MODULE -3 Air Compressor

- **Reciprocating compressors**
- **Construction, Working**
- **Effect of clearance volume**
- **Multi-staging**
- **Volumetric efficiency**
- **Isothermal efficiency**

Introduction

- Compressor is a mechanical device which pressurizes a working fluid.

- Air is free of cost, but compressed air is not free.

- Compressed air is a form of stored energy that is used for inflating tires or powering small tools, large industrial units that are used in manufacturing and construction.

Power into Kinetic Energy

Working of Air compressor

The working of an air compressor involves several steps, which are outlined below:

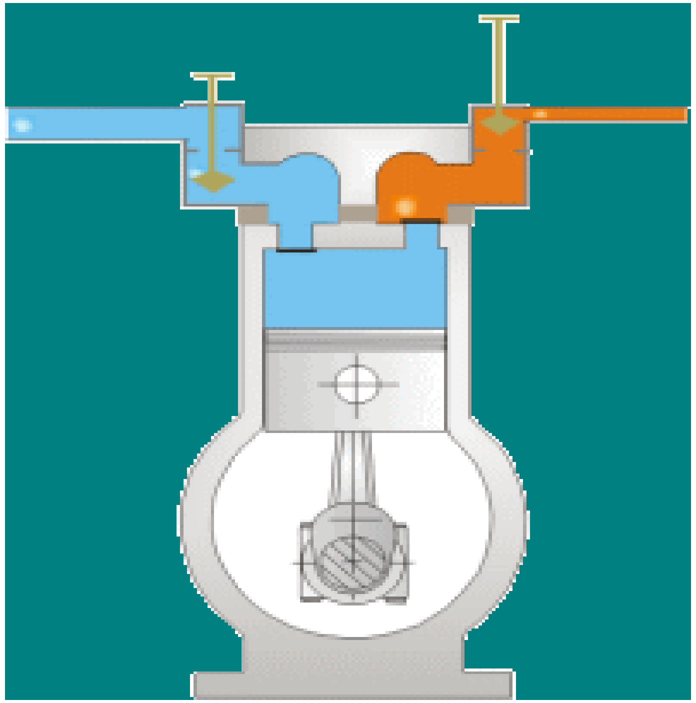
1.Intake of air: The compressor draws in air from the surroundings through an intake valve.

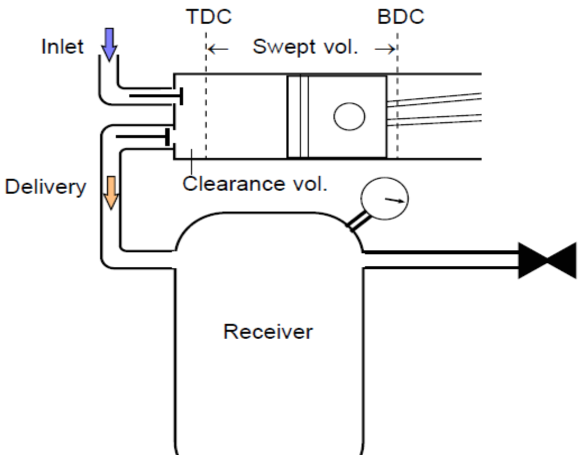
2.Compression of air: The air is compressed by reducing its volume, which is achieved by increasing the pressure of the air. This is done by the compressor pump, which uses pistons, rotors, or other mechanisms to squeeze the air and force it into a smaller space.

3.Storage of compressed air: The compressed air is stored in a tank or receiver, which is designed to hold the air at a specific pressure.

4.Regulation of pressure: The air pressure in the tank is regulated by a **pressure switch**, which turns the compressor on and off as needed to maintain a specific pressure level.

5.Exhaust of air: When the compressed air is used, it is released from the tank and distributed to the desired application. The exhaust air may also be used for other purposes, such as cooling the compressor or powering other equipment.





Applications of Compressed Air

- The compressed air can be filled in the tubes of automobiles.

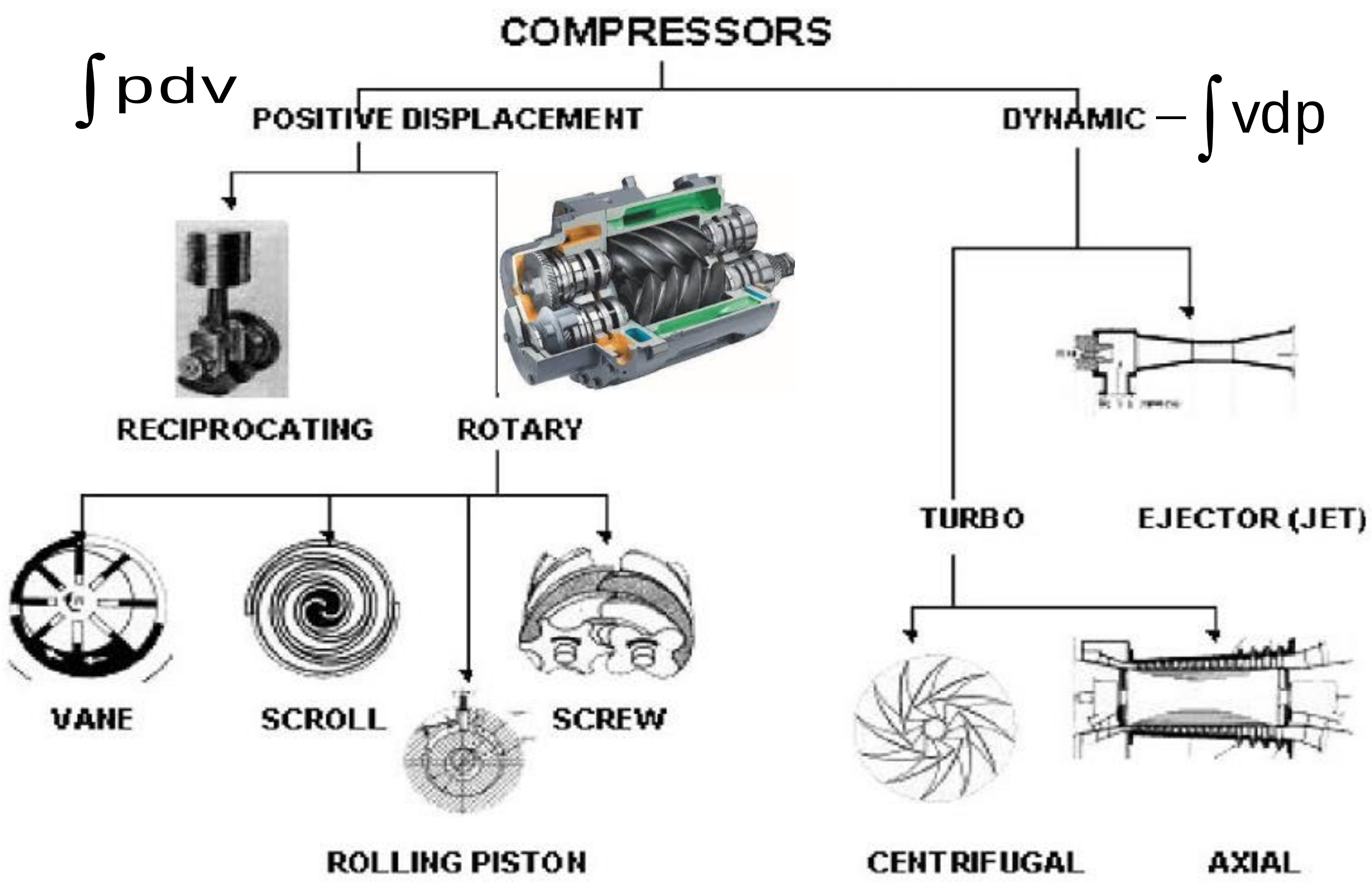
- It can be used for operating various electronic instruments, for operation of pneumatic machines, cleaning purposes.

- Compressed-air is used for machine and tool operation, drilling, painting, soot blowing, food processing, instrument operations, and Pressures range from 25 psig (1.72 bar) to 60,000 psig (4137 bar).

- Gas compressors are used for refrigeration, air conditioning, heating, pipeline conveying, natural gas gathering, catalytic cracking, polymerization, and in other chemical processes.

Classification of Air-Compressor

(i) According to method of energy transfer and pressure generation i.e. positive displacement and roto dynamic compressors



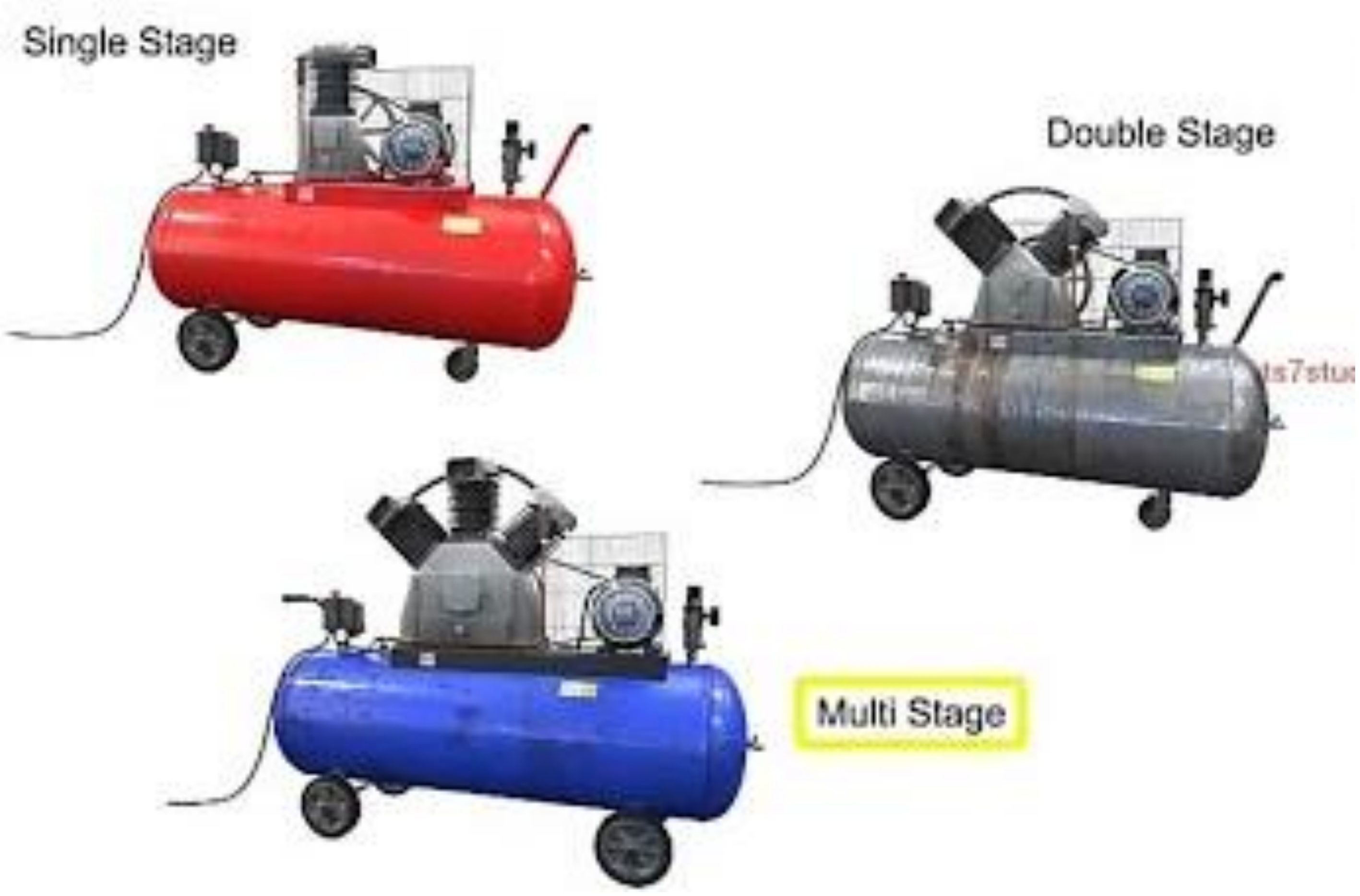
Positive displacement compressors work on the principle of increasing the pressure of air by reducing the volume of air in an enclosed chamber.

Dynamic compressors works on the principle of imparting the energy by rotating vanes of impeller on air flowing through casing that increases pressure in air.

Classification of Air-Compressor

(ii) According to number of stages

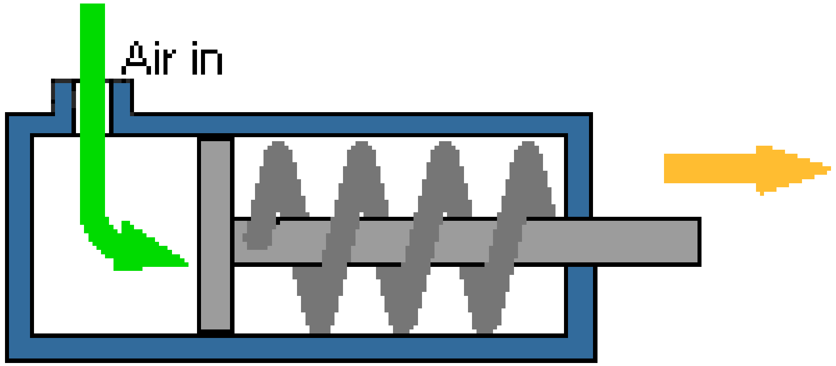
Single stage, double stage, three stage (or) multiple stage



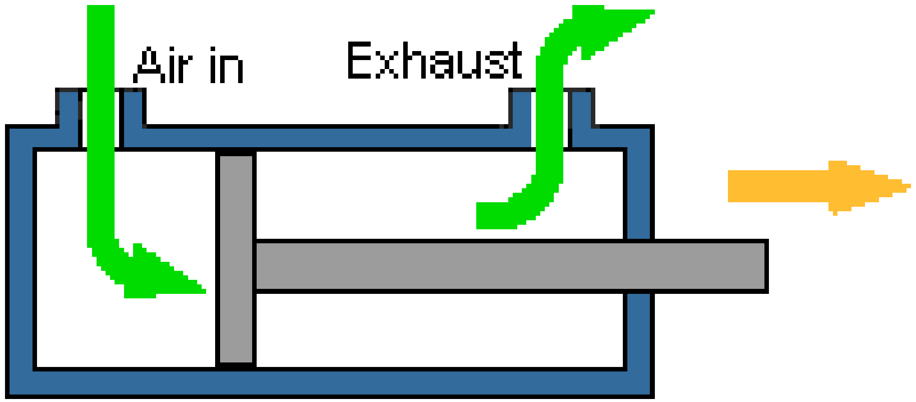
Classification of Air-Compressor

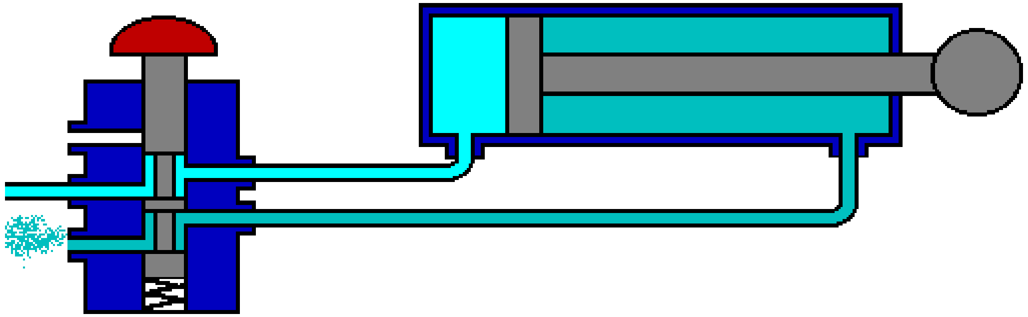
(iii) According to action

Single Acting



Double Acting

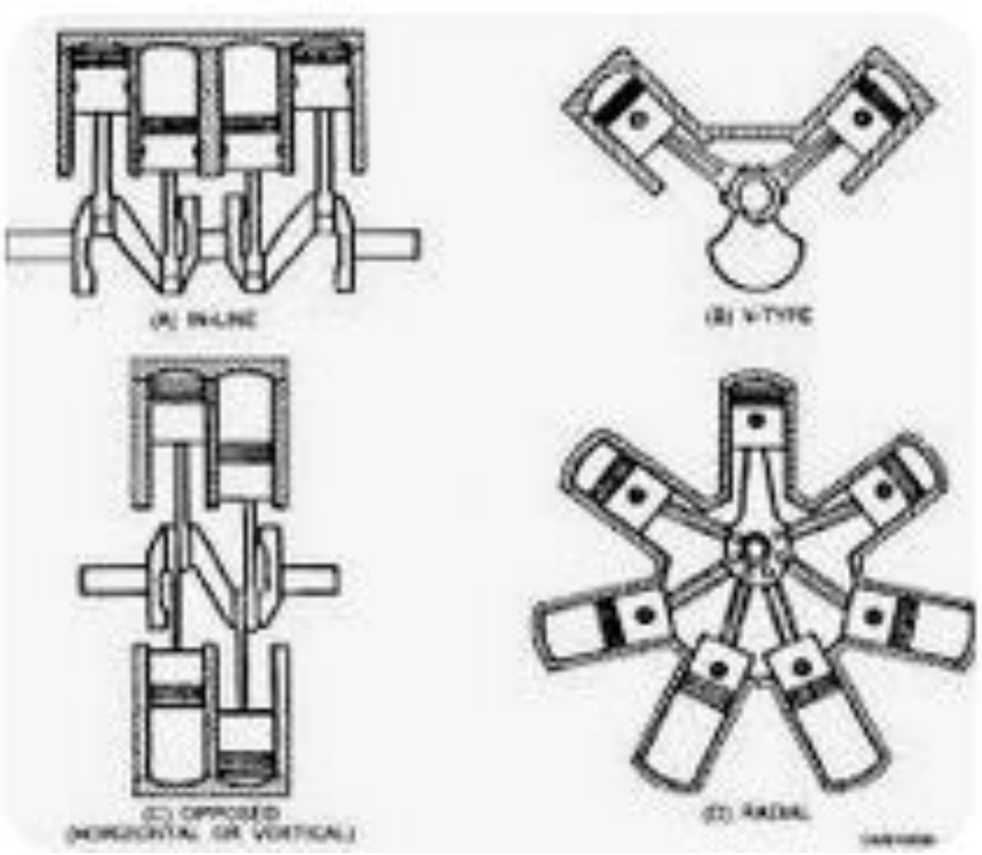




Classification of Air-Compressor

(iv) According to position of cylinder w.r.t. crankshaft

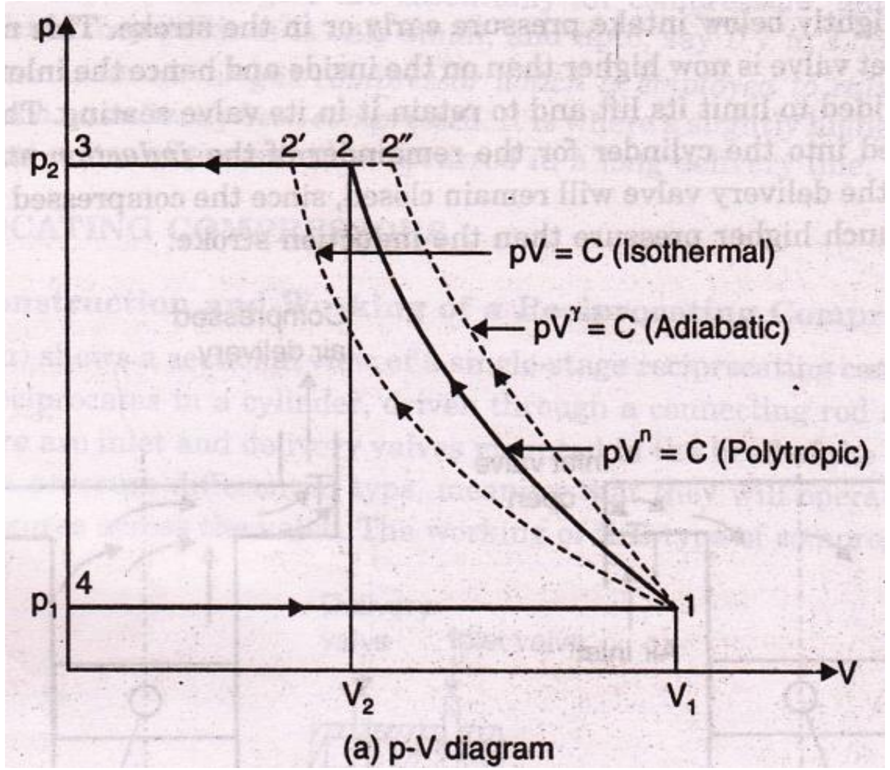
Cylinders inline, vertical, radial position, V-type cylinder arrangement



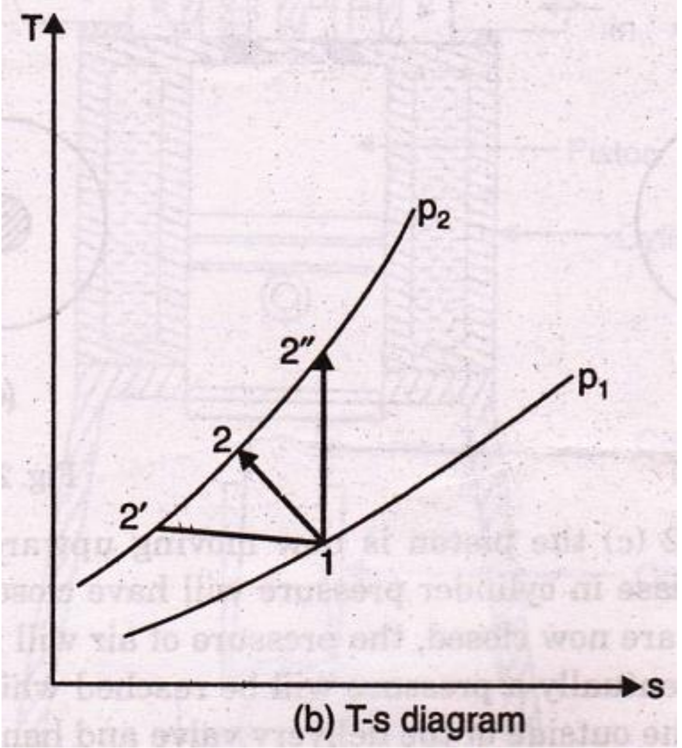
Compressor Characteristics

Types of Compressors	Pressure Ratio			Efficiency
	Industrial	Aerospace	Research	
Positive Displacemnt	Up to 30	–	–	75–82%
Centrifugal	1.2–1.9	2.0–7.0	13	75–87%
Axial	1.05–1.3	1.1–1.45	2.1	80–91%

Efficiencies



(a) p-V diagram



(b) T-s diagram

$$Isothermal\,efficiency, \quad \eta_{iso} = \frac{w_{iso}}{w_{actual}} \qquad Isentropic\,efficiency, \quad \eta_{isen} = \frac{w_{isentropic}}{w_{actual}}$$

$$Mechanical\,efficiency, \quad \eta_{mech} = \frac{w_{actual}}{w_{shaft}}$$

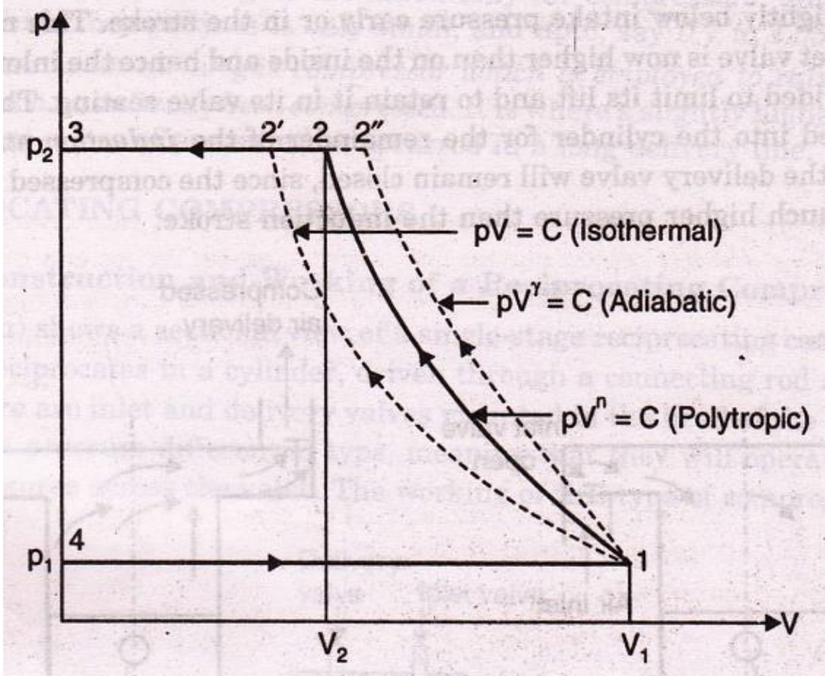
Work Done –without clearance Volume

Total shaft work done/cycle, W = Area 41234

$$W = \text{Area under 4-1} - \text{Area under 1-2} - \text{Area under 2-3}$$

$$= p_1V_1 - \frac{p_2V_2 - p_1V_1}{n-1} - p_2V_2$$

$$= (p_1V_1 - p_2V_2) - \left(\frac{p_2V_2 - p_1V_1}{n-1}\right) = (p_1V_1 - p_2V_2) + \left(\frac{p_1V_1 - p_2V_2}{n-1}\right)$$



(a) p-V diagram

$$= \left(1 + \frac{1}{n-1}\right)(p_1V_1 - p_2V_2)$$

$$W = \left(\frac{n}{n-1}\right)(p_1V_1 - p_2V_2)$$

$$W = \frac{n}{n-1}(p_1V_1 - p_2V_2) = \frac{n}{n-1} \cdot p_1V_1\left(1 - \frac{p_2V_2}{p_1V_1}\right)$$

$$p_1V_1^n = p_2V_2^n$$

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{1/n}$$

Work Done –without clearance Volume

$$W = \frac{n}{n-1}\, p_1 V_1 \left\{1 - \frac{p_2}{p_1}\left(\frac{p_1}{p_2}\right)^{1/n}\right\} = \frac{n}{n-1}\, p_1 V_1 \left\{1 - \frac{p_2}{p_1}\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}\right\}$$

$$= \frac{n}{n-1}\, p_1 V_1 \left\{1 - \left(\frac{p_2}{p_1}\right)^{1-\frac{1}{n}}\right\} = \frac{n}{n-1}\, p_1 V_1 \left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right\}$$

$$W = \frac{n}{n-1}\, p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}$$

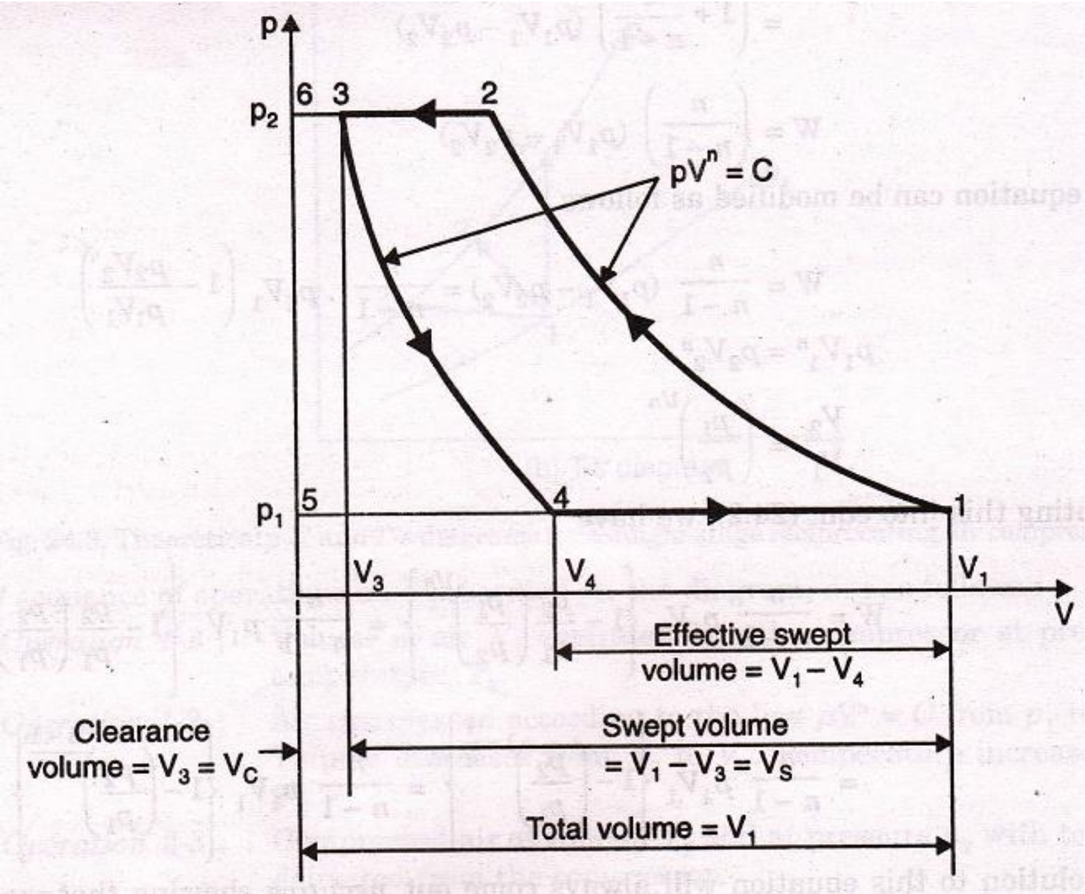
$$= \frac{n}{n-1}\, mRT_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{or} \quad T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

or

$$W_{in} = \frac{n}{n-1}\, m\, R\, (T_2 - T_1)$$

Work Done –with clearance Volume



$$W = \frac{n}{n-1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} - \frac{n}{n-1} p_4 V_4 \left\{ \left(\frac{p_3}{p_4} \right)^{\frac{n-1}{n}} - 1 \right\}$$

$$W = \frac{n}{n-1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} - \frac{n}{n-1} p_1 V_4 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\}$$

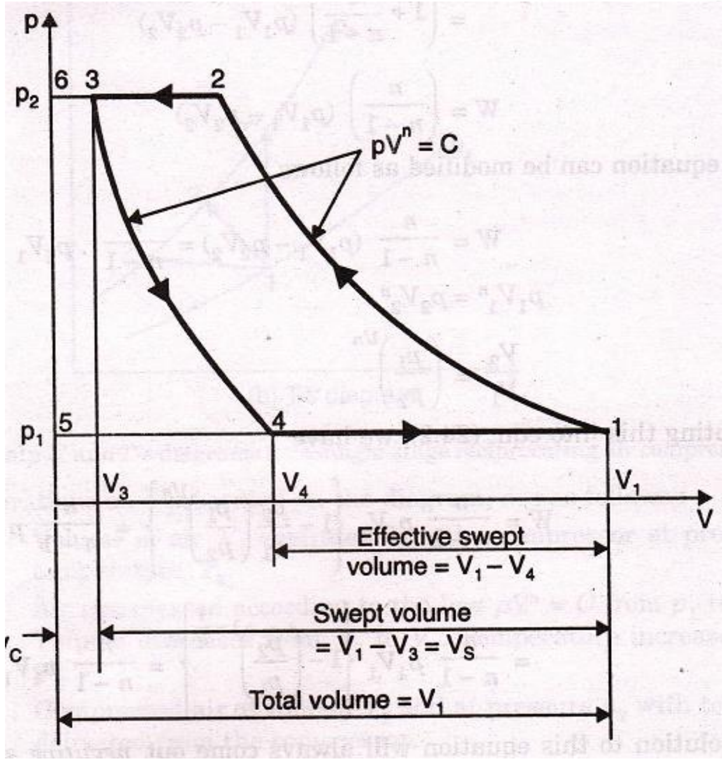
$$= \frac{n}{n-1} p_1 (V_1 - V_4) \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\}$$

$$\text{Volumetric efficiency} = \frac{\text{Effective swept volume}}{\text{Swept volume}} = \frac{V_1 - V_4}{V_1 - V_3}$$

$$\eta_{vol} = \frac{FAD}{V_S} = \frac{V_a}{V_S} = \frac{V_1 - V_4}{V_1 - V_3}$$

$$\frac{\text{Clearance volume}}{\text{Swept volume}} = \frac{V_3}{V_1 - V_3} = \frac{V_c}{V_s} = k$$

Work Done –with clearance Volume



Volumetric efficiency, $\eta_{vol.} = \dfrac{V_1 - V_4}{V_1 - V_3}$

$$= \frac{(V_1 - V_3) + (V_3 - V_4)}{(V_1 - V_3)} = 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3}$$

$$= 1 + \frac{V_3}{V_1 - V_3} - \frac{V_4}{V_1 - V_3} \cdot \frac{V_3}{V_3} = 1 + \frac{V_3}{V_1 - V_3} - \frac{V_3}{V_1 - V_3} \cdot \frac{V_4}{V_3}$$

$$= 1 + k - k \cdot \frac{V_4}{V_3}$$

$$p_3 V_3^n = p_4 V_4^n$$

$$\frac{V_4}{V_3} = \left(\frac{p_3}{p_4}\right)^{1/n}$$

$$= 1 + k - k \left(\frac{p_3}{p_4}\right)^{1/n}$$

$$(\because \quad p_3 = p_2, p_4 = p_1)$$

$$\eta_{vol.} = 1 + k - k \left(\frac{p_2}{p_1}\right)^{1/n}$$

$$\eta_{vol.} = 1 + k - k \left(\frac{V_1}{V_2}\right)$$

Volumetric efficiency lowered by any of the following conditions

(i) Very high speed

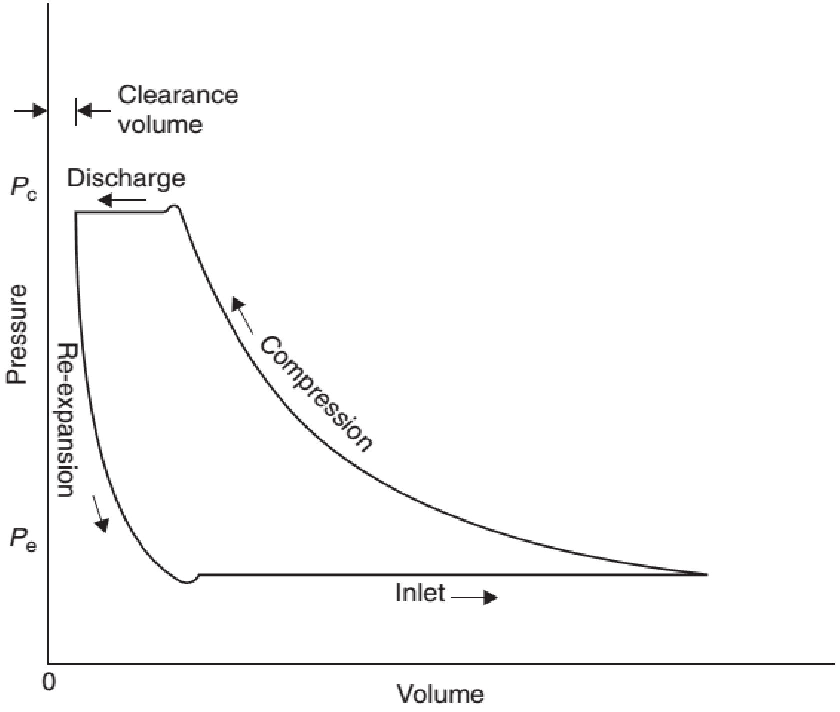
(ii) Leakage past the piston

(iii) Too large clearance volume

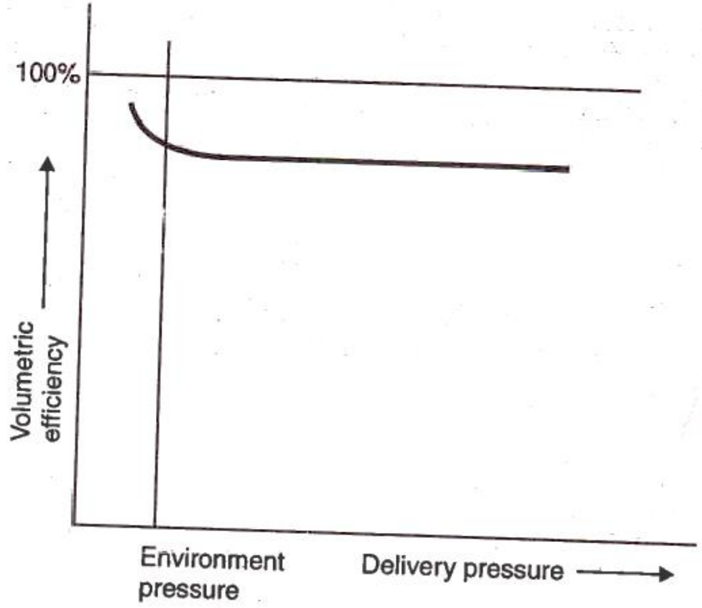
(iv) Obstruction at inlet valves

(v) Overheating of air by contact with hot cylinder walls

(vi) Inertia effect of air in suction pipe



Actual P-V diagram



Variation of volumetric efficiency with delivery pressure

Numerical on Air compressor

1. A single acting, single-cylinder reciprocating air compressor has a cylinder diameter of 200 mm and a stroke of 300 mm. Air enters a cylinder at 1 bar; 27°C. It is then compressed polytropically to 8 bar according to the law $pv^{1.3}$ = const. If the speed of the compressor is 250 rpm, calculate the mass of air compressed per minute, and the power input to compressor and motor (brake) power, (neglecting losses due to clearance, leakage and cooling) if mechanical efficiency is 80% .

Numerical on Air compressor

1. A single acting, single-cylinder reciprocating air compressor has a cylinder diameter of 200 mm and a stroke of 300 mm. Air enters a cylinder at 1 bar; 27°C. It is then compressed polytropically to 8 bar according to the law $pv^{1.3}$ = const. If the speed of the compressor is 250 rpm, calculate the mass of air compressed per minute, and the power input to compressor and motor (brake) power, (neglecting losses due to clearance, leakage and cooling) if mechanical efficiency is 80% .

$$Given\,data:$$

$$d = 200\,mm = 0.2\,m$$

$$L = 300\,mm = 0.3\,m$$

$$P_1 = 1\,bar$$

$$P_2 = 8\,bar$$

$$N = 250\,rpm$$

$$T_1 = 27^\circ C = 300K$$

$$n = 1.3$$

$$To\,Find: \quad (i)\,m_a = \frac{P_1V_1}{RT_1}(kg/cycles) \times N(cycles/\min)$$

$$(ii)\,W_{comp} = \frac{n}{n-1} m_a\,R(T_2 - T_1)(kW)$$

$$(iii)\,\eta_{mech} = \frac{W_{comp}}{W_{motor}} \times 100\%$$

$$Assumptions:$$

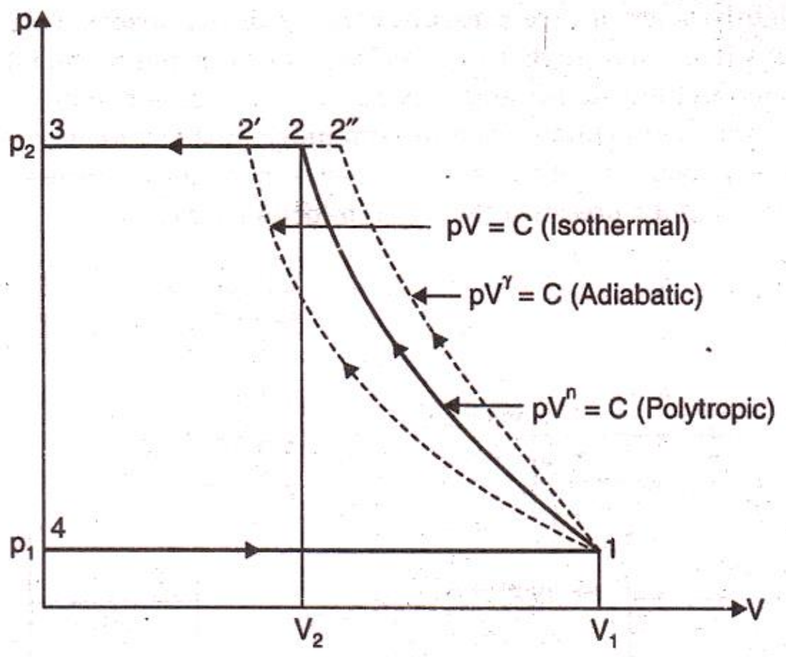
$$1.\,Negligible\,clearance\,volume\,\text{in}\,the\,cylinder$$

$$2.\,Air\,as\,an\,ideal\,gas\,with\,R = 0.287\,kJ/kg.K$$

Numerical on Air compressor

$$Solution:$$

$$(i)\, m_a = \frac{P_1V_1}{RT_1}(kg/cycles)$$



(a) p-V diagram

$$V_1 = V_s = \frac{\pi}{4}d^2L = 9.424\times10^{-3}m^3$$

$$m_a = \frac{P_1V_1}{RT_1}\times N = \frac{(100\,kPa)\times(9.424\times10^{-3}m^3)}{(0.287\,kJ/kg-K)\times(300\,K)} = 0.0109\,kg/cycles$$

$$m_a = 0.0109\,(kg/cycles)\times 250(cycles/\min)$$

$$= 2.74\,kg/\min$$

Numerical on Air compressor

$(ii)\, Work\ input\ to\ the\ compressor:$

$$W_{in} = \frac{n}{n-1} m_a R (T_2 - T_1)(kW)$$

$\because$ *where,*

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = (300K) \times \left(\frac{8}{1}\right)^{\frac{1.3-1}{1.3}} = 484.75K$$

$$= \frac{1.3}{1.3-1} \times (2.74\, kg/\min) \times (0.287\, kJ/kg.k) \times (484.75 - 300)k$$

$$= 629.56\, kJ/\min = 10.49\, (kJ/s = kW)$$

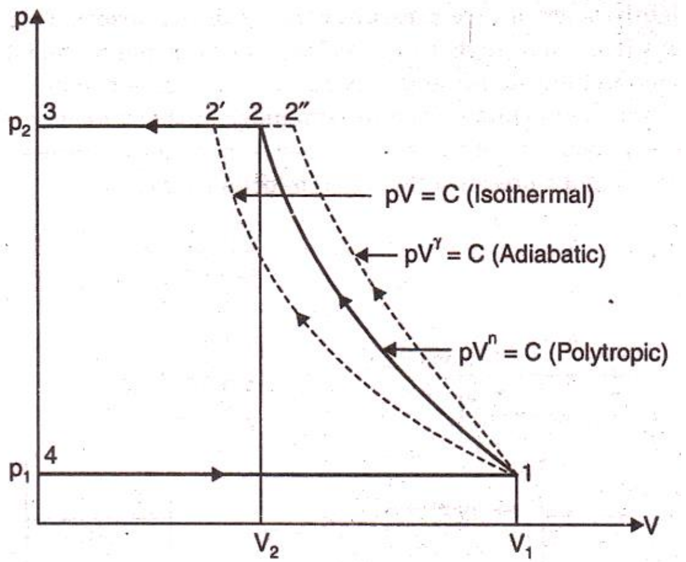
Numerical on Air compressor

2. A single acting, single-stage reciprocating air compressor of 250 mm bore and 350 mm stroke runs at 200 rpm. The suction and delivery pressures are 1 bar and 6 bar respectively. Calculate the theoretical power required to run the compressor and isothermal efficiency under each of he following conditions of compression: (Neglect the effect of clearance)

(a)Isothermal process

(b)Polytropic process (n =1.3) and

(c)Isentropic process (γ=1.4).



(a) p-V diagram

Numerical on Air compressor

2. A single acting, single-stage reciprocating air compressor of 250 mm bore and 350 mm stroke runs at 200 rpm. The suction and delivery pressures are 1 bar and 6 bar respectively. Calculate the theoretical power required to run the compressor and isothermal efficiency under each of he following conditions of compression: (Neglect the effect of clearance)

(a) Isothermal
(b) Polytropic n =1.3 and
(c) Isentropic process γ=1.4.

Given data :

$$d = 250\,mm = 0.25\,m$$

$$L = 350\,mm = 0.35\,m$$

$$P_1 = 1\,bar = 100\,kPa$$

$$P_2 = 6\,bar = 600\,kPa$$

$$N = 200\,rpm$$

To Find :

(i) *Theoritical power required to run the compressor,*

(a) *Isothermal compression,*

(b) *Polytropic compression,*

(c) *Isentropic compression.*

(ii) *Isothermal efficency in each case*

Numerical on Air compressor

Formula :

1. *Theoritical power required :*

(i) *Isothermal compression process*

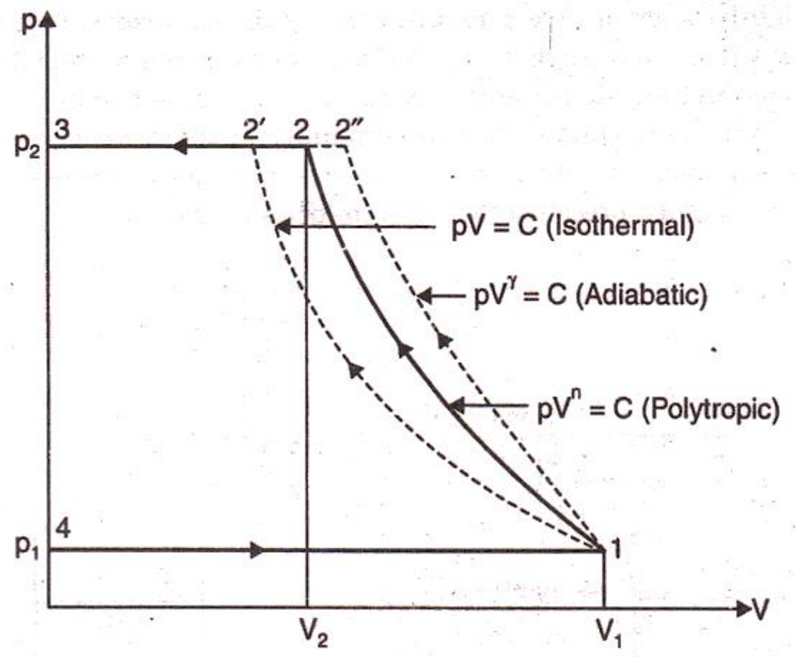
$$W_{iso} = p_1 V_1 \ln\left(\frac{p_2}{p_1}\right) \qquad where, V_1 = V_s = \frac{\pi}{4} d^2 \times L \times \frac{N}{60} (m^3/s)$$

(ii) *Polytropic compression process*

$$W_{poly} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

(iii) *Isentropic or Adiabatic compression process*

$$W_{isen} = \frac{\gamma}{\gamma - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$



(a) p-V diagram

2. *Isothermal efficency*

$$\eta_{iso} = \frac{W_{iso}}{W_{actual}} \times 100$$

Numerical on Air compressor

Solution :

Theoritical power required :

(i) *Isothermal compression process*

$$W_{iso} = p_1 V_1 \ln\left(\frac{p_2}{p_1}\right) = 100 \times 0.0572 \times \ln\left(\frac{6}{1}\right) = 10.261\ kJ/s\,(or)\,kW$$

$$where, V_1 = V_s = \frac{\pi}{4} d^2 \times L \times \frac{N}{60} = 0.0572(m^3/s) \qquad why, V_1\ in\ m^3/s$$

(ii) *Polytropic compression process*

$$W_{poly} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] = \frac{1.3}{1.3-1} \times 100 \times 0.0572 \left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right] = 12.7\,kW$$

(iii) *Isentropic compression process*

$$W_{Isen} = \frac{\gamma}{\gamma-1} p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = \frac{1.4}{1.4-1} \times 100 \times 0.0572 \left[\left(\frac{6}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right] = 13.38\,kW$$

Numerical on Air compressor

$(ii)\ Isothermal\ efficency$

$$\eta_{iso} = \frac{W_{iso}}{W_{actual}} \times 100$$

$(a)\ For\ isothermal\ compression,$

$$\eta_{iso} = \frac{W_{iso}}{W_{iso}} \times 100 = \frac{10.26}{10.26} \times 100 = 100\%$$

$(b)\ For\ polytropic\ compression,$

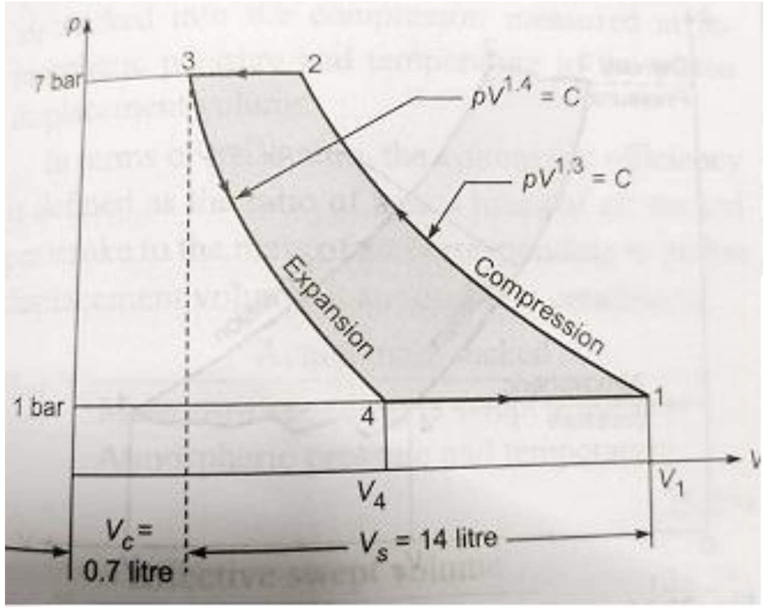
$$\eta_{iso} = \frac{W_{iso}}{W_{poly}} \times 100 = \frac{10.26}{12.70} \times 100 = 80.8\%$$

$(c)\ For\ isentropic\ compression,$

$$\eta_{iso} = \frac{W_{iso}}{W_{isen}} \times 100 = \frac{10.26}{13.38} \times 100 = 76.67\%$$

Numerical on Air compressor

3. An ideal single-stage, single-acting, reciprocating air compressor has a displacement volume of 14 litre and a clearance volume of 0.7 litre. It receives a air at a pressure of 1 bar and delivers it at a pressure of 7 bar. The compression is polytropic process with a index of 1.3 and expansion process is isentropic with an index of 1.4. Calculate the net indicated work of a cycle.



Numerical on Air compressor

3. An ideal single-stage, single-acting, reciprocating air compressor has a displacement volume of 14 litre and a clearance volume of 0.7 litre. It receives a air at a pressure of 1 bar and delivers it at a pressure of 7 bar. The compression is polytropic process with a index of 1.3 and expansion process is isentropic with an index of 1.4. Calculate the net indicated work of a cycle.

Given data :

$$P_1 = 1\,bar = 100\,kPa$$

$$P_2 = 7\,bar = 700\,kPa$$

$$V_s = 14\,litre = 14 \times 10^{-3}\,m^3$$

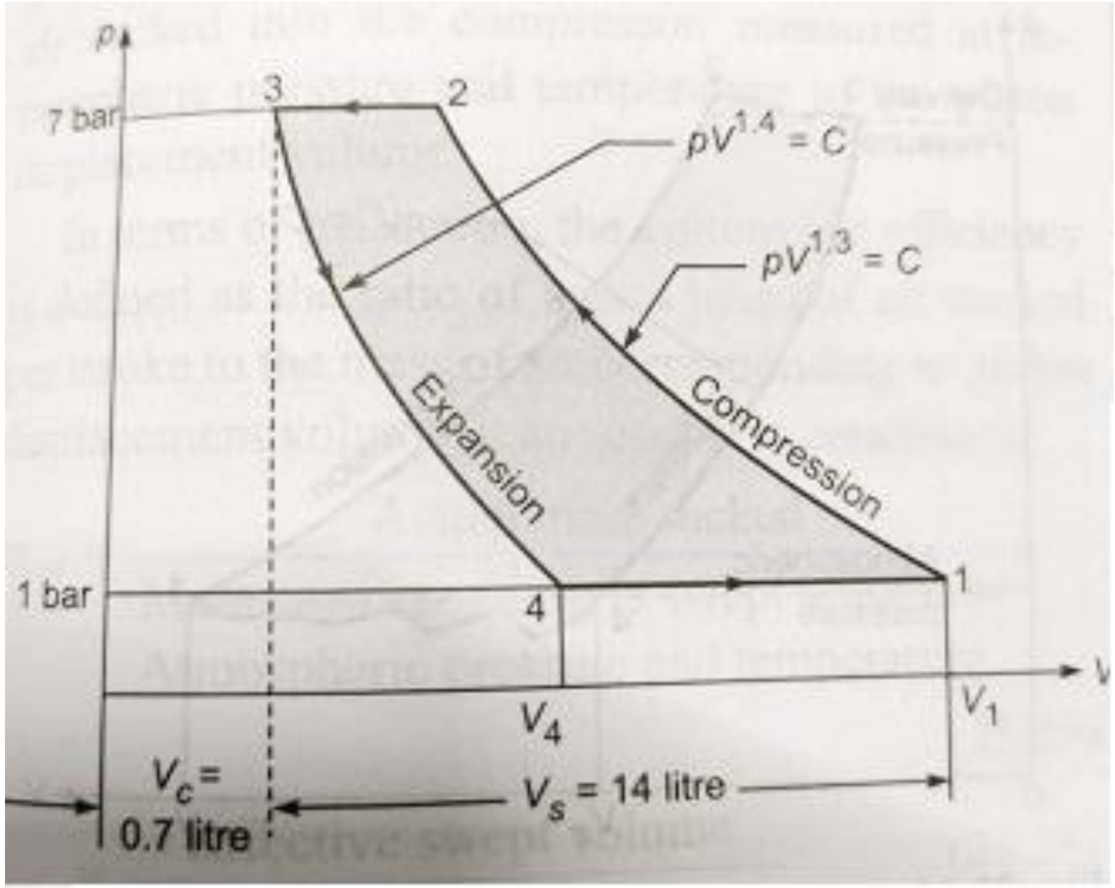
$$V_c = 0.7\,litre = 0.7 \times 10^{-3}\,m^3$$

$$n_c = 1.3$$

$$n_e = 1.4$$

To Find :

Indicated work input per cycle



Numerical on Air compressor

Formula :

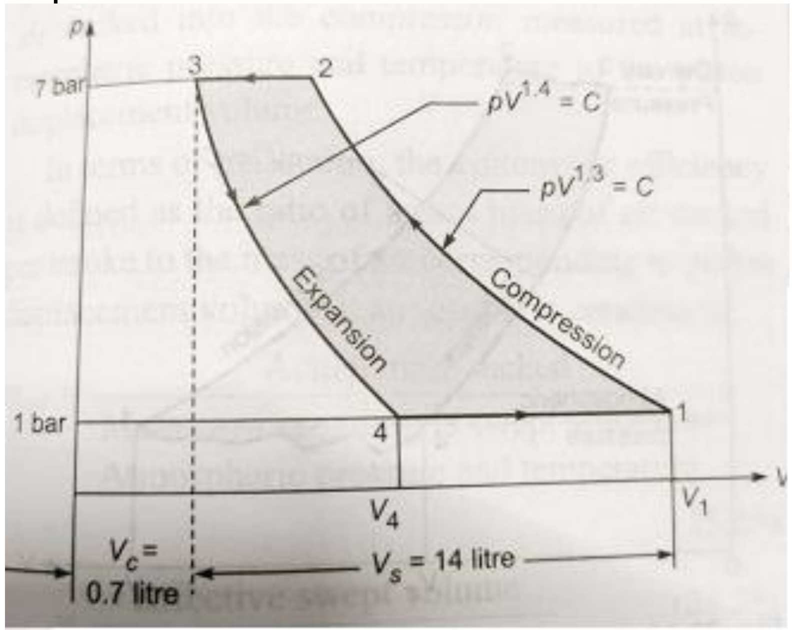
Indicated work input per cycle :

$$W_{in} = W_c - W_e$$

$$= \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] - \frac{n_e}{n_e - 1} p_4 V_4 \left[\left(\frac{p_3}{p_4} \right)^{\frac{n_e - 1}{n_e}} - 1 \right] in\, kJ$$

$$where,\ V_1 = V_s + V_c\ (m^3)$$

$$V_4 = V_3 \left(\frac{p_3}{p_4} \right)^{\frac{1}{n_e}} = V_3 \left(\frac{p_2}{p_1} \right)^{\frac{1}{n_e}}\ (m^3)$$



Numerical on Air compressor

Solution:

Indicated work input per cycle:

$$W_{in} = \frac{n_c}{n_c - 1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_c - 1}{n_c}} - 1 \right] - \frac{n_e}{n_e - 1} p_4 V_4 \left[\left(\frac{p_3}{p_4} \right)^{\frac{n_e - 1}{n_e}} - 1 \right]$$

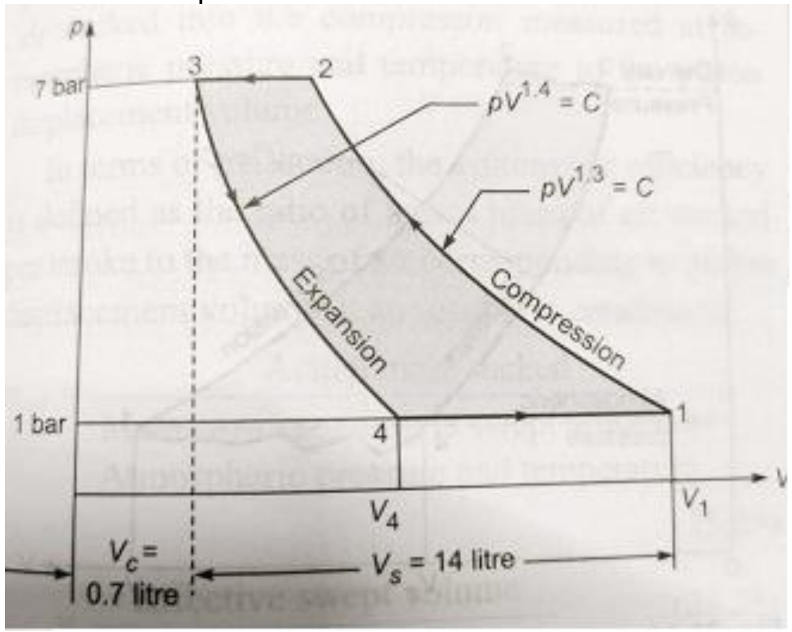
$$W_{in} = \frac{1.3}{1.3 - 1} \times 100 \times 0.0147 \left[\left(\frac{7}{1} \right)^{\frac{1.3-1}{1.3}} - 1 \right] - \frac{1.4}{1.4 - 1} \times 100 \times 0.00281 \left[\left(\frac{7}{1} \right)^{\frac{1.4-1}{1.4}} - 1 \right]$$

$$W_{in} = 3.61 - 0.731 = 2.88\, kJ / cycle$$

where,

$$V_1 = V_s + V_c = 14 + 0.7 = 14.7\, litre = 0.0147\, m^3$$

$$V_4 = V_3 \left(\frac{p_3}{p_4} \right)^{\frac{1}{n_e}} = V_3 \left(\frac{p_2}{p_1} \right)^{\frac{1}{n_e}} = 0.0007 \times \left(\frac{7}{1} \right)^{\frac{1}{1.4}} = 0.00281 m^3$$



Numerical on Air compressor

5. A single stage, single-acting air compressor operating at 1000 rpm delivers air at 25 bar. For this purpose the induction and free air conditions can be taken as 1.013 bar and 15^oC, and the FAD as 0.25 m^3/min. The clearance volume is 3% of the swept volume and the stroke to bore ratio is 1.2.

Calculate i) The bore and stroke

ii) Volumetric efficiency and

iii) The indicated power.

(Take the index of compression and re-expansion as 1.3)

Numerical on Air compressor

$Given\ data:$

$FAD = V_a = 0.25\, m^3/\min$

$P_1 = 1.013\, bar$

$P_2 = 25\, bar$

$N = 1000\, rpm$

$\frac{L}{d} = 1.2$

$To\ Find:$

$(i)\ d, L$

$(ii)\ \eta_{vol}$

$(iii)\ I.P.$

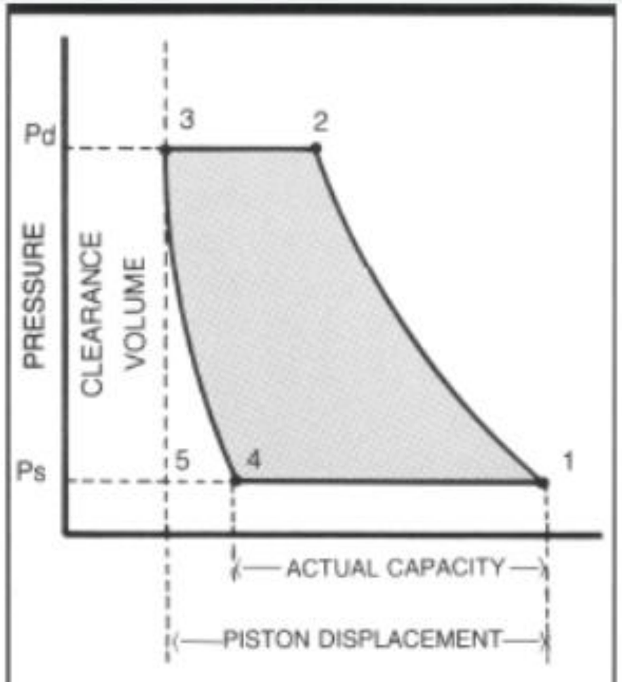


FIGURE 7: Volumetric efficiency is the ratio of actual capacity to piston displacement.

Numerical on Air compressor

$$To\,Find:$$

$$(i)\,D, L \quad (ii)\eta_{vol} \quad (iii)\,I.P.$$

$$\eta_{vol} = 1 + k - k\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$\eta_{vol} = \frac{FAD}{V_S} = \frac{V_a}{V_S} = \frac{V_1 - V_4}{V_1 - V_3}$$

$$here, k = \frac{V_c}{V_s}$$

$$Here, V_S = \frac{\pi}{4} d^2 \times L$$

$$(iii)\,I.P. = \frac{n}{n-1} p_1 (V_1 - V_4)\left(\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right) \quad (if\ n_c = n_e)$$

$$here, V_1 - V_4 = V_a = FAD$$

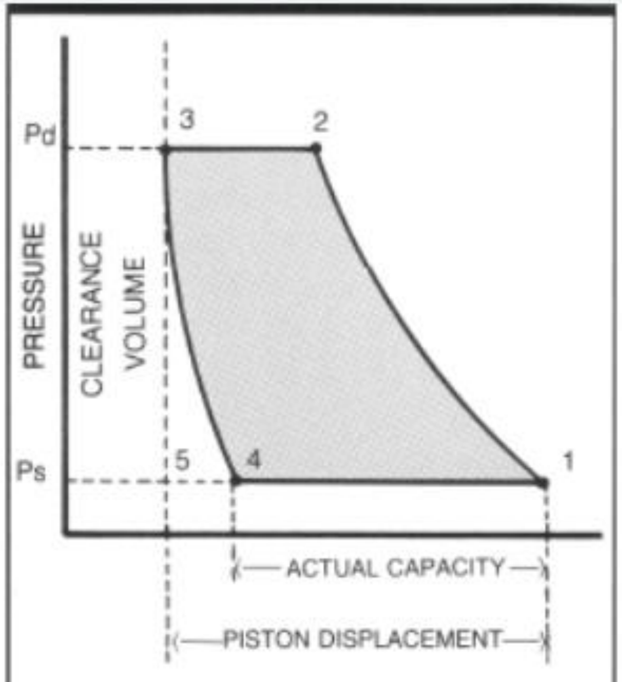


FIGURE 7: Volumetric efficiency is the ratio of actual capacity to piston displacement.

Answer:

Calculate i) the bore and stroke **73.2 mm, 87.8 mm**

ii) the volumetric efficiency **67.7 %**

and iii) the indicated power **2 kW**. Take the index of compression and re-expansion as 1.3.

Numerical on Air compressor

4. A single stage, single acting reciprocating compressor delivers 15 m^3 of free air per minute, pressure increased from 1 bar to 8 bar. The speed of compressor is 300 rpm. If the clearance is 1/16th of the swept volume, determine the diameter and stroke of the compressor. Take L/D =1.5 and n =1.3.

$$Given\,data:$$

$$V_a = 15 m^3/\min$$

$$P_1 = 1 bar$$

$$P_2 = 8 bar$$

$$N = 300\,rpm$$

$$\frac{V_c}{V_s} = \frac{1}{16}$$

$$To\,Find:$$

$$(i)\,D$$

$$(ii)\,L$$

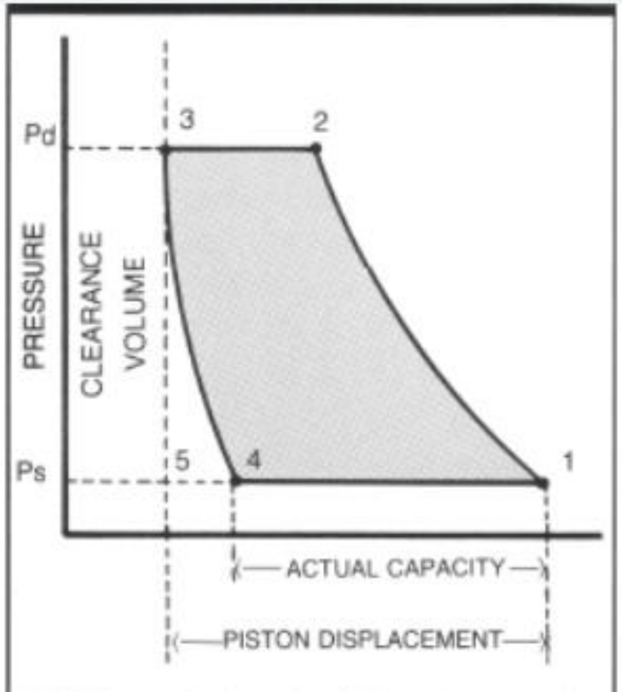


FIGURE 7: Volumetric efficiency is the ratio of actual capacity to piston displacement.

Numerical on Air compressor

Formula :

$$\eta_{vol} = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$$

$$\eta_{vol} = \frac{V_a}{V_s} \Rightarrow$$

$$V_s = \frac{V_a}{\eta_{vol}} (m^3/\min)$$

$$V_s = \frac{V_s\,(m^3/\min)}{N\,(rev/\min)} = \ldots\ldots\ldots\ldots m^3$$

$$V_s = \frac{\pi}{4} D^2 \times L \ldots\ldots\ldots(1)$$

$$\because \frac{L}{D} = 1.5 \quad \ldots\ldots\ldots\ldots(2)(given)$$

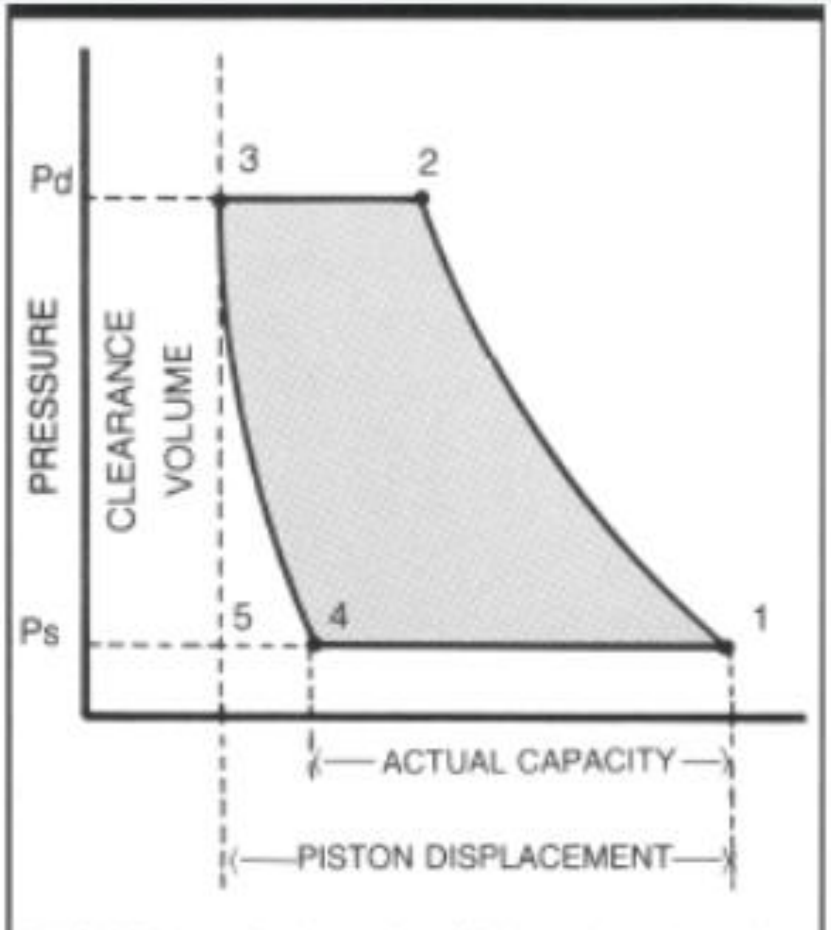


FIGURE 7: Volumetric efficiency is the ratio of actual capacity to piston displacement.

Numerical on Air compressor

$Solution:$

$$\eta_{vol} = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right] = 75.3\%$$

$$\eta_{vol} = \frac{V_a}{V_s} \Rightarrow$$

$$V_s = \frac{V_a}{\eta_{vol}} = \frac{15\,m^3/\min}{0.753} = \frac{19.92\,m^3/\min}{300\,rev/\min} = 0.0664\,m^3$$

$$V_s = \frac{\pi}{4}d^2 \times L = 0.0664\,m^3 \text{.........}(1)$$

$$\because \frac{L}{D} = 1.5 \qquad \text{...........}(2)$$

$$D = 0.383\,m$$

$$L = 0.575\,m$$

Numerical on Air compressor

5. A single-stage, single-acting reciprocating air-compressor delivers 0.6 kg of air per minute at 6 bar. The temperature and pressure at the end of suction stroke are 30°C and 1 bar. The bore and stroke of the compressor are 100 mm and 150 mm, respectively. The clearance is 3% of the swept volume. Assuming the index of compression and expansion to be 1.3 find (i) Volumetric efficiency, (ii) Power required to drive the compressor (iii) Speed of the compressor (rpm) if Mechanical efficiency is 85%.

Numerical on Air compressor

5. A single-stage, single-acting reciprocating air-compressor delivers 0.6 kg of air per minute at 6 bar. The temperature and pressure at the end of suction stroke are 30°C and 1 bar. The bore and stroke of the compressor are 100 mm and 150 mm, respectively. The clearance is 3% of the swept volume. Assuming the index of compression and expansion to be 1.3 find (i) Volumetric efficiency, (ii) Power required to drive the compressor (iii) Speed of the compressor (rpm) if Mechanical efficiency is 85%.

Given Data :

$m = 0.6\,kg/\min$

$P_2 = 6\,bar\ and\ P_1 = 1\,bar$

$T_1 = 30^\circ C = 303\,K$

$D = 100\,mm = 0.1\,m$

$L = 150\,mm = 0.15\,m$

$V_c = 0.03 V_s$

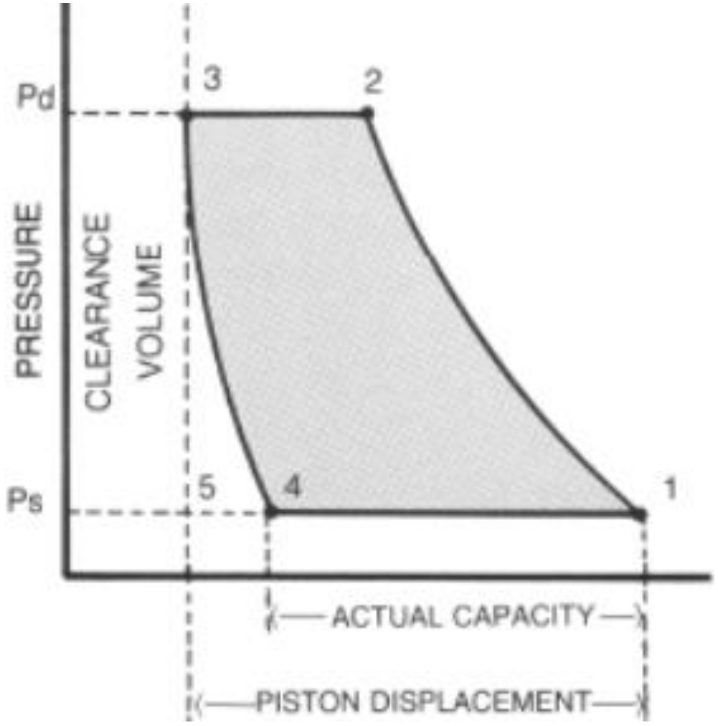
$\eta_{mech} = 85\%$

To Find :

(i) η_{vol}

(ii) $Power\,(kW)$

(iii) $N\,(rpm$



Numerical on Air compressor

$$Formula:$$

$$(i)\ \eta_{vol} = 1 - k\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right] = 1 + k - k\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} \quad (where, k = \frac{V_c}{V_s})$$

$$(ii)\ \ \eta_{mech} = \frac{Indicated\ work}{Power\ required\ to\ drive}$$

$$Indicated\ work, W = \frac{n}{n-1} mRT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$(iii)\ F.A.D(V_1) = \frac{mRT_1}{p_1} = \frac{\pi}{4} D^2 \times L \times N \times \eta_{vol}$$

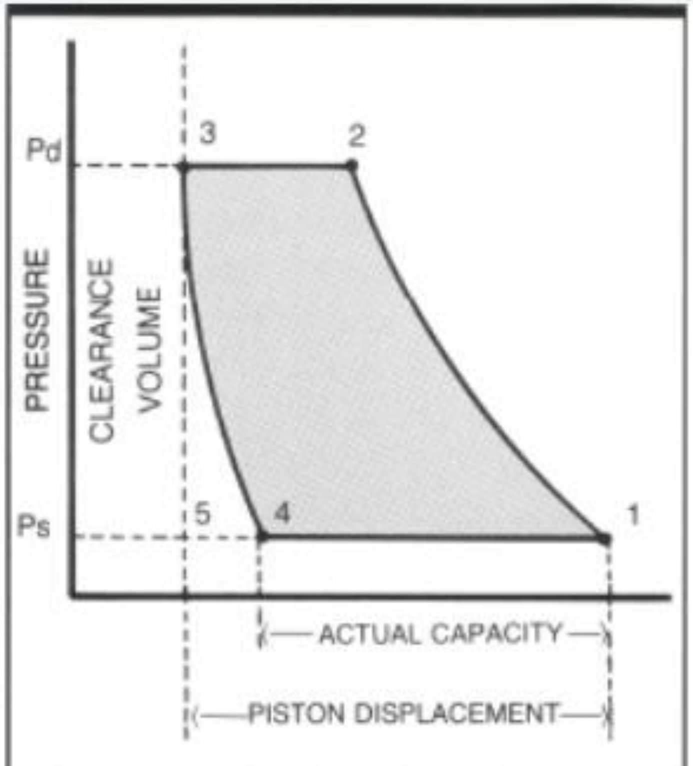


FIGURE 7: Volumetric efficiency is the ratio of actual capacity to piston displacement.

Numerical on Air compressor

$Solution:$

$$(i)\,\eta_{vol}=1-k\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}-1\right]=1+k-k\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}=91.09\%$$

$$(ii)\ Indicated\ work, W=\frac{n}{n-1}mRT_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}-1\right]=1.92\,kW$$

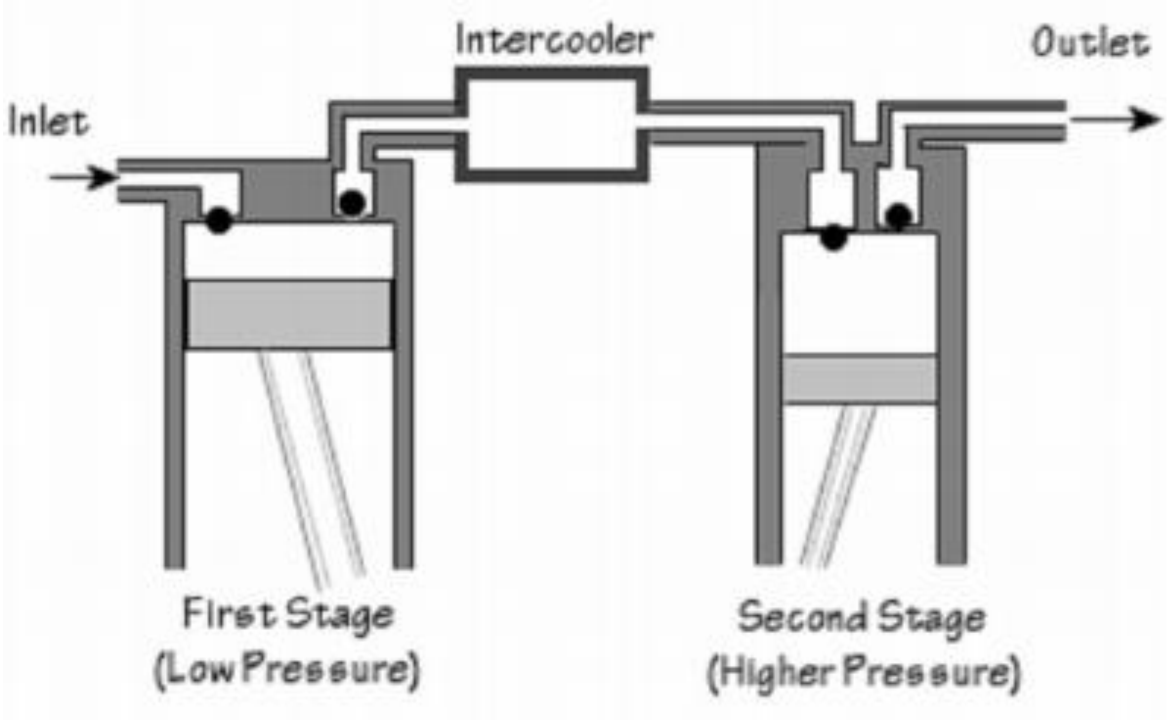
$$Power\ required\ to\ drive=\frac{Indicated\ work}{\eta_{mech}}=2.26\,kW$$

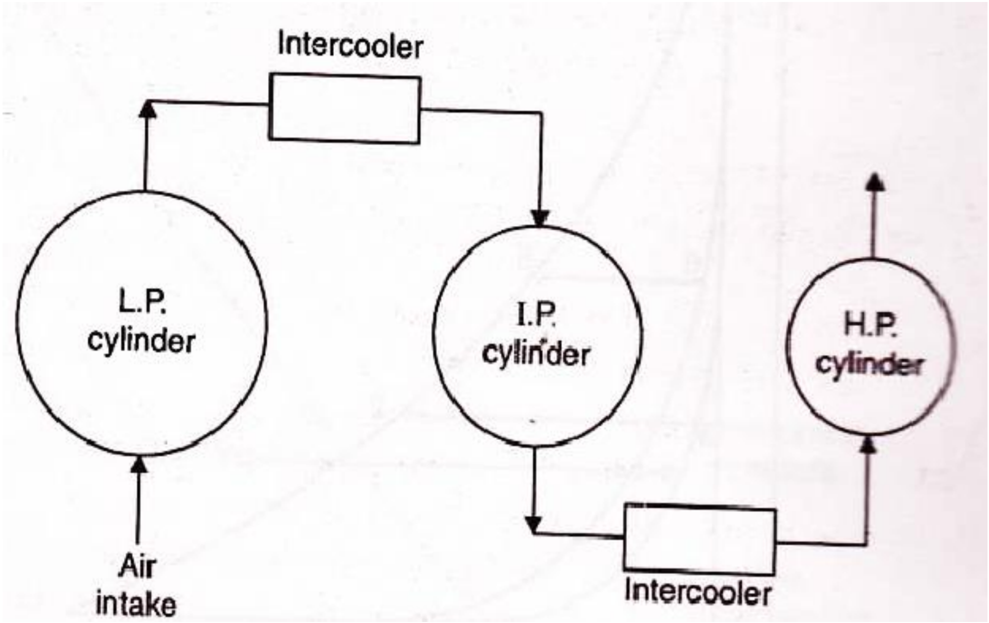
$$(iii)\,F.A.D=\frac{mRT_1}{p_1}=\frac{\pi}{4}D^2\times L\times N\times\eta_{vol}$$

$$\frac{0.6\times 287\times 303}{1\times 10^5}=\frac{\pi}{4}(0.1)^2\times 0.15\times N\times 0.9109$$

$$N=486.2\,rpm$$

Multi Stage Compressor





Advantages of Multistage compression

- Improved Efficiency: Multi-stage compression can improve the efficiency of the compression process. By compressing the gas or air in multiple stages, the compressor can operate more efficiently, reducing energy consumption and saving money on energy costs.

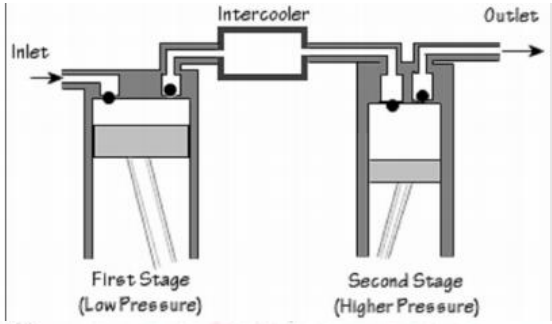
- Reduced Heat: Compressing gas or air generates heat, which can be problematic in certain applications. Multi-stage compression reduces the amount of heat generated during compression, reducing the risk of damage to equipment and improving safety.

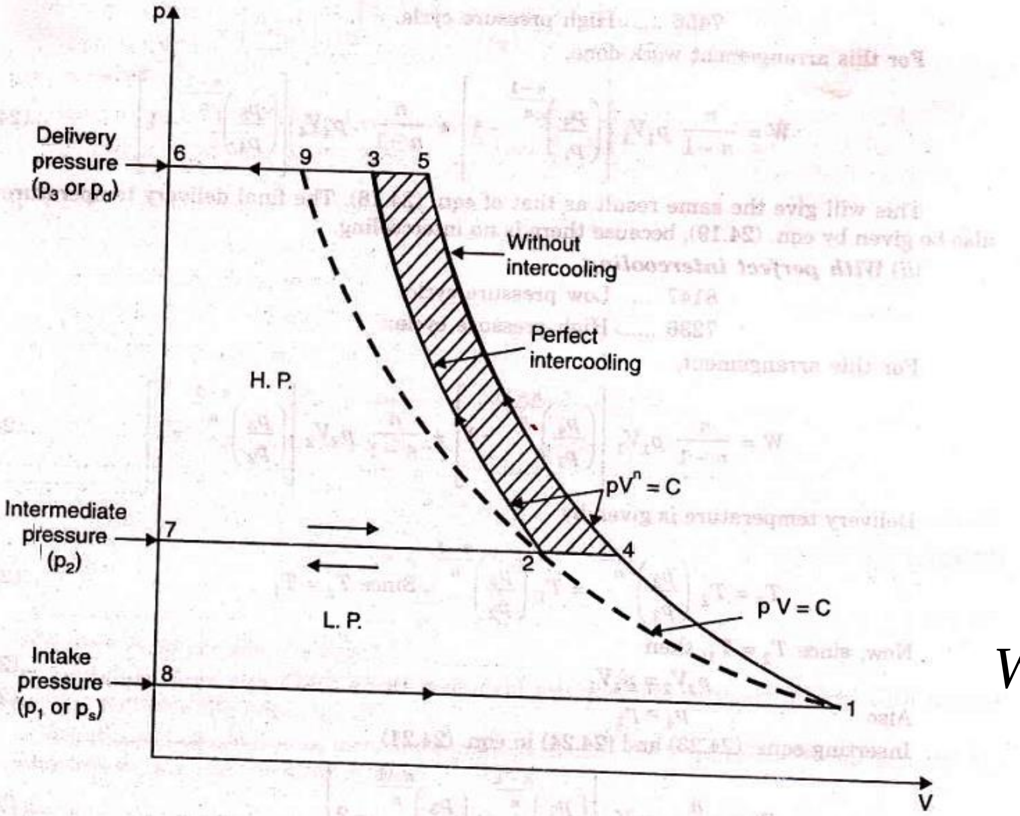
- Higher Pressure Ratios: Multi-stage compression allows for higher pressure ratios than single-stage compression. This is because each stage can be designed to operate at a different pressure ratio, allowing for greater flexibility in the compression process.

- Better Control: Multi-stage compression allows for better control over the compression process. Each stage can be controlled independently, allowing for greater precision and accuracy in the compression process.

- Longer Equipment Life: Multi-stage compression can extend the life of the compressor equipment. By reducing the workload on each stage, the compressor equipment is subjected to less wear and tear, resulting in longer equipment life and reduced maintenance costs.

Work of 2-Stage Compressor





(i) If operates in Single stage compressor

$$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{p_5}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

(ii) If operates in two-stage compressor

(a). Without inter-cooling

8147-----Low pressure cycle

7456------High pressure cycle

$$W_{input} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{p_4}{p_1} \right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{n-1} P_4 V_4 \left[\left(\frac{p_5}{p_4} \right)^{\frac{n-1}{n}} - 1 \right]$$

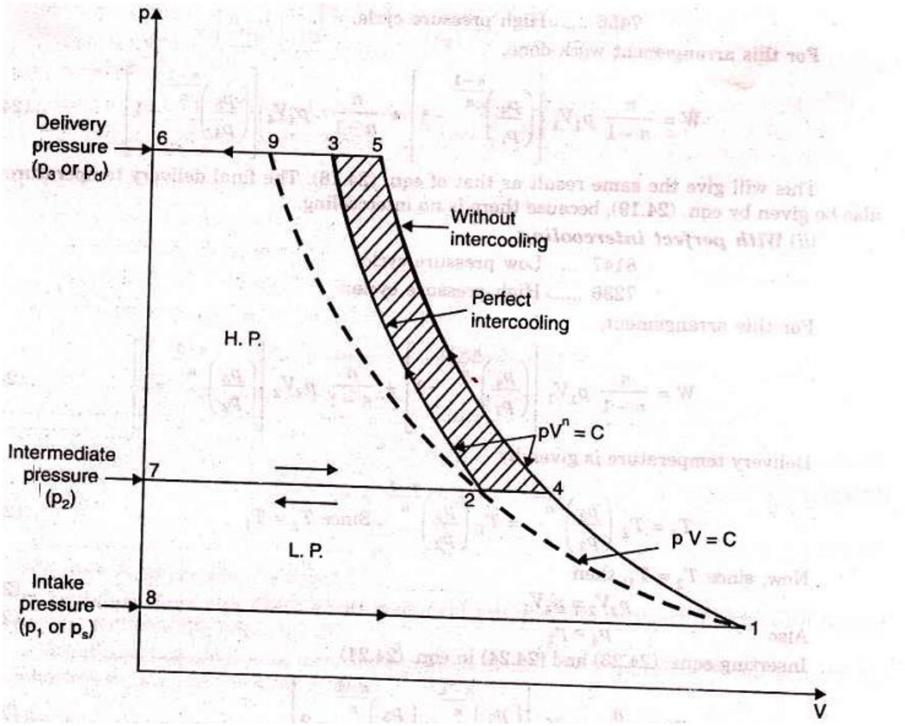
Work of 2-Stage Compressor

(ii) If operates in two-stage compressor

(b). With perfect inter-cooling

8147-----Low pressure cycle

7236------High pressure cycle



$$W_{input} = \frac{n}{n-1} P_1V_1\left[\left(\frac{p_4}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1} P_2V_2\left[\left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 1\right]$$

Delivery tempearture is

$$T_3 = T_2\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} = T_1\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} \quad (\because T_2 = T_1)$$

$$Then, P_2V_2 = P_1V_1$$

$$Also, P_2 = P_4$$

$$W_{input} = \frac{n}{n-1} P_1V_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} - 2\right]$$

Condition for Minimum Work

Hence, for *minimum*, W, $\dfrac{dW}{dp_2} = \dfrac{d\left[\left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\dfrac{p_3}{p_2}\right)^{\frac{n-1}{n}}\right]}{dp_2} = 0$

Differentiating, with respect to p_2,

$$\frac{1}{(p_1)^{\frac{n-1}{n}}} \times \left(\frac{n-1}{n}\right) p_2^{\left(\frac{n-1}{n}\right)-1} + (p_3)^{\frac{n-1}{n}} \times -\left(\frac{n-1}{n}\right) (p_2)^{-\left(\frac{n-1}{n}\right)-1} = 0$$

$$\frac{1}{p_1^{\frac{n-1}{n}}} \times \left(\frac{n-1}{n}\right) (p_2)^{-1/n} = (p_3)^{\frac{n-1}{n}} \times \left(\frac{n-1}{n}\right) p_2^{\frac{-2n+1}{n}}$$

$$\frac{p_2^{-1/n}}{p_2^{\left(\frac{-2n+1}{n}\right)}} = (p_1 p_3)^{\frac{n-1}{n}}$$

$$p_2^{-1/n} \times p_2^{-\left(\frac{-2n+1}{n}\right)} = (p_1 p_3)^{\frac{n-1}{n}}$$

$$p_2^{-1/n} \; p_2^{\frac{2n-1}{n}} = (p_1 p_3)^{\frac{n-1}{n}}$$

$$p_2^{\frac{2n-2}{n}} = (p_1 p_3)^{\frac{n-1}{n}}$$

$\therefore$

$$p_2^2 = p_1 p_3$$

$$p_2 = \sqrt{p_1 p_3}$$

$$\frac{p_2}{p_1} = \frac{p_3}{p_2}$$

pressure ratio per stage is equal.

Minimum Work Input Required for Two/Multi Stage Compression

Two-Stage Compressor

$$W = \frac{2n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$W = \frac{2n}{n-1} p_1 V_1 \left[\left\{ \frac{(p_1 p_3)^{1/2}}{p_1} \right\}^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{2n}{n-1} p_1 V_1 \left[\left\{ \left(\frac{p_3}{p_1} \right)^{1/2} \right\}^{\frac{n-1}{n}} - 1 \right]$$

$$W_{\min} = \left(\frac{2n}{n-1} \right) m_a R T_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$\frac{p_2}{p_1} = \frac{\sqrt{p_1 p_3}}{p_1} = \left(\frac{p_3}{p_1} \right)^{\frac{1}{2}}$$

Total minimum power:

$$W_{\min} = \left(\frac{2n}{n-1} \right) m_a R T_1 \left[\left(\frac{p_3}{p_1} \right)^{\frac{n-1}{2n}} - 1 \right]$$

Work required for x-Stages

$$W = \frac{xn}{n-1} p_1 V_1 \left[\left(\frac{p_{(x+1)}}{p_1} \right)^{\frac{n-1}{xn}} - 1 \right]$$

This expression can be extended to 'x' stages of compression

$$W_{\min} = \left(\frac{xn}{n-1} \right) m_a R T_1 \left[\left(\frac{p_{(x+1)}}{p_1} \right)^{\frac{n-1}{xn}} - 1 \right]$$

Numerical on Air compressor-Minimum work

5. Mass of air enters the L.P cylinder of two-stage reciprocating air compressor is 2 kg/s. The overall pressure ratio is 9. The air at inlet to the compressor is 100 kPa and 35°C. The index of compression in each cylinder is 1.3. Find the intercooler pressure for perfect intercooling. Also find the minimum power required for compression and percentage saving over single-stage compression.

Numerical on Air compressor-Minimum work

5. Mass of air enters the L.P cylinder of two-stage reciprocating air compressor is 2 kg/s. The overall pressure ratio is 9. The air at inlet to the compressor is 100 kPa and 35°C. The index of compression in each cylinder is 1.3. Find the intercooler pressure for perfect intercooling. Also find the minimum power required for compression and percentage saving over single-stage compression.

Given data:

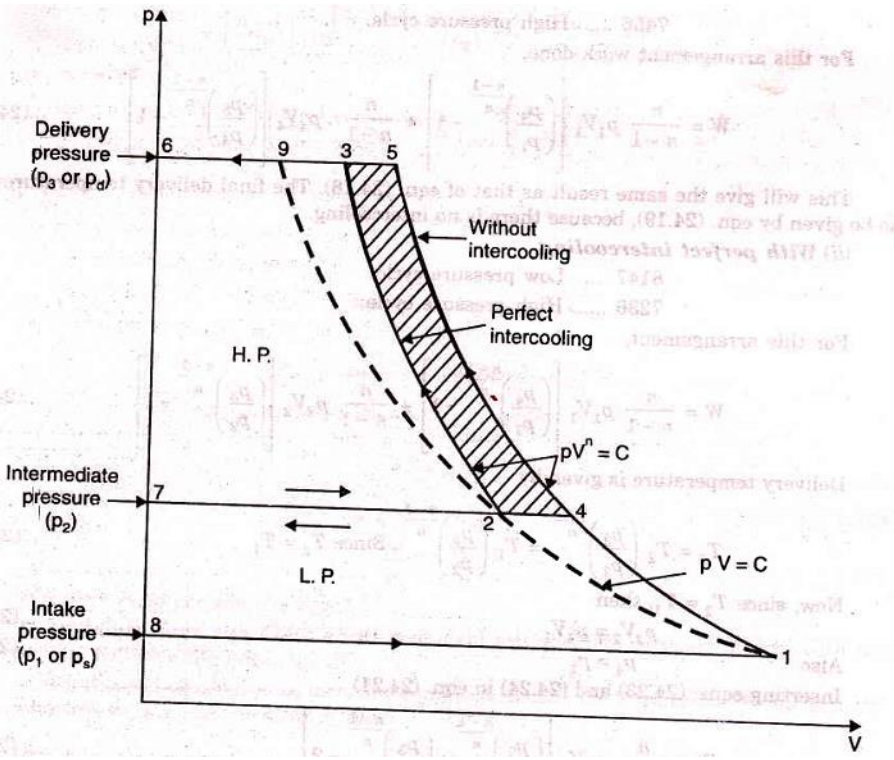
$$m_a = 2\,kg/s$$

$$T_1 = 35^oC = 308\,K$$

$$P_1 = 100kPa$$

$$\frac{P_3}{P_1} = 9$$

$$n = 1.3$$



Numerical on Air compressor-Minimum work

$Given\ data:$

$m_a = 2\, kg/s$

$T_1 = 35^o C = 308\, K$

$P_1 = 100 kPa$

$\frac{P_3}{P_1} = 9$

$n = 1.3$

$To\ Find:$

$(i)\ p_2 = \sqrt{p_1 \times p_3}$

$$(ii) W_{min,2-stage} = \left(\frac{2n}{n-1}\right) m_a R T_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right]$$

$$(iii)\ Percentage\ saving = \frac{W_{single} - W_{2-stage}}{W_{single}} \times 100\%$$

$where,$

$$W_{Single} = \left(\frac{n}{n-1}\right) m_a R T_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Numerical on Air compressor-Minimum work

Solution :

$$(i)\ p_2 = \sqrt{p_1 \times p_3} = 3 bar$$

$$(ii) W_{\min,2-stage} = \left(\frac{2n}{n-1}\right) m_a R T_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right] = 442.1 kW$$

$$(iii)\ Percentage\ saving = \frac{W_{single} - W_{2-stage}}{W_{single}} \times 100 = 12.61\%$$

where,

$$W_{Single} = \left(\frac{n}{n-1}\right) m_a R T_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}} - 1\right] = 505.92 kW$$

Numerical on Air compressor-Minimum work

6. The pressure limits of a 3-stage compressor are 1.05 bar and 40 bar. The compressor takes in air of 3 m^3 per minute. The law of compression is $pv^{1.25}$ = constant. Calculate the following on minute basis,

(i) Indicated work done assuming the conditions of maximum efficiency
(ii) Isothermal work between the pressure limits
(iii) Isothermal efficiency
(iv) Indicated work if the compressor is one-stage
(v) Percentage work saving due to 3-stage compressor

Given data :

$$P_1 = 1.05\, bar$$

$$P_3 = 40\, bar$$

$$V_a = 3\, m^3 / \min = V_1$$

$$pv^{1.25} = const$$

Numerical on Air compressor-Minimum work

$$(i)\,Work\ done,\ operating\ in\ 3-stage:$$

$$W_{actual,3-stage}=\left(\frac{3n}{n-1}\right)p_1V_1\left[\left(\frac{p_d}{p_s}\right)^{\frac{n-1}{3n}}-1\right]$$

$$(ii)\,W_{iso}=p_1V_1\ln\left(\frac{p_d}{p_s}\right)$$

$$(iii)\,\eta_{iso}=\frac{W_{iso}}{W_{actual}}$$

$$(iv)\,Work\ done,\ If\ operating\ in\ single\ stage:$$

$$W_{actual,single}=\left(\frac{n}{n-1}\right)p_1V_1\left[\left(\frac{p_d}{p_s}\right)^{\frac{n-1}{n}}-1\right]$$

$$(v)\,Percentage\ work\ saved=\left(\frac{W_{1-stage}-W_{3-stage}}{W_{1-stage}}\right)\times 100$$

Numerical on Air compressor-Minimum work

$$(i)\, Work\ done,\ operating\ in\ 3-stage:$$

$$W_{actual,3-stage}=\left(\frac{3n}{n-1}\right)p_1V_1\left[\left(\frac{p_d}{p_s}\right)^{\frac{n-1}{3n}}-1\right]=12,97,725.7\,(Nm/\min)=J/\min$$

$$(ii)\, W_{iso}=p_1V_1\times\ln\left(\frac{p_d}{p_s}\right)=1,14,6628\,(Nm/\min)=J/\min$$

$$(iii)\, \eta_{iso}=\frac{W_{iso}}{W_{actual}}=88.3\%$$

$$(iv)\, Work\ done,\ If\ operating\ in\ single\ stage:$$

$$W_{actual,\sin gle}=\left(\frac{n}{n-1}\right)p_1V_1\left[\left(\frac{p_d}{p_s}\right)^{\frac{n-1}{n}}-1\right]=16,86,780.2\ (Nm/\min)=J/\min$$

$$(v)\, Percentage\ work\ saved=\left(\frac{W_{1-stage}-W_{3-stage}}{W_{1-stage}}\right)\times100=23\%$$

Numerical on Air compressor-Minimum work

7. A 2-stage single-acting reciprocating compressor takes in air at the rate of 0.2 m^3/s. The intake pressure and temperature of air are 0.1 *Mpa* and 16°*C*. The air is compressed to a final pressure of 0.7 *Mpa*. The intermediate pressure is ideal and the intercooling is perfect. The compression index in both the stages is 1.25 and compressor runs at 600 rpm. Neglecting clearance ($V_c = 0$), determine (i) Intermediate pressure (ii) The total volume of each cylinder (iii) Power required to drive the compressor (iv) Rate of heat rejection in the intercooler take, c_p=1.005 *kJ/kg-K* and R = 0.287 *kJ/kg-K*.

Numerical on Air compressor-Minimum work

7. A 2-stage single-acting reciprocating compressor takes in air at the rate of 0.2 m^3/s. The intake pressure and temperature of air are 0.1 *Mpa* and 16°*C*. The air is compressed to a final pressure of 0.7 *Mpa*. The intermediate pressure is ideal and the intercooling is perfect. The compression index in both the stages is 1.25 and compressor runs at 600 rpm. Neglecting clearance ($V_c=0$), determine (i) Intermediate pressure (ii) The total volume of each cylinder (iii) Power required to drive the compressor (iv) Rate of heat rejection in the intercooler take, c_p=1.005 *kJ/kg-K* and R =0.287 *kJ/kg-K.*

$$Given\, data:$$

$$V_1 = 0.2\, m^3/s$$

$$T_1 = 16^o C = 289\, K$$

$$P_1 = 0.1\, MPa$$

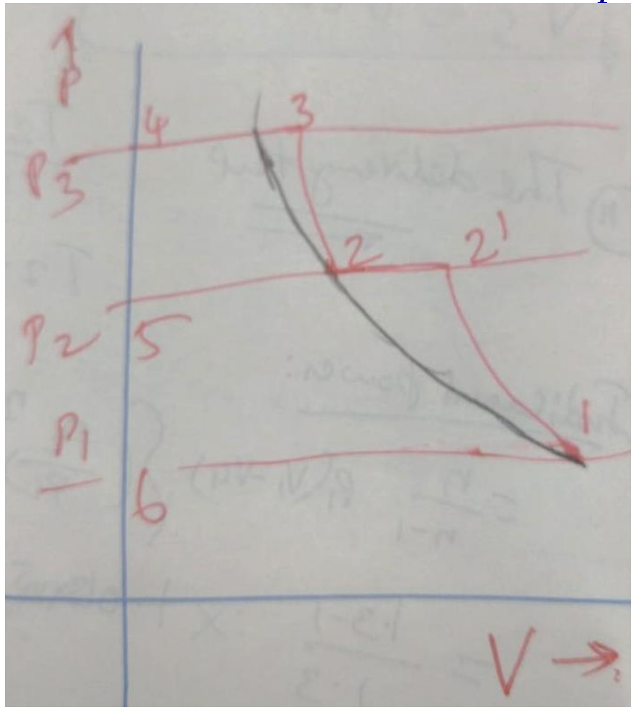
$$P_3 = 0.7\, MPa$$

$$n = 1.25$$

$$N = 600\, rpm$$

$$c_p = 1.005\, kJ/kg.K$$

$$R = 0.287\, kJ/kg.K$$



Numerical on Air compressor-Minimum work

$(i)\ Intermediate\ pressure\ (p_2) = \sqrt{p_1 p_3}$

$(ii)\ The\ total\ volume\ of\ each\ cylinder\ (Vs_1, Vs_2)$

$$V_1 = Vs_1 \times \frac{N}{60} (m^3/s) \Rightarrow Vs_1\ ?$$

$$p_1 Vs_1 = p_2 Vs_2 \qquad \Rightarrow Vs_2\ ?$$

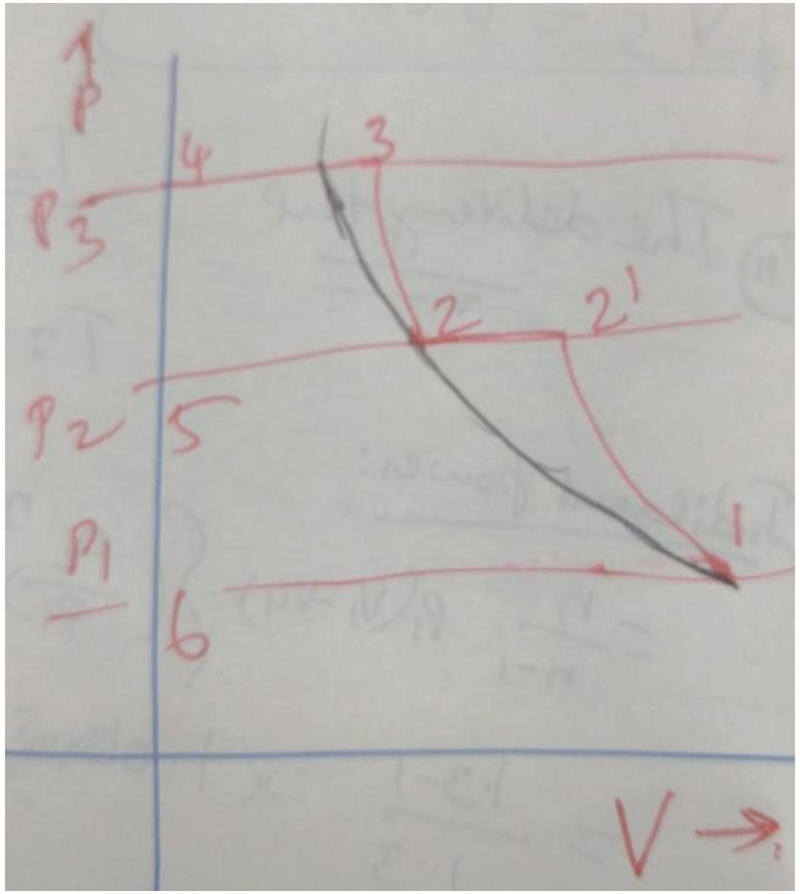
$(iii)\ Power\ required:$

$$W_{2-stage} = \left(\frac{2n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_d}{p_s}\right)^{\frac{n-1}{2n}} - 1\right]$$

$(iv)\ The\ rate\ of\ heat\ rejection\ in\ the\ intercooler$

$$Q = mc_p (T_{2^1} - T_2) \qquad (wkt: T_2 = T_1)$$

$$m = \frac{p_1 V_1}{RT_1} \quad and \quad T_{2^1} = T_1 \left(\frac{p_{2^1}}{p_1}\right)^{\left(\frac{n-1}{n}\right)} \quad (wkt: p_2 = p_{2^1})$$



Numerical on Air compressor-Minimum work

$$(i)\ Intermediate\ pressure\,(p_{inter}) = \sqrt{p_s p_d} = 0.2646\,Mpa$$

$$(ii)\ The\ volume\ of\ each\ cylinder\ (Vs_1, Vs_2)$$

$$V_1 = Vs_1 \times \frac{N}{60}(m^3/s) \Rightarrow Vs_1 = 0.02\,m^3$$

$$p_1 Vs_1 = p_2 Vs_2 \qquad \Rightarrow Vs_2 = 0.00756\,m^3$$

$$(iii)\ Power\ required:$$

$$W_{2-stage} = \left(\frac{2n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_d}{p_s}\right)^{\frac{n-1}{2n}} - 1\right] = 42.96\,kW$$

$$(iv)\ The\ rate\ of\ heat\ rejection\ in\ the\ intercooler$$

$$m = \frac{p_1 V_1}{RT_1} = 0.241\,kg/s \ \ and \quad T_2^1 = T_1 \left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} = 351.1\,K \ \ (\because p_2 = p_{2^1})$$

$$Q = mc_p(T_2^1 - T_1) = 15\,kW$$

Numerical on Air compressor-Revision

1. What is the percentage theoretical saving in work done by compressing air in two stages from 1 bar to 6 bar instead of in one stage? Assume the compression index is 1.3 and the intercooling restores air to its original temperature.

Numerical on Air compressor

$$W = \frac{n}{n-1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\}$$

$$= \frac{n}{n-1} mRT_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\}$$

$$W = \frac{2n}{n-1} p_1 V_1 \left[\left\{ \frac{(p_1 p_3)^{1/2}}{p_1} \right\}^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{2n}{n-1} p_1 V_1 \left[\left\{ \left(\frac{p_3}{p_1} \right)^{1/2} \right\}^{\frac{n-1}{n}} - 1 \right]$$

Numerical on Air compressor

Answer:

Single stage work done = 221.8 V_1 kJ

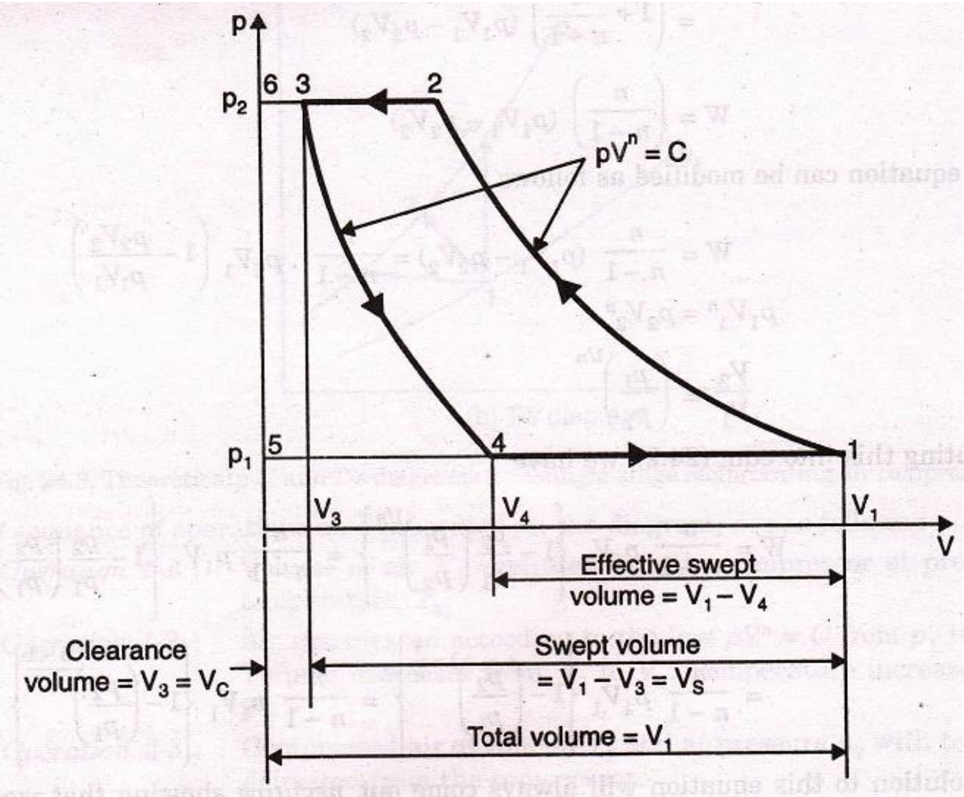
Double stage work done = 119 V_1 kJ

Theoretical saving = 10.2%

Numerical on Air compressor-Revision

A single stage, single-acting air compressor operating at 1000 rpm delivers air at 25 bar. For this purpose the induction and free air conditions can be taken as 1.013 bar and 15 deg.C, and the FAD as 0.25 m^3/min. The clearance volume is 3% of the swept volume and the stroke to bore ratio is 1.2:1. Calculate i) the bore and stroke ii) the volumetric efficiency and iii) the indicated power. Take the index of compression and re-expansion as 1.3.

Numerical on Air compressor



$$\eta_{vol} = \frac{FAD}{V_S} = \frac{V_a}{V_S} = \frac{V_1 - V_4}{V_1 - V_3}$$

Numerical on Air compressor

Calculate i) the bore and stroke **73.2 mm, 87.8 mm**

ii) the volumetric efficiency **67.7 %**

and iii) the indicated power **2 kW**. Take the index of compression and re-expansion as 1.3.

Numerical on Air compressor

Numerical on Air compressor

$$W_{min} = \left(\frac{2n}{n-1}\right) m_a R T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$\frac{p_2}{p_1} = \frac{\sqrt{p_1 p_3}}{p_1} = \left(\frac{p_3}{p_1}\right)^{\frac{1}{2}}$$

Total minimum power :

$$W_{min} = \left(\frac{2n}{n-1}\right) m_a R T_1 \left[\left(\frac{p_3}{p_1}\right)^{\frac{n-1}{2n}} - 1\right]$$

This expression can be extended to 'x' stages of compression

$$W_{min} = \left(\frac{xn}{n-1}\right) m_a R T_1 \left[\left(\frac{p_{(x+1)}}{p_1}\right)^{\frac{n-1}{xn}} - 1\right]$$

Numerical on Air compressor

$$(i)\ Intermediate\ pressure\,(p_{inter}) = \sqrt{p_s p_d} = 0.2646\,Mpa$$

$$(ii)\ The\ volume\ of\ each\ cylinder\ (Vs_1, Vs_2)$$

$$V_1 = Vs_1 \times \frac{N}{60}(m^3/s) \Rightarrow Vs_1 = 0.02\,m^3$$

$$p_1 Vs_1 = p_2 Vs_2 \qquad \Rightarrow Vs_2 = 0.00756\,m^3$$

$$(iii)\ Power\ required:$$

$$W_{2-stage} = \left(\frac{2n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_d}{p_s}\right)^{\frac{n-1}{2n}} - 1\right] = 42.96\,kW$$

$$(iv)\ The\ rate\ of\ heat\ rejection\ in\ the\ intercooler$$

$$m = \frac{p_1 V_1}{RT_1} = 0.241\,kg/s\ \ and \quad T_2^1 = T_1\left(\frac{p_2}{p_1}\right)^{\left(\frac{n-1}{n}\right)} = 351.1\,K\ \ (\because\ p_2 = p_{2^1})$$

$$Q = mc_p(T_2^1 - T_1) = 15\,kW$$