MODULE - II

AC Circuits

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PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS

Purely resistive a.c. circuit

Purely inductive a.c. circuit

Purely capacitive a.c. circuit

Purely Resistive AC Circuit

- The purely resistive ac circuit is as shown in fig. 1(a). It consists of an ac voltage source $v = V_m \sin \omega t$, and a resistor R connected across it.

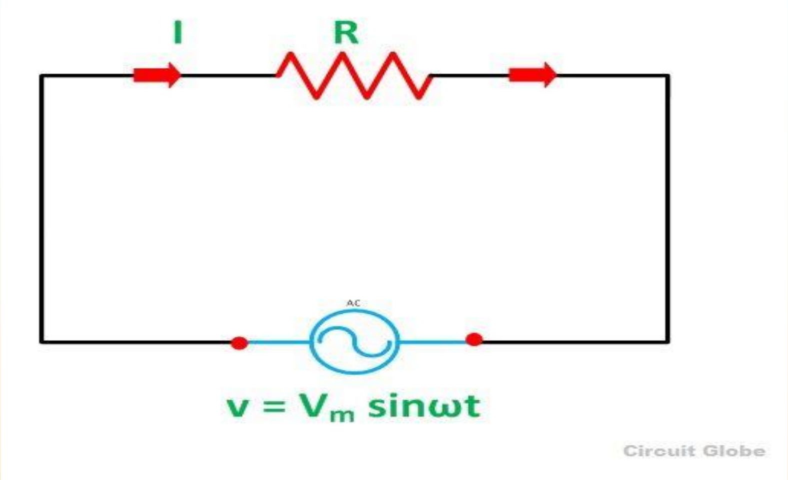


Fig. 1(a): Purely resistive ac circuit

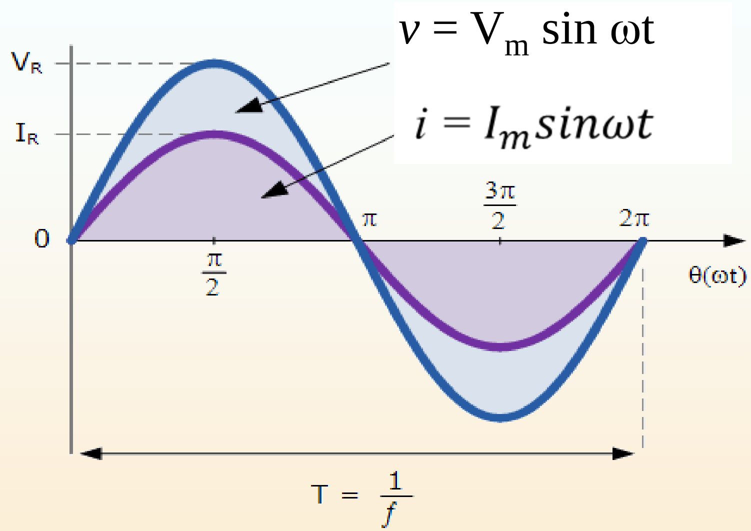


Fig. 1(b): Voltage and current waveform

Voltage-current relations for a resistor

- Referring to fig. 1(a), the instantaneous voltage across the resistor (v_R) is same as the source voltage.

$$\therefore v_R = v = V_m \sin \omega t \qquad .....(1)$$

- Applying the ohm's law the expression for the instantaneous current flowing through the resistor is given by,

$$i = \frac{V}{R} = \frac{V_m sin\omega t}{R}$$

$$or \qquad i = I_m sin\omega t \qquad .....(2)$$

Where

$$I_m = \frac{V_m}{R}$$

As seen from the equation (1) and (2), there is no phase difference between the voltage and current in case of resistive circuits

Continued….

- From current equation (2), we conclude that:
  1. The current flowing through a purely resistive ac circuit is sinusoidal.
  2. The current through the resistive circuit and the applied voltage are in phase with each other.

- **Phasor Diagram:**

- The phasor diagram for a purely resistive ac circuit is as shown in fig. 1(c).

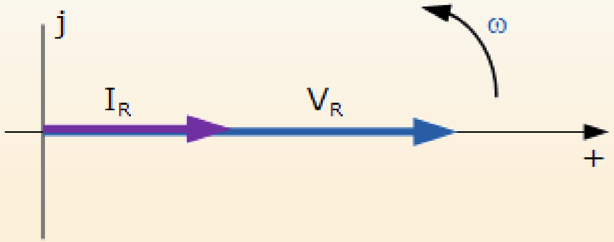
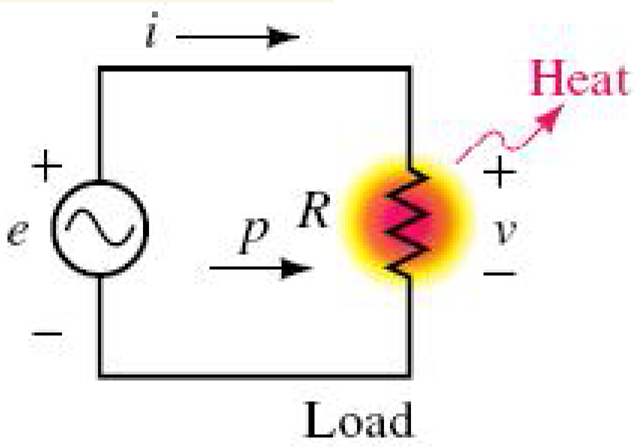


Fig. 1(c): phasor diagram

Power in a resistor

The instantaneous value of power in the resistance is given by the product of the instantaneous value of voltage and current in the resistance.

$$p = vi = (V_m \sin \omega t)(I_m \sin \omega t) = V_m I_m \sin^2 \omega t$$

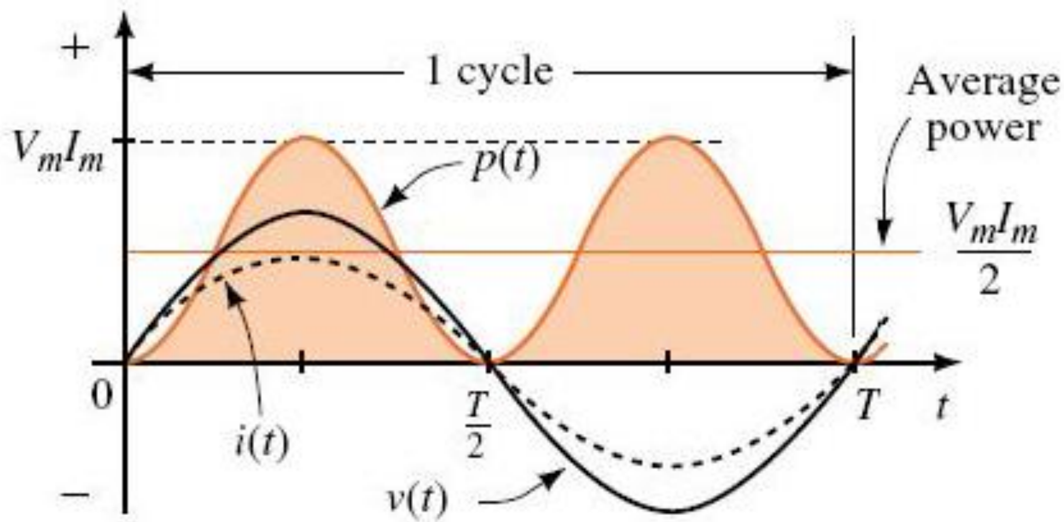


(a)

$$p = \frac{V_m I_m}{2}(1 - \cos 2\,\omega t)$$

$$P = V_m I_m / 2$$

$$P = VI \quad \text{(watts)}$$



(b)

Average or Real power

$$\therefore \text{ Power, } P = \frac{1}{\pi}\int_0^{\pi} p\, d\theta = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\theta\, d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{1 - \cos 2\theta}{2}\, d\theta$$

$$= \frac{V_m I_m}{2\pi}\int_0^{\pi} (1 - \cos 2\theta)\, d\theta$$

$$= \frac{V_m I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$= \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = VI$$

$$\therefore P = VI$$

Impedance of the purely resistive circuit

- The impedance Z is expressed in the rectangular form as:

$$Z = R + jX$$

where R is the resistive part while X is the reactive part.

- When the load is purely resistive, the reactive part is zero.

$$\therefore Z = R\ \Omega$$

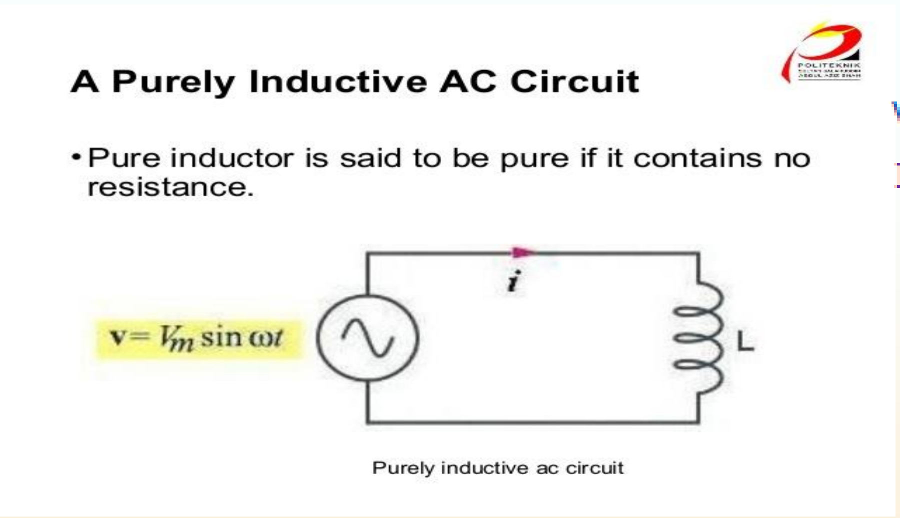
- In the polar form it is given by,

$$Z = R \angle 0^0\ \Omega$$

$$\mathbf{Z}_R = \frac{\mathbf{V}_R}{\mathbf{I}} = \frac{V_R\angle\theta}{I\angle\theta} = \frac{V_R}{I}\angle 0^\circ = R$$

Purely Inductive AC Circuit

- Fig. 2(a). shows a purely inductive ac circuit.
- The pure inductance has zero ohmic resistance. It is a coil with only pure inductance of L Henries (H).



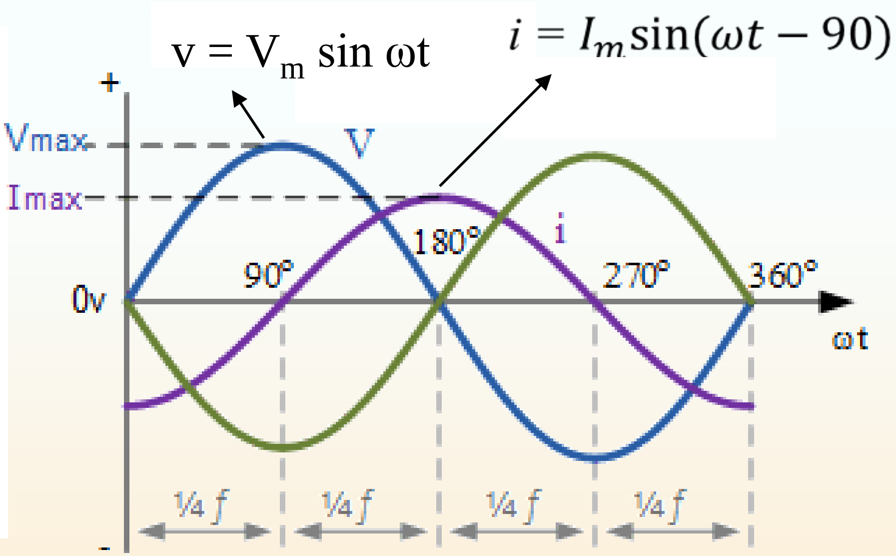


Fig. 2(a): A purely inductive ac circuit **Fig 2 (b): current and voltage waveform**

Voltage-current relations for a inductor

- Let the instantaneous voltage applied to the purely inductive ac circuit be given by,

$$v = V_m \sin \omega t \qquad \text{......(1)}$$

- As shown in fig. 2(b), the instantaneous current is given by,

$$v = L\frac{di}{dt} \qquad i = \frac{1}{L}\int v dt = \frac{Vm}{L}\int sin\omega t\, dt \quad = \frac{Vm}{\omega L}(-\cos\omega t)$$

$$= \frac{V_m}{\omega L}(-\sin(90 - \omega t))$$

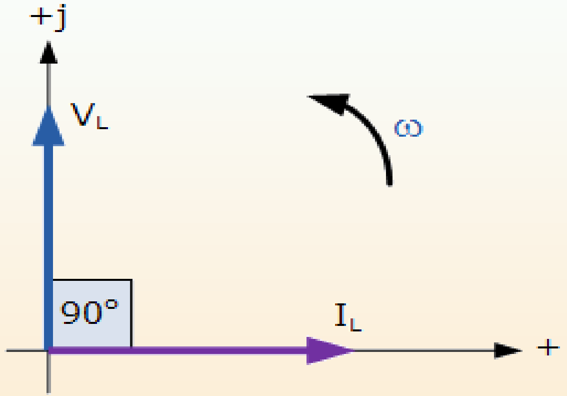
$$= \frac{V_m}{\omega L}\sin(\omega t - 90)$$

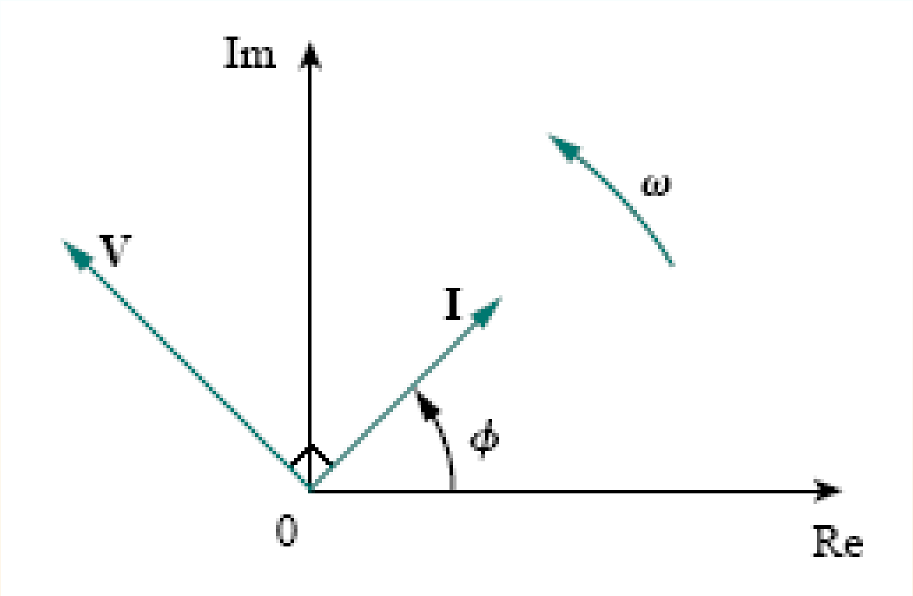
$$or \qquad i = I_m \sin(\omega t - 90) \qquad \text{.....(2)}$$

Where

$$I_m = \frac{V_m}{X_L} \ \ and\ X_L = \omega L \ = \text{impedance offered by pure inductance}$$

As seen from the equation (1) and (2), the current lags behind the voltage by 90 degrees in a pure inductive circuits





Phasor diagram for inductor.

Power in an inductor

The instantaneous value of power in the inductance is given by the product of the instantaneous value of voltage and current in the inductance.

Instantaneous power

$$p = v \times i$$

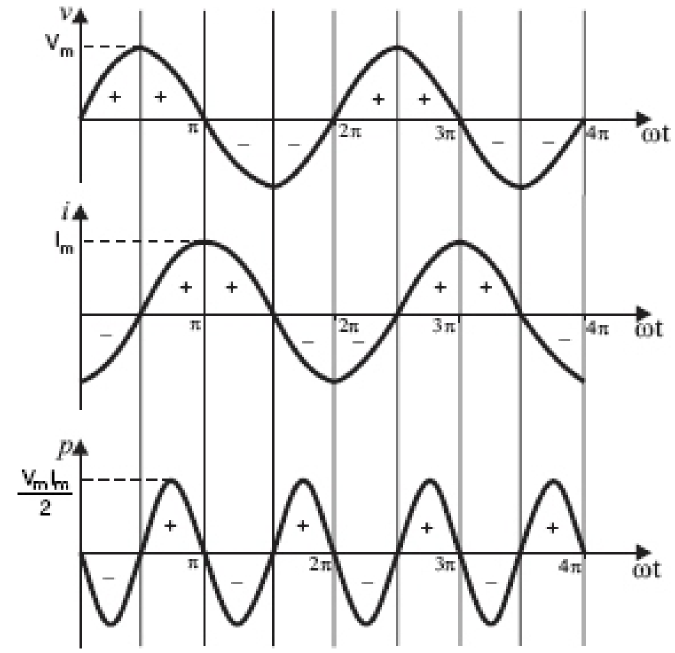
$$= V_m \sin \omega t \times I_m \sin(\omega t - 90^\circ)$$

$$= V_m I_m \sin \omega t\,(-\cos \omega t)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$\therefore \quad p = -V_m I_m \frac{\sin 2\omega t}{2}$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t = -\frac{V_m I_m}{2} \sin 2\theta$$



Waveform of voltage, current and power i

The instantaneous power is also a sinusoidal quantity whose frequency is double that of voltage or current

Average or Real power

$$\text{Power, P} = \frac{1}{\pi}\int_0^{\pi} p\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{-V_m I_m}{2} \sin 2\theta\, d\theta$$

$$= \frac{-V_m I_m}{2\pi}\int_0^{\pi} \sin 2\theta\, d\theta$$

$$= \frac{-V_m I_m}{2\pi}\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{-V_m I_m}{2\pi}\left[-\frac{\cos 2\pi}{2} + \frac{\cos 0}{2}\right]$$

$$= \frac{-V_m I_m}{2\pi}\left[-\frac{1}{2} + \frac{1}{2}\right] = 0$$

Alternatively, the expression for power can be obtained from complex power.

$$\text{Complex Power, } \bar{S} = \bar{V}\bar{I}^* = V\angle 0^\circ \times (I\angle -90^\circ)^*$$
$$= V\angle 0^\circ \times I\angle +90^\circ$$
$$= VI\angle 90^\circ$$

We know that, $|\bar{S}| = S = VI$ and $\angle \bar{S} = \phi = 90^\circ$

$$\therefore \text{Power, } P = S\cos\phi = VI\cos 90^\circ = 0$$

$$\text{Reactive Power, } Q = S\sin\phi = VI\sin 90^\circ = VI$$

The inductance consumes only reactive power and *"the active power in the pure inductance is zero."* The reactive power of inductance is positive which means that it absorbs reactive power.

Impedance of a purely inductive circuit:

- When circuit is purely inductive, the resistive part is zero i.e. R = 0.

$$\therefore Z = j\,X_L\ \Omega$$

- In polar form, it is given by,

$$Z = X_L \angle 90^0\ \Omega$$

$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}} = \frac{V_L\angle 0^\circ}{I\angle -90^\circ} = \frac{V_L}{I}\angle 90^\circ = \omega L\angle 90^\circ = j\omega L$$

Purely Capacitive AC Circuit

- The fig. 3(a) shows the purely capacitive AC circuit.
- A pure capacitor has its leakage resistance equal to infinity.

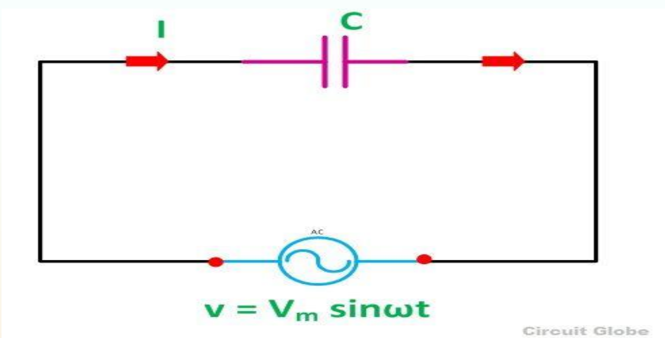


Fig.3(a): A purely Capacitive Circuit

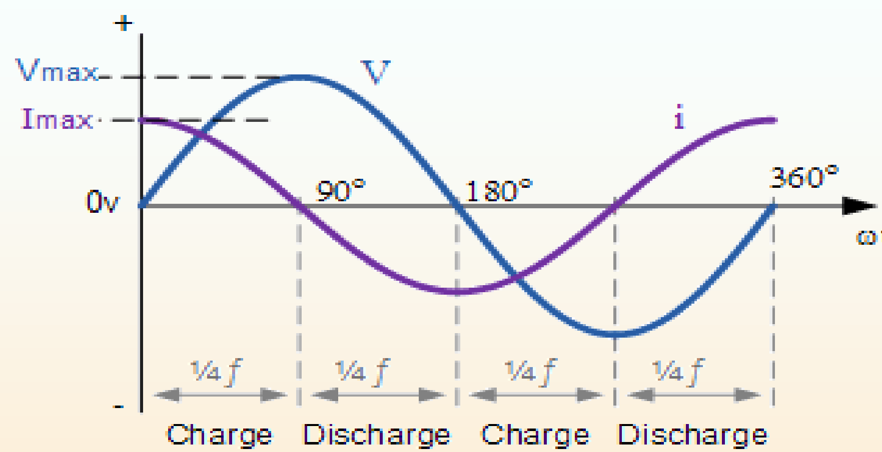


Fig.3(b): current and voltage waverform

Voltage-current relations for a Capacitor

- Let instantaneous voltage can be given by,

$$v = V_m \sin \omega t \qquad \text{.....(1)}$$

- Then from fig. 3(b), instantaneous current is given by,

$$i = C\frac{dv}{dt}$$

$$i = C\frac{d(V_m sin\omega t)}{dt}$$

$$= CV_m\omega(\cos\omega t)$$

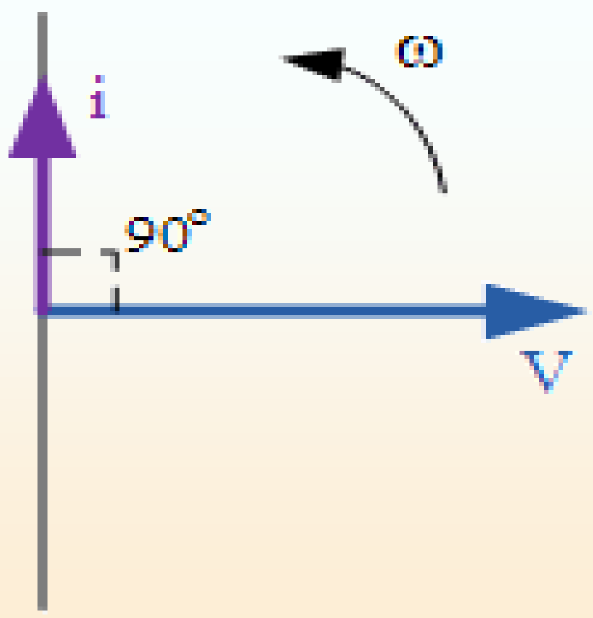
$$= \frac{V_m}{X_C}\sin(\omega t + 90)$$

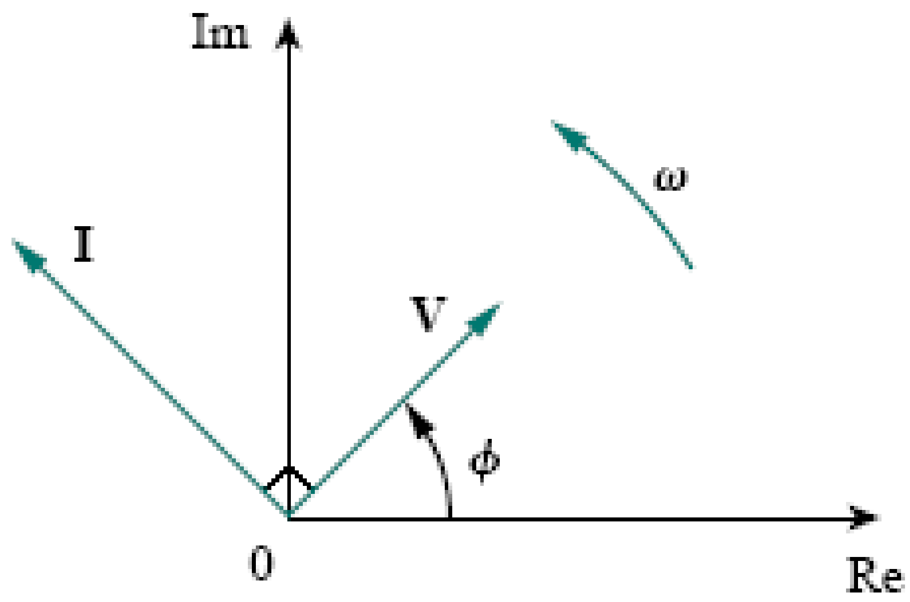
$$or \qquad i = I_m \sin(\omega t + 90) \qquad \text{.....(2)}$$

Where

$$I_m = \frac{V_m}{X_C} \;\; and \; X_C = \frac{1}{\omega C}$$ = impedance offered by pure capacitance

As seen from the equation (1) and (2), the current leads the voltage by 90 degrees in a pure capacitive circuits





Power in a capacitor

Instantaneous power

$$\therefore \; p = v \times i$$

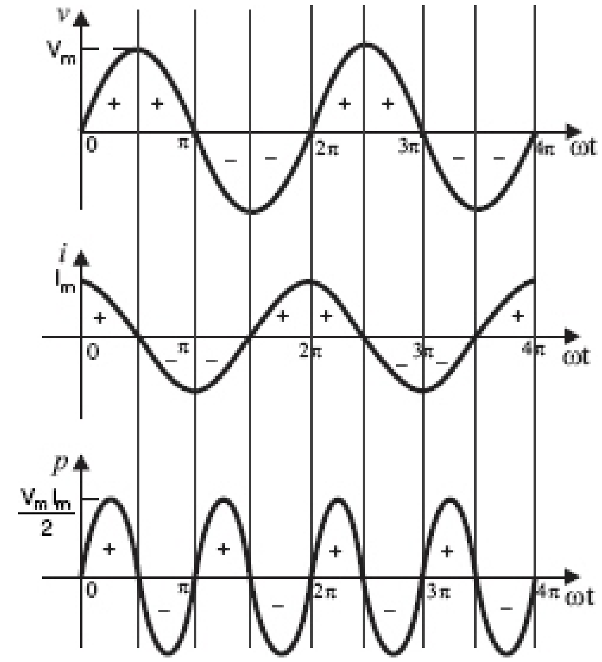
$$= V_m \sin \omega t \times I_m \sin(\omega t + 90^\circ)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= V_m I_m \frac{2 \sin \omega t \cos \omega t}{2}$$

$$= \frac{V_m I_m}{2} \sin 2\omega t \quad \boxed{\text{where}, \theta = \omega t}$$

$$= \frac{V_m I_m}{2} \sin 2\theta \qquad \ldots\ldots (3.37)$$



Waveform of voltage, current and power

the instantaneous power is also a sinusoidal quantity whose frequency is double that of voltage or current

Average or Real power

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$$= \frac{1}{\pi}\int_0^{\pi} \frac{V_m I_m}{2} \sin 2\theta\, d\theta = \frac{V_m I_m}{2\pi}\int_0^{\pi} \sin 2\theta\, d\theta$$

$$= \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\pi}{2} + \frac{\cos 0}{2}\right]$$

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Alternatively, the expression for power can be obtained from complex power.

$$\text{Complex Power, } \bar{S} = \bar{V}\bar{I}^* = V\angle 0^\circ \times (I\angle 90^\circ)^*$$
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We know that, $|\bar{S}| = S = VI$ and $\angle\bar{S} = \phi = -90^\circ$

$$\therefore \text{Power, } P = S\cos\phi = VI\cos(-90^\circ) = 0$$

$$\text{Reactive power, } Q = S\sin\phi = VI\sin(-90^\circ) = -VI$$

The capacitance has only reactive power and "*the active power in the pure capacitance is zero*". The reactive power of capacitance is negative which means that it delivers reactive power.

Impedance of a purely capacitive circuit:

- When circuit is purely capacitive, the resistive part is zero i.e. R = 0.

$$\therefore Z = -j X_C \ \Omega$$

- In polar form, it is given by,

$$Z = X_L \angle -90^0 \ \Omega$$

$$\mathbf{Z}_C = \frac{\mathbf{V}_C}{\mathbf{I}} = \frac{V_C\angle 0^\circ}{I\angle 90^\circ} = \frac{V_C}{I}\angle -90^\circ = \frac{1}{\omega C}\angle -90^\circ = -j\frac{1}{\omega C}$$