thermal agitation of the molecules results in zero net average dipole moment per molecule. (c) A dipole such as HCl placed in a field experiences a torque that tries to rotate it to align p_o with the field E. (d) In the presence of an applied field, the dipoles try to rotate to align with the field against thermal agitation. There is now a net average dipole moment per molecule along the field.

- Certain molecules possess permanent dipole moments. For example, the HCl molecule shown in Figure 7.10a has a permanent dipole moment p_o from the Cl- ion to the H+ ion.
- In the liquid or gas phases, these molecules, in the absence of an electric field, are randomly oriented as a result of thermal agitation, as shown in Figure 7.10b.
- **When an electric field E is applied**, E tries to align the dipoles parallel to itself, as depicted in Figure 7.10c. The Cl− and H+ charges experience forces in opposite directions. But the nearly rigid bond between Cl− and H+ holds them together, which means that the molecule experiences a torque τ about its center of mass. This torque acts to rotate the molecule to align p_o with E.
- The polarization would be $P = Np_0$, Thus the material exhibits net polarization, which leads to a dielectric constant that is determined by this orientational polarization.

where, N is the number of molecules per unit volume

Average dipole moment in Orientational polarization

$$p_{av} = \frac{1}{3}\frac{p_0^2\, E}{kT}$$

Dipolar orientational polarizability

$$\alpha_d = \frac{1}{3}\frac{p_0^2}{kT}$$

For proof of above, refer to Kasap book Section 7.3.2

- It is apparent that, in contrast to the electronic and ionic polarization, dipolar orientational polarization is strongly temperature dependent.
- α_d decreases with temperature, which means that the relative permittivity ε_r also decreases with temperature.
- We cannot use the simple Lorentz local field approximation for dipolar dielectrics. So, the Clausius–Mossotti equation does not work with dipolar dielectrics and the calculation of the local field is quite complicated.

3. INTERFACIAL POLARIZATION

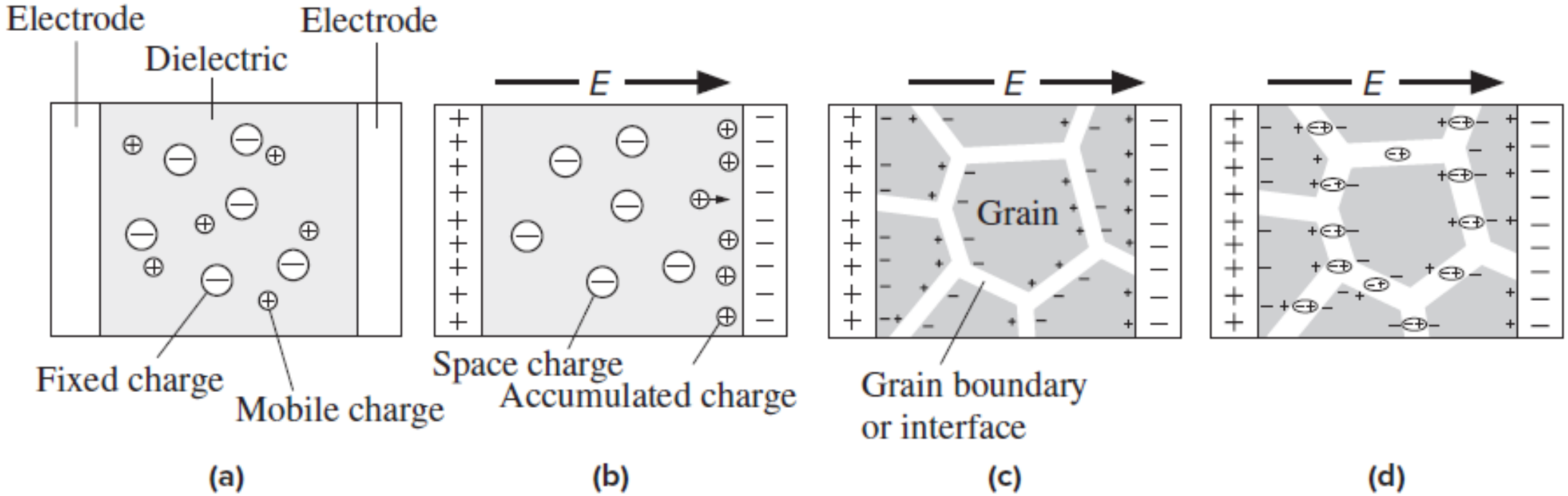


Figure 7.11 (a) A crystal with equal number of mobile positive ions and fixed negative ions. In the absence of a field, there is no net separation between all the positive charges and all the negative charges. (b) In the presence of an applied field, the mobile positive ions migrate toward the negative electrode and accumulate there. There is now an overall separation between the negative charges and positive charges in the dielectric. The dielectric therefore exhibits interfacial polarization. (c) Grain boundaries and interfaces between different materials frequently give rise to interfacial polarization. In this simple example, electrons and holes within grains drift and become trapped at the grain boundaries. (d) Positive and negative ions within a grain boundary can jump to neighboring vacant sites, aided by the field, and thereby form dipoles within the grain boundary.

- **Interfacial polarization** occurs whenever there is an accumulation of charge at an interface between two materials or between two regions within a material.
- The simplest example is interfacial polarization due to the accumulation of charges in the dielectric near one of the electrodes, as depicted in Figure 7.11a and b.
- The term interfacial polarization arises because the positive charges accumulating at the interface and the remainder of negative charges in the bulk together constitute dipole moments that appear in the polarization vector P.

TOTAL POLARIZATION

- In the presence of electronic, ionic, and dipolar polarization mechanisms, the average induced dipole moment per molecule will be the sum of all the contributions in terms of the local field,

$$p_{\text{av}} = \alpha_e E_{\text{loc}} + \alpha_i E_{\text{loc}} + \alpha_d E_{\text{loc}}$$

- The dielectric constant ε_r under **electronic** and **ionic polarizations,** however, can be obtained from *Clausius–Mossotti equation*

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3\varepsilon_o}(N_e\alpha_e + N_i\alpha_i)$$

Go through Example 7.4 in Kasap Textbook

FREQUENCY DEPENDENCE: DIELECTRIC CONSTANT AND DIELECTRIC LOSS

- The static dielectric constant is an effect of polarization under DC conditions.
- When the applied field, is a sinusoidal signal, then the polarization of the medium under these AC conditions leads to an AC dielectric constant that is generally different than the static case.
- The sinusoidally varying field changes magnitude and direction continuously, and it tries to line up the dipoles one way and then the other way and so on.
- If the instantaneous induced dipole moment 'p' per molecule can instantaneously follow the field variations, then any instant. $\boldsymbol{p} = \boldsymbol{\alpha_d}\,\mathbf{E}$
- The polarizability $\boldsymbol{\alpha_d}$ has its expected maximum value from DC conditions.

$$\alpha_d = \frac{1}{3}\,\frac{p_0^2}{kT}$$

- There are two factors opposing the immediate alignment of the dipoles with the field.
- First is that thermal agitation tries to randomize the dipole orientations.
- Second, the molecules rotate in a viscous medium by virtue of their interactions with neighbors.
- If the field changes too rapidly, then the dipoles cannot follow the field and, as a consequence, remain randomly oriented.
- At high frequencies, $\boldsymbol{\alpha_d}$ will be zero as the field cannot induce a dipole moment. At low frequencies, of course, the dipoles can respond rapidly to follow the field and $\boldsymbol{\alpha_d}$ has its maximum value.
- It is clear that $\boldsymbol{\alpha_d}$ changes from its maximum value to zero as the frequency of the field is increased.

- Corresponding to DC conditions, the applied field across the dipolar gaseous medium is suddenly decreased from E*o* to E at a time we define as zero, as shown in Figure 7.12.
- The field E is smaller than E*o*, so the induced DC dipole moment per molecule should be smaller and given by $\boldsymbol{\alpha_d(0)E}$ where $\boldsymbol{\alpha_d(0)}$ is $\boldsymbol{\alpha_d}$ at $\omega = 0$, DC conditions.
- Therefore, the induced dipole moment per molecule has to decrease, or ***relax***, from $\boldsymbol{\alpha_d(0)E_o}$ to $\boldsymbol{\alpha_d(0)E}$.

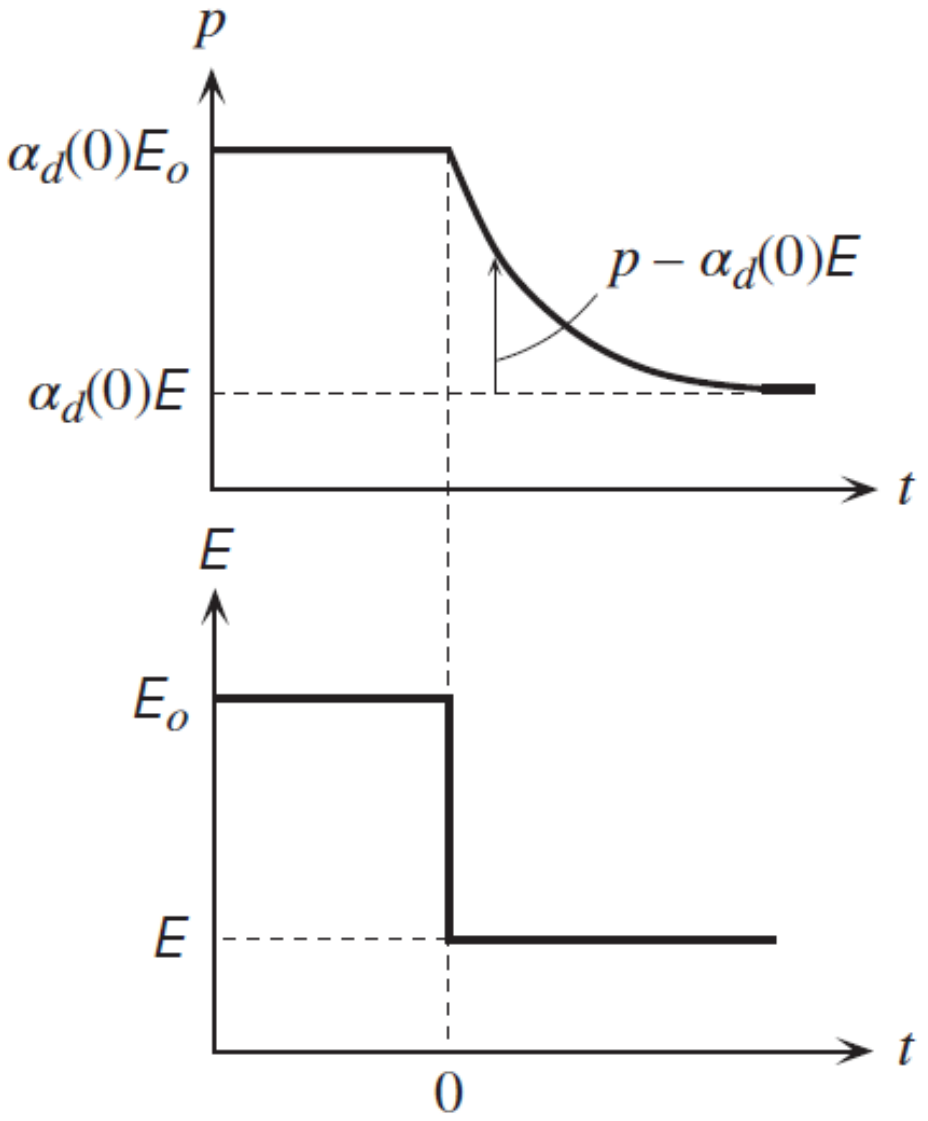


Figure 7.12 The applied dc field is suddenly changed from E_o to E at time $t = 0$.

The induced dipole moment p has to decrease from $\alpha_d(0)E_o$ to a final value of $\alpha_d(0)E$. The decrease is achieved by random collisions of molecules in the gas.

- In a gas medium the molecules would be moving around randomly and their collisions with each other and the walls of the container randomize the induced dipole per molecule.
- Thus the decrease, or the **relaxation process,** in the induced dipole moment is archived by random collisions.
- Assuming that τ is the average time, called the relaxation time, between molecular collisions, then this is the mean time it takes per molecule to randomize the induced dipole moment.
- If $\boldsymbol{p}$ is the instantaneous induced dipole moment, then $\boldsymbol{p} - \boldsymbol{\alpha_d(0)E}$ is the *excess* dipole moment, which must eventually disappear to zero through random collisions as $\boldsymbol{t} \rightarrow \infty$. It would take an average $\boldsymbol{\tau}$ seconds to eliminate the excess dipole moment $\boldsymbol{p} - \boldsymbol{\alpha_d(0)E}$.

The rate at which the induced dipole moment is changing is then $-[p - \alpha_d(0)E]/\tau$, where the negative sign represents a decrease.

$$\frac{dp}{dt} = -\frac{p - \alpha_d(0)E}{\tau}$$

Dipole relaxation equation

The above Eq. can be used to obtain the dipolar polarizability under AC conditions. For an AC field, we would write $E = E_0 \sin(wt) = E_0 \exp(jwt)$

Substitute AC 'E' field in above Eq. $$\frac{dp}{dt} = -\frac{p}{\tau} + \frac{\alpha_d(0)}{\tau} E_o \exp(j\omega t)$$

By integrating on both sides $$p = \alpha_d(\omega) E_o \exp(j\omega t)$$

$$\alpha_d(\omega) = \frac{\alpha_d(0)}{1 + j\omega\tau}$$

Orientational polarizability and frequency

- The above equation represents the orientational polarizability under AC field.
- Polarizability $\boldsymbol{\alpha_d(\omega)}$ is a complex number that indicates that p and E are out of phase.
- Put differently, if N is the number of molecules per unit volume, $P = Np$ and E are out of phase, as indicated in Figure 7.13a.
- At low frequencies, $\omega\tau \ll 1$, $\boldsymbol{\alpha_d(\omega)}$ is nearly $\boldsymbol{\alpha_d(0)}$, and p is in phase with E. The rate of relaxation $1/\tau$ is much faster than the frequency of the field or the rate at which the polarization is being changed; **p then closely follows E**.
- At very high frequencies, $\omega\tau \gg 1$, the rate of relaxation $1/\tau$ is much slower than the frequency of the field and **p can no longer follow the variations in the field**.

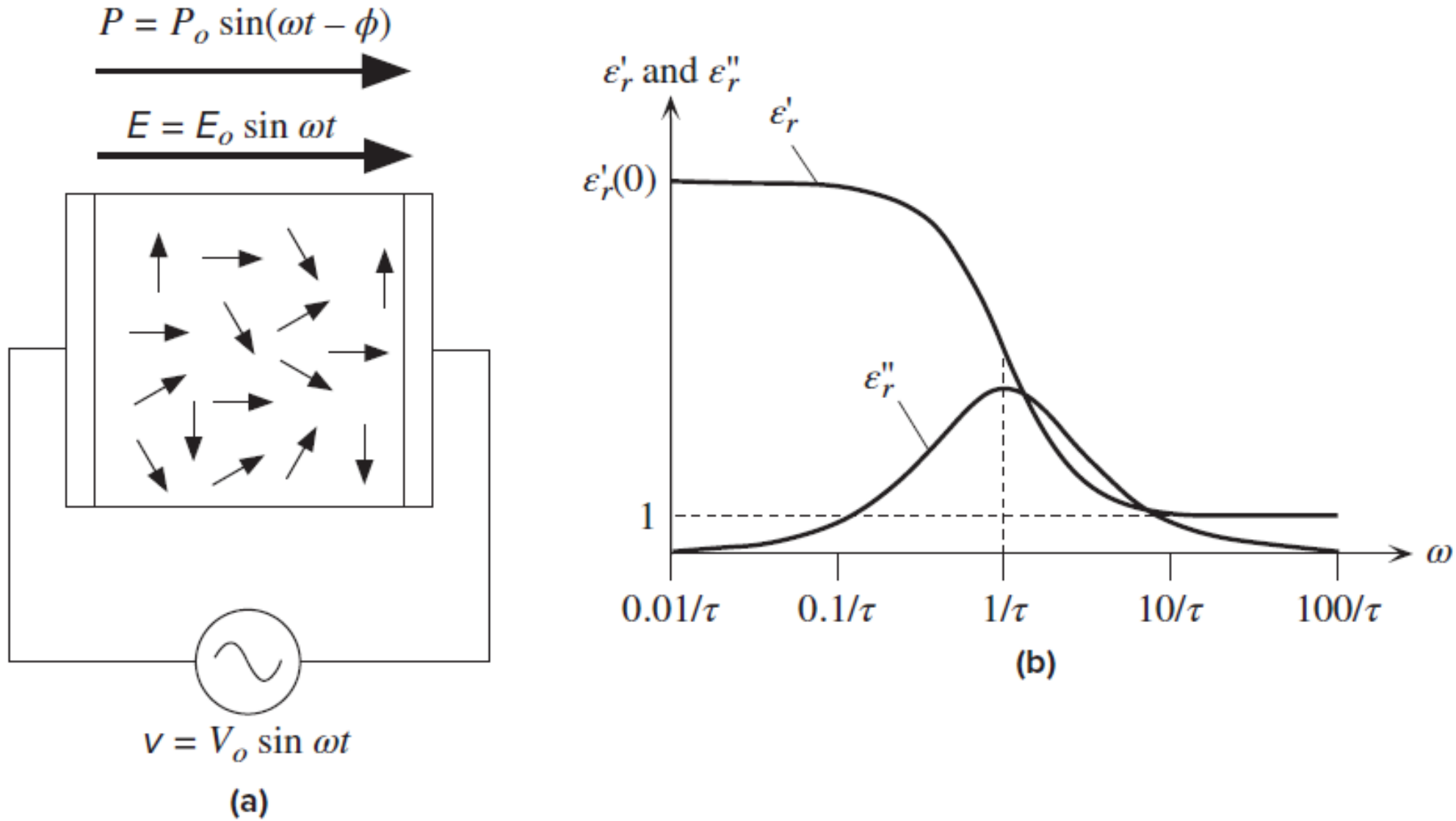


Figure 7.13 (a) An ac field is applied to a dipolar medium. The polarization $P(P = Np)$ is out of phase with the ac field. (b) The relative permittivity is a complex number with real (ε_r') and imaginary (ε_r'') parts that exhibit relaxation at $\omega \approx 1/\tau$.

- The dielectric constant ε_r can be obtained by substituting $\boldsymbol{\alpha_d(\omega)}$, which is a complex number since $\boldsymbol{\alpha_d(\omega)}$ is a complex number.

$$\varepsilon_r = 1 + \frac{N\alpha_d(w)}{\varepsilon_0}$$

$$\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \qquad \textit{Complex relative permittivity}$$

- where ε_r' is the real part and ε_r'' is the imaginary part, both being frequency dependent, as shown in Figure 7.13b.
- The real part ε_r' decreases from its maximum value $\varepsilon_r'(0)$, corresponding to $\boldsymbol{\alpha_d(0)}$, to 1 at high frequencies when $\boldsymbol{\alpha_d = 0}$, as $\omega \rightarrow \infty$ in Equation 7.26.
- The imaginary part $\varepsilon_r''(w)$ is zero at low and high frequencies but peaks when $\omega\tau = 1$ or when $\omega = 1/\tau$.
- The real part ε_r' represents the relative permittivity that we would use in calculating the capacitance, as for example in $C = (\varepsilon_r\varepsilon_0 A)/d$.

- The imaginary part $\varepsilon_r''(w)$ represents the energy lost in the dielectric medium as the dipoles are oriented against random collisions one way and then the other way and so on by the field.

Consider the capacitor in Figure 7.14, which has this dielectric medium between the plates.

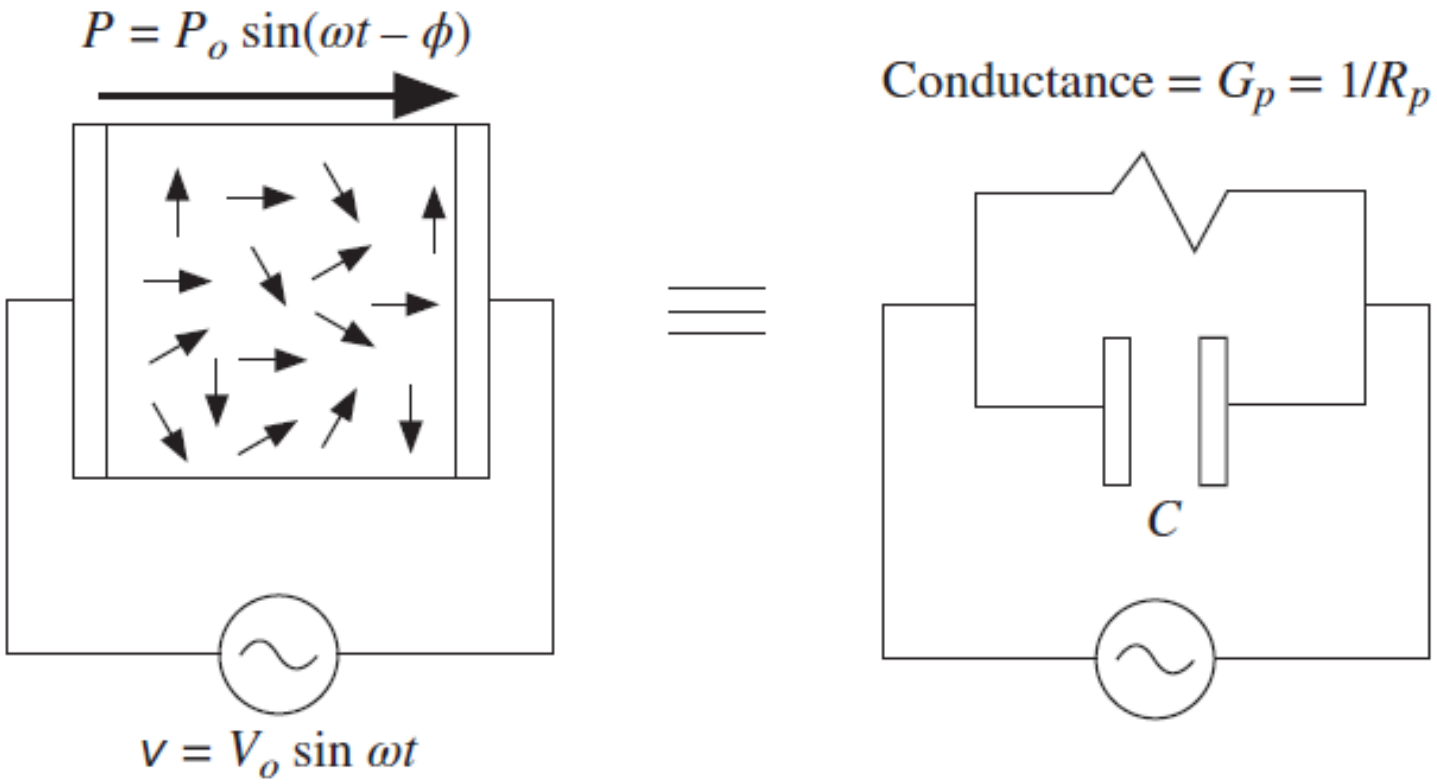


Figure 7.14 The dielectric medium behaves like an ideal (lossless) capacitor of capacitance C, which is in parallel with a conductance G_p.

Then the admittance Y, i.e., the reciprocal of impedance of this capacitor, with ε_r given in Equation 7.27 is

Substitute $\varepsilon_r(w) = \varepsilon_r'(w) - j\varepsilon_r''(w)$

Admittance of a parallel Plate capacitor

$$Y = \frac{jwA\varepsilon_0\varepsilon_r(w)}{d} = \frac{jwA\varepsilon_0\varepsilon_r'(w)}{d} + \frac{wA\varepsilon_0\varepsilon_r''(w)}{d}$$

$$Y = jwC + G_P$$

Equivalent ideal capacitance

$$C = \frac{A\varepsilon_0\varepsilon_r'}{d}$$

Equivalent parallel conductance

$$G_P = \frac{wA\varepsilon_0\varepsilon_r''}{d}$$

Input Power $$IV = YV^2 = jwCV^2 + \frac{V^2}{R_P}$$

- Thus the power dissipated in the dielectric medium is related to ε_r'' and peaks when $\omega = 1/\tau$.
- The rate of energy storage by the field is determined by ω, whereas the rate of energy transfer to molecular collisions is determined by $1/\tau$.
- When $\omega = 1/\tau$, the two processes, energy storage by the field and energy transfer to random collisions, **are then occurring at the same rate**, and hence energy is being transferred to heat most efficiently.
- The peak in ε_r'' versus ω is called a relaxation peak, which is at a frequency when the dipole relaxations are at the right rate for maximum power dissipation. This process is known as dielectric resonance.

In engineering applications of dielectrics in capacitors, we would like to minimize ε_r'' for a given ε_r'. We define the relative magnitude of ε_r'' with respect to ε_r' through a quantity, tan δ, called the **loss tangent** (or **loss factor**), as

Loss tangent

$$\tan\delta = \frac{\varepsilon_r''}{\varepsilon_r'}$$

We can easily find the **Dielectric loss per unit volume in the medium**. The resistance R_P represents the dielectric loss.

$$W_{\text{vol}} = \frac{\text{Power loss}}{\text{Volume}} = \frac{V^2}{R_P} \times \frac{1}{dA} = \frac{V^2}{\dfrac{d}{\omega A \varepsilon_o \varepsilon_r''}} \times \frac{1}{dA} = \frac{V^2}{d^2}\omega\varepsilon_o\varepsilon_r''$$

Using $E = V/d$ ⟶

$$W_{\text{vol}} = \omega E^2 \varepsilon_o \varepsilon_r' \tan\delta$$

We can represent the general features of the frequency dependence of the real and imaginary parts of the dielectric constant as in Figure 7.15.

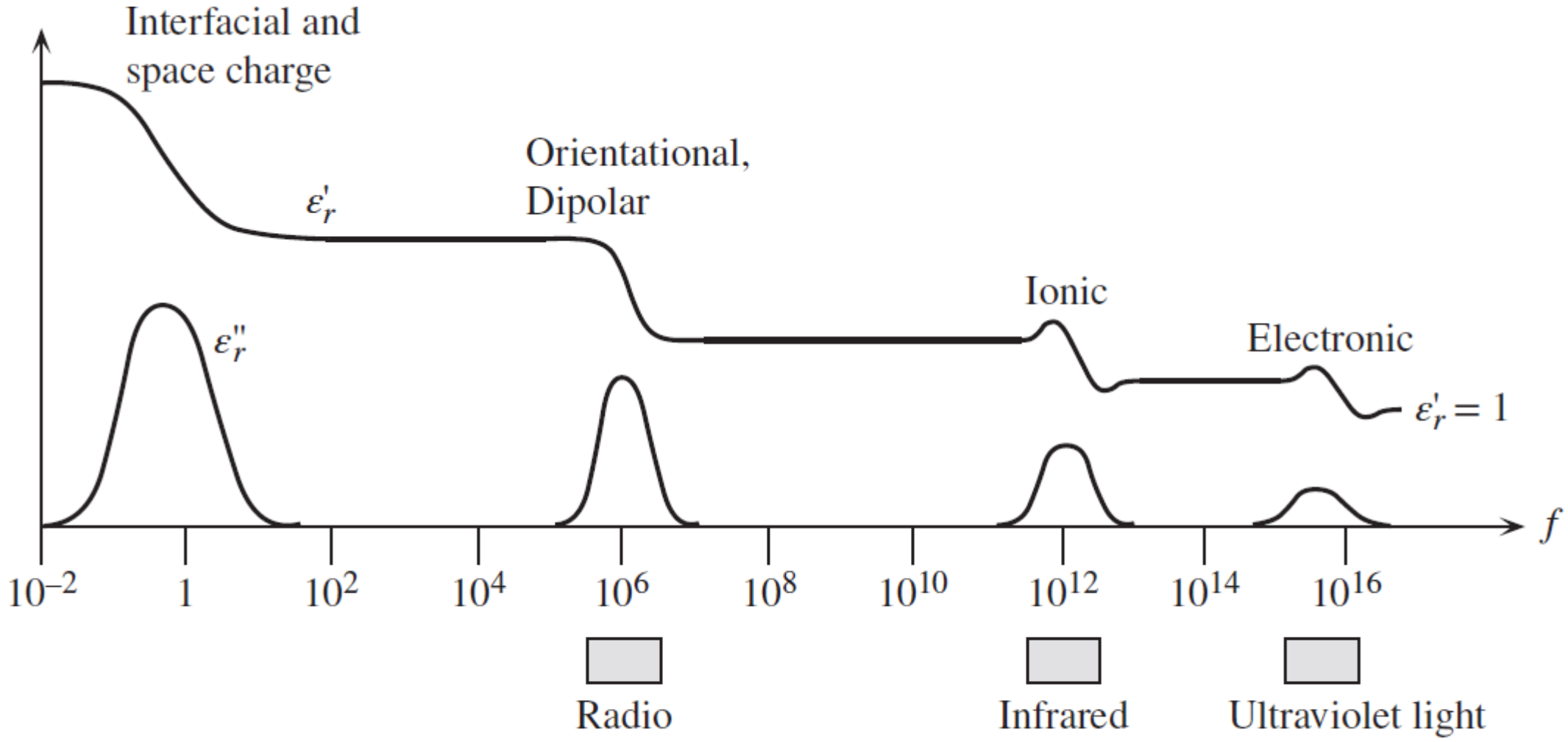
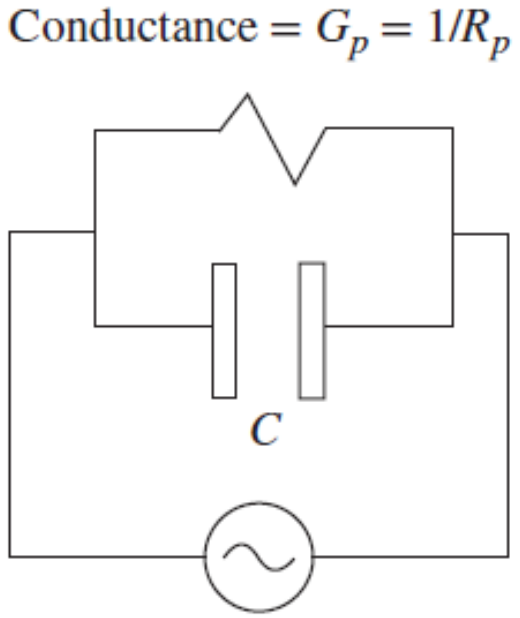


Figure 7.15 The frequency dependence of the real and imaginary parts of the dielectric constant in the presence of interfacial, orientational, ionic, and electronic polarization mechanisms.

Go through Example 7.5 in Kasap Textbook

DIELECTRIC LOSS PER UNIT CAPACITANCE AND THE LOSS ANGLE δ

The power loss in the capacitor is due to R_P. If V is the rms value of the voltage across the capacitor, then the power dissipated per unit capacitance W_{cap} is



$$W_{\text{cap}} = \frac{V^2}{R_P} \times \frac{1}{C} = V^2 \frac{\omega \varepsilon_o \varepsilon_r'' A}{d} \times \frac{d}{\varepsilon_o \varepsilon_r' A} = V^2 \frac{\omega \varepsilon_r''}{\varepsilon_r'}$$

$$W_{\text{cap}} = V^2 \omega \tan \delta$$

Consider the rms currents through R_P and C are I_{loss} and I_{cap} , respectively, and their ratio is given by

$$\frac{I_{\text{loss}}}{I_{\text{cap}}} = \frac{V}{R_P} \times \frac{1}{\frac{j\omega C}{V}} = \frac{\omega \varepsilon_o \varepsilon_r'' A}{d} \times \frac{d}{j\omega \varepsilon_o \varepsilon_r' A} = -j \tan \delta$$

As expected, the two are 90° out of phase $(-j)$ and the loss current (through R_P) is a factor tan δ, of the capacitive current (through C).

The ratio of I_{cap} and the total current, $I_{total} = I_{cap} + I_{loss}$, is

$$\frac{I_{\text{cap}}}{I_{\text{total}}} = \frac{I_{\text{cap}}}{I_{\text{cap}} + I_{\text{loss}}} = \frac{1}{1 + \frac{I_{\text{loss}}}{I_{\text{cap}}}} = \frac{1}{1 - j \tan \delta}$$

Go through Example 7.6 and 7.7 in Kasap Textbook


Thank You!