Experiment 2–A

Evaluation of Volume by Integrals (Solids of Revolution)

Volume of solid of revolution – Disc method

The solid figure formed by revolving a plane curve about an axis is called Solid of revolution.

If the solid is formed by revolving the curve $y = f(x)$ about a line $y = c$ (parallel to the $x-$axis), then the volume of the solid formed is given by $V = \int_a^b \pi[y-c]^2\,dx$.

Note: For $c = 0$, the axis of revolution will be the $x-$axis itself.

Example 1.
The volume of the solid generated by the revolving the curve $y = \sqrt{x}$ about the line $y = 1$ from $x = 1$ to $x = 4$ is given by the following code:

MATLAB Syntax used:
The code below consists of two parts.
The first part evaluates the volume of the solid generated by revolving the curve $y = f(x)$ about the line $y = yr$ (axis of revolution) for $x \in (a,b)$.
In the second part we give the visualization of the solid of revolution in the 3-D space.
Some of the MATLAB commands used in the code are explained here.

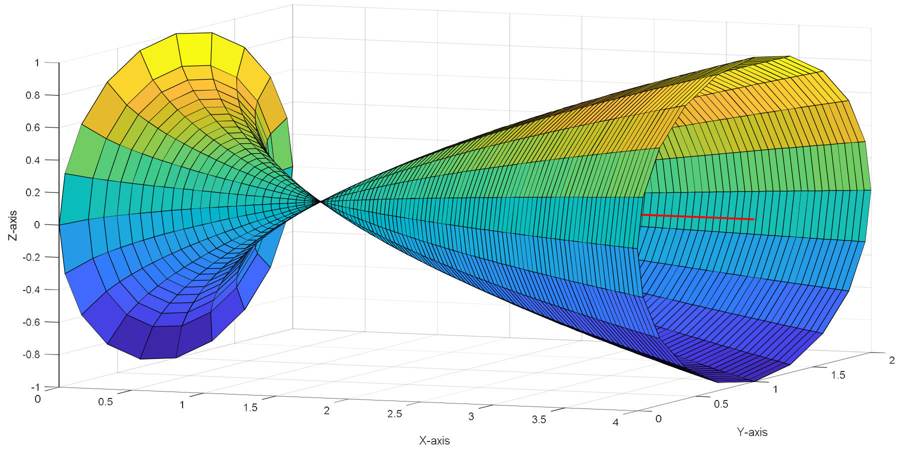
`int(f(x),a,b)`	evaluates the integration of $f(x)$ between the limits a and b
`fx=matlabFunction(f)`	This converts Symbolic Function f to Anonymous Function *fx*
`[X,Y,Z]=cylinder(r);`	This returns the *x*-, *y*-, and *z*-coordinates of a cylinder using *r* to define a profile curve. cylinder treats each element in `r` as a radius at equally spaced heights along the unit height of the cylinder. The cylinder has 20 equally spaced points around its circumference.
`surf(X,Y,Z)`	This creates a three-dimensional surface plot. The function plots the values in matrix Z as heights above a grid in the *x*-*y* plane defined by X and Y.

```
%Evaluation of Volume of solid of revolution
clear
clc
syms x
f(x)=sqrt(x);          % Given function
yr=1;                  % Axis of revolution y=yr
I=[0,4];               % Interval of integration
a=I(1);b=I(2);
vol=pi*int((f(x)-yr)^2,a,b);
disp('Volume of solid of revolution is: ');
disp(vol); % Visualization if solid of revolution
fx=matlabFunction(f);
xv = linspace(a,b,101); % Creates 101 points from a to b
[X,Y,Z] = cylinder(fx(xv)-yr);
Z = a+Z.*(b-a);     % Extending the default unit height of the
%cylinder profile to the interval of integration.
```

```
surf(Z,Y+yr,X)    % Plotting the solid of revolution about y=yr
hold on;
plot([a b],[yr yr],'-r','LineWidth',2); % Plotting the line y=yr
view(22,11);       % 3-D graph viewpoint specification
xlabel('X-axis');ylabel('Y-axis');zlabel('Z-axis');
```

Output

```
Volume of solid of revolution is:
(4*pi)/3
```



Exercise:

1. Find the volume of the solid generated by revolving about the $x-$axis the region bounded by the curve $y=\frac{4}{x^2+4}$, the $x-$axis, and the lines $x=0$ and $x=2$.
2. Find the volume of the solid generated by revolving about the $x-$axis the region bounded by the curve $y=\sqrt{x}$, the $x-$axis, and the line y= $x-2$.
3. Find the volume of the solid generated by revolving about the line y=1, the region bounded by the curve $y=\sqrt{x}$, the lines y=1 and x=3.

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