Final Assessment Test– Jan/Feb 2023

VIT®
Vellore Institute of Technology

Course: BMAT101L - Calculus

Class NBR(s):5453 / 5456 / 6425

Time: Three Hours

Slot: D2+TD2

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Determine the value of 'c' for which the function $f(x) = \sqrt{x(1-x)}$ in [0,1] satisfies Mean Value theorem. Also check whether this function satisfies Rolle's theorem in [0,2] **[10]** +10

2. Find the area of the region R bounded by the parabola $4y = x^2$ and the line $y = 2x$ in the first quadrant. **[10]** +10
   If R is revolved about Y-axis to form a solid, find the volume of the solid.

3. a) If $u = tan^{-1}\left(\frac{y}{x}\right)$ where $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$, find $\frac{du}{dt}$ **[5+5]**
   b) Show that $u = 3x + 2y - z, v = x - 2y + z$ and $w = x(x + 2y - z)$ are functionally dependent.

4. Expand using Taylor series for $f(x, y) = cosx\, cosy$ at the origin upto 3rd order and hence find f (0.1,0.1). **[10]**

5. Use Lagrange multiplier method, to find the largest volume of the rectangular parallelepiped that can be inscribed in the ellipsoid **[10]**
   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

6. Change the order of integration and evaluate **[10]**
   $$\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$$

7. Evaluate using spherical coordinates **[10]**
   $$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$$
   -2

8. Using Beta and Gamma functions, evaluate **[5+5]**
   a) $\int_0^{\frac{\pi}{2}} \sqrt{cot\theta}\, d\theta$ and b) $\int_0^1 \frac{xdx}{\sqrt{1-x^5}}$

9. a) Determine the directional derivative of $\varphi = 2xy - 3y^2$ at (5,5) in the direction $4\hat{\imath} + 3\hat{\jmath}$ **[10]** +10
   b) Find div$\vec{F}$ and curl$\vec{F}$ for
   $\vec{F}$ = grad$[x^3y + y^3z + z^3x - x^2y^2z^2]$

10. Show that $\vec{F} = (2x + yz)\hat{\imath} + (4y + zx)\hat{\jmath} - (6z - xy)\hat{k}$ is both solenoidal and irrotational. Find also its scalar potential. **[10]**

11. Verify Green's theorem for $\oint_C [3x^2 - 8y^2]dx + [4y - 6xy]dy$ where C is the boundary of the region $x = 0, y = 0$ and $x + y = 1$ **[10]** +10

12. Using Stoke's theorem evaluate $\int_C \vec{F}.\vec{dr}$ where $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ and C is the boundary of the plane $3x + 2y + z = 6$ in the first quadrant. **[10]**