# BEEE203L
# CIRCUIT THEORY

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# MODULE – IV

# Analysis of Transient Response of Circuits

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- Review of Laplace transformation;
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- Evaluation of initial and final conditions in RL,RC,RLC circuits with AC excitation

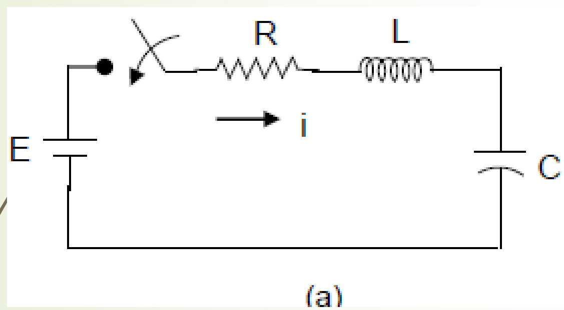
# Transient Response of RLC CIRCUIT



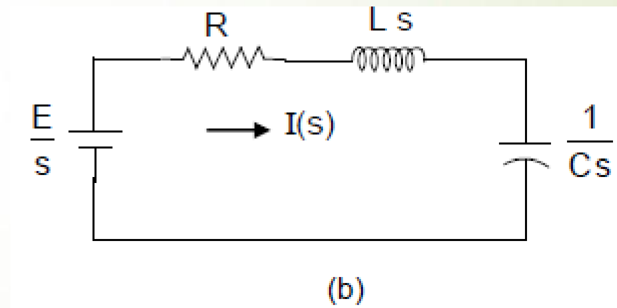
Consider the RLC series circuit shown in Fig. (a).

Assume that there is no initial charge on the capacitor and there is no initial current through the inductor.

The switch is closed at time t = 0. Transform circuit for time t > 0 is shown in Fig. (b).



(a)



(b)

- Using the transform circuit, expression for the current is obtained as

$$I(s) = \frac{E/s}{R + Ls + \frac{1}{Cs}} = \frac{EC}{RCs + LCs^2 + 1}$$

$$= \frac{E/L}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

- The roots of the denominator polynomial are

$$s_1, s_2 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = \alpha \pm \beta$$

$$\text{where } \alpha = -\frac{R}{2L} \text{ and } \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

- Depending on whether

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC},\ \left(\frac{R}{2L}\right)^2 = \frac{1}{LC} \text{ or } \left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$

the discriminant value will be positive, zero or negative and **three different cases of solutions** are possible.

- The value of R, for which the discriminant is zero, is called the critical resistance, RC.

$$\frac{R_c^{\ 2}}{4L^2} = \frac{1}{LC};$$

$$R_c = 2\sqrt{\frac{L}{C}}$$

# Case1: $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$ i.e. $R > R_C$

- The two roots s1 and s2 are real and distinct.

  $s_1 = \alpha + \beta$ and $s_2 = \alpha - \beta$

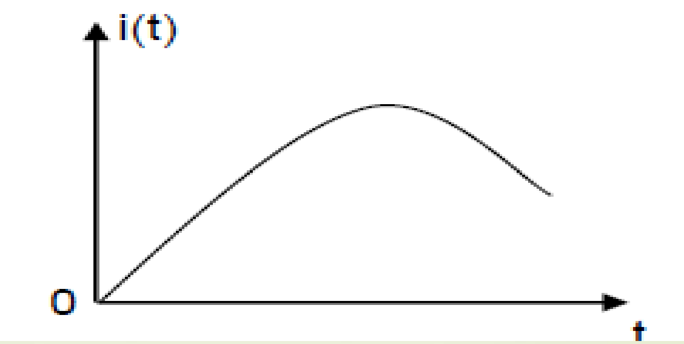
Then,

$$I(s) = \frac{K_1}{s-(\alpha+\beta)} + \frac{K_2}{s-(\alpha-\beta)}$$

Taking inverse LT, we get

$$i(t) = K_1 e^{(\alpha+\beta)t} + K_2\, e^{(\alpha-\beta)t} = e^{\alpha t}\,[K_1 e^{\beta t} + K_2\, e^{-\beta t}]$$

Its plot is shown in Fig. In this case the current is said to be over-damped.



# Case2: $(\frac{R}{2L})^2 = \frac{1}{LC}$ i.e. $R = R_C$

- Then $\beta = 0$ and hence the roots equal

  $s_1 = s_2 = \alpha$

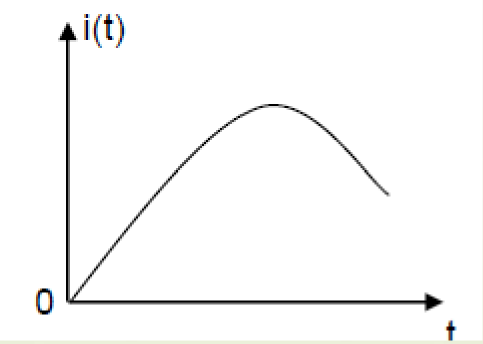
Then,

$$I(s) = \frac{E/L}{(s-\alpha)^2} = \frac{K}{(s-\alpha)^2}$$

Taking inverse LT, we get

$$i(t) = Kte^{\alpha t}$$

Its plot is shown in Fig. In this case the current is said to be critically-damped.



# Case3: $(\frac{R}{2L})^2 < \frac{1}{LC}$ i.e. $R < R_C$

- For this case, the roots are complex conjugate,

  $s_1 = \alpha + j\beta$ and $s_2 = \alpha - j\beta$

Then,

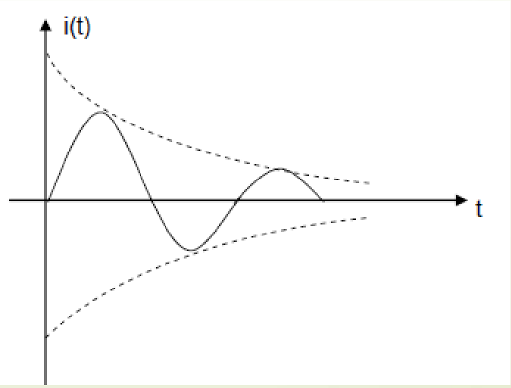
$$I(s) = \frac{E/L}{(s-\alpha-j\beta)(s-\alpha+j\beta)} = \frac{E/L}{(s-\alpha)^2+\beta^2} = \frac{E}{L\beta}\frac{\beta}{(s-\alpha)^2+\beta^2}$$

$$= A\frac{\beta}{(s-\alpha)^2+\beta^2}$$

Taking inverse LT, we get

$$i(t) = Ae^{\alpha t}\,\text{Sin}\beta t$$

This is under damped case, the current is oscillatory and at the same time it decays.

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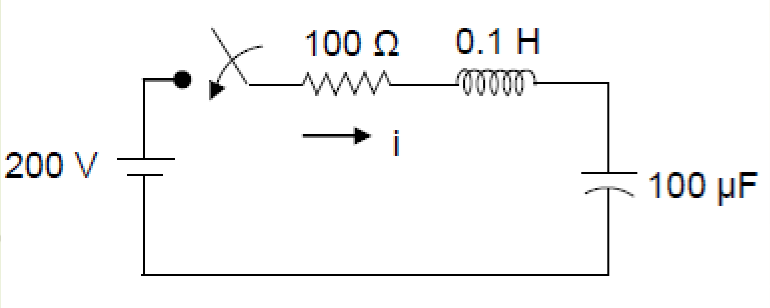
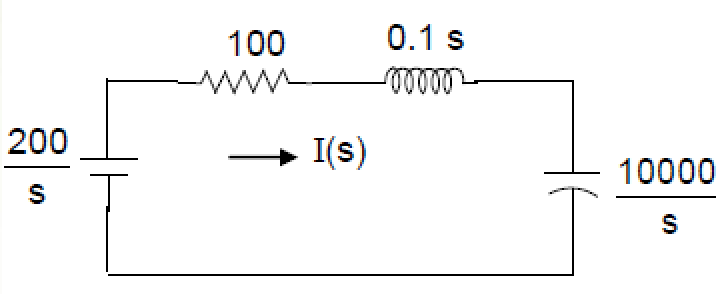
**P.4.19** For the RLC circuit shown, find the expression for the transient current when the switch is closed at time t = 0. Assume initially relaxed circuit conditions.




## Solution

$$\text{Current } I(s) = \frac{200/s}{100 + 0.1s + \dfrac{10000}{s}} = \frac{200}{0.1s^2 + 100s + 10000}$$

$$= \frac{2000}{s^2 + 1000s + 100000} = \frac{E/L}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}}$$

$$\frac{2000}{s^2 + 1000s + 100000} = \frac{A}{(s+112.71)} + \frac{B}{(s+887.29)}$$

When s = – 887.29

2000 = B (– 774.58) or B = –2.582

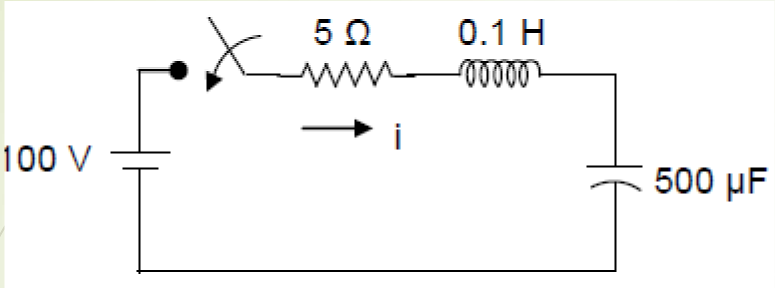
When S = –112.71

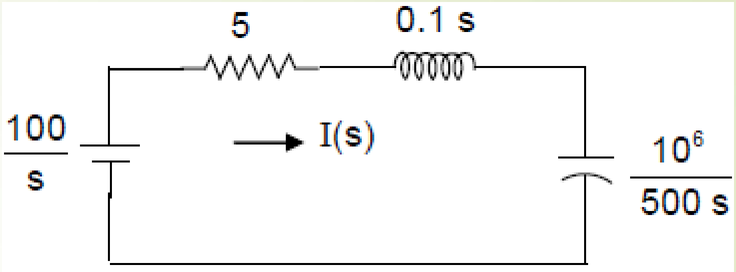
2000 = A (774.58) or A = 2.582

So $I(S) = \frac{2.582}{s+112.71} - \frac{2.582}{s+887.29}$

**P.4.20** Taking the initial conditions as zero, find the transient current in the circuit shown in Fig. when the switch is closed at time t = 0.






**Solution** The transform circuit is shown in

$$\text{Current } I(s) = \frac{100/s}{5 + 0.1s + \dfrac{10^6}{500\,s}} = \frac{100}{0.1s^2 + 5s + 2000}$$

$$= \frac{1000}{s^2 + 50s + 20000}$$

$$s_1, s_2 = \frac{-50 \pm \sqrt{2500 - 80000}}{2} = -25 \pm j139.1941$$

So $\alpha = -25$ ; $\beta = 139.19$

$$I(s) = \frac{E/L}{(s - \alpha - j\beta)(s - \alpha + j\beta)} = \frac{E/L}{(s-\alpha)^2 + \beta^2} = \frac{E}{L\beta}\,\frac{\beta}{(s-\alpha)^2 + \beta^2}$$

$$= A\,\frac{\beta}{(s-\alpha)^2 + \beta^2}$$

So $i(t) = Ae^{\alpha t} \sin\beta t$

$A = E/(L\beta) = 100/(0.1*139.19) = 7.184$

So $i(t) = 7.184e^{-25t} \sin 139.19t$