Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

School of Electronics Engineering
CAT II
Programme Name & Branch: B.Tech - ECE
Circuit Theory (BECE 203L)
Winter Semester 22-23

Slot: D2+TD2+TDD2	**Exam Duration: 90 mins**	**Maximum Marks: 50**

Answer all the questions

Q1. The switch in the network of Figure 1 is opened at $t = 0$. Find $i(t)$ for $t > 0$, if $L = \frac{1}{2}\ H$ and $C = 1\ F$. 10 M

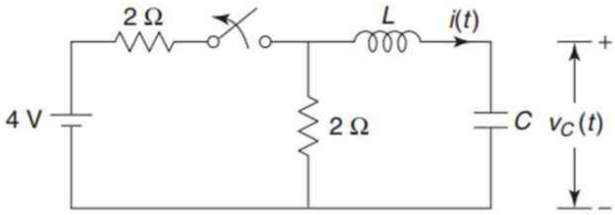


Figure 1

Q2. Find the trigonometric Fourier series of the periodic waveform in Figure 2. 10 M

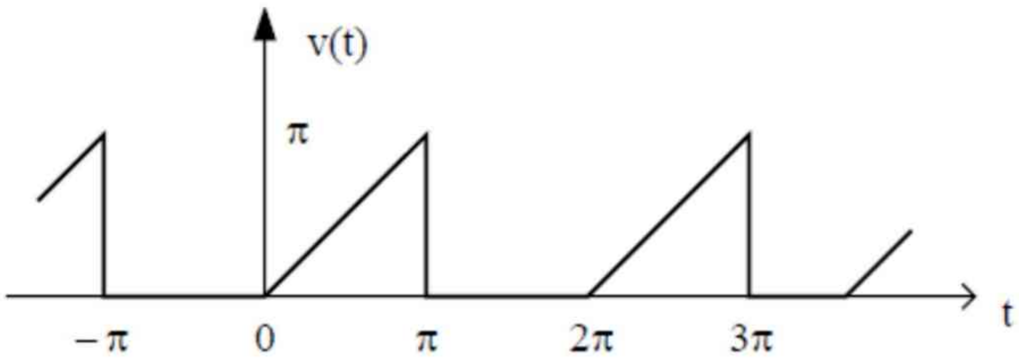


Figure 2

Q3. Let $i_s(t) = 2 + \sum_{k=1}^{\infty} \frac{4}{n\pi} sin(n\pi t),\ n = 2k - 1$ is applied to the circuit in Figure 3. Find v_o using Fourier series approach. 10 M

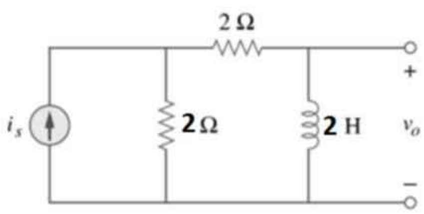


Figure 3

Q4. Use the Fourier transform to find $i_o(t)$ in the circuit shown in Figure 4. The current source $i_g(t)$ is the signum function $20\ sgn\ (t)\ A$. 10 M

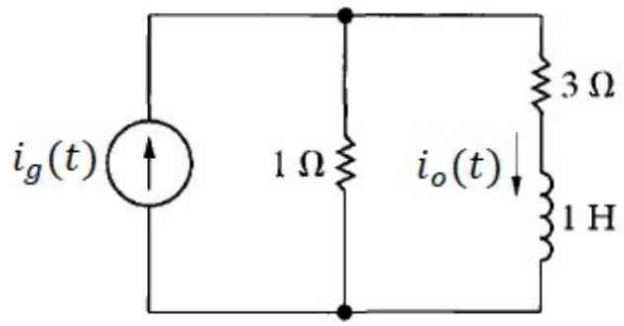


Figure 4

Q5. The switch in the network shown in Figure 5 is closed at $t = 0$. Determine the voltage across the capacitor using Laplace transform. Assume zero initial conditions. 10 M

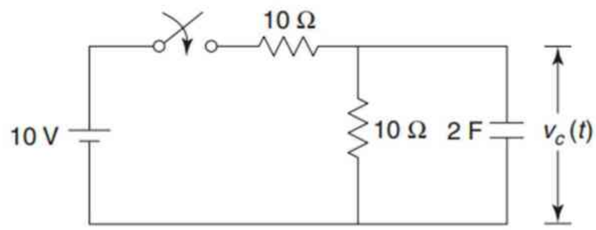


Figure 5

Question 1

Solution At $t = 0^-$, the network has attained steady-state condition. Hence, the inductor acts as a short circuit and the capacitor acts as an pen circuit.

$$v_C(0^-) = 4 \times \frac{2}{2+2} = 2\text{ V}$$

$$i(0^-) = 0$$

Since current through the inductor and voltage across the capacitor cannot change instantaneously,

$$v_C(0^+) = 2\text{ V}$$

$$i(0^+) = 0$$

Case I *When* $R = 2\ \Omega$, $L = \frac{1}{2} H$, $C = 1 F$

$$\alpha = \frac{R}{2L} = \frac{2}{2\times\frac{1}{2}} = 2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2}\times 1}} = \frac{1}{\sqrt{0.5}} = 1.414$$

$$\alpha > \omega_0$$

This indicates an overdamped case.

$$i(t) = A_1 e^{s_1 t} - A_2 e^{s_2 t}$$

where, $s_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -2 - \sqrt{4-2} = -2 - \sqrt{2} = -3.414$

and $s_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -2 + \sqrt{2} = -0.586$

$$i(t) = k_1 e^{-3.414t} + k_2 e^{-0.586t}$$

At $t = 0$, $i(0) = 0$

$$k_1 + k_2 = 0 \qquad \text{...(i)}$$

Also $v_L(0^+) + v_C(0^+) + v_R(0^+) = 0$

$$v_L(0^+) = -v_R(0^+) - v_C(0^+) = -2i(0^+) - v_C(0^+) = -2\text{ V} \qquad \text{...(ii)}$$

$$v_L(0^+) = L\frac{di}{dt}(0^+)$$

$$\frac{di}{dt}(0^+) = \frac{v_L(0^+)}{L} = -\frac{2}{0.5} = -4\text{ A/s}$$

Differentiating the equation of $i(t)$ and putting the condition at $t = 0$,

$$-3.414\,k_1 - 0.586\,k_2 = -4 \qquad \text{...(iii)}$$

Solving Eqs (i) and (iii), we get

$$k_1 = 1.414 \quad \text{and} \quad k_2 = -1.414$$

$$i(t) = 1.414(e^{-3.414t} - e^{-0.586t}) \qquad \text{for } t > 0$$

Question 2

Since the waveform does not exhibit any symmetry, we will have to determine the coefficients a_0, a_n and b_n. The a_0 coefficient is

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} v(t)\,dt$$

where, of course, $v(t) = t$ in the interval $0 \le t \le \pi$ and zero elsewhere and $\omega_0 = \frac{2\pi}{T}$

$$a_0 = \frac{1}{2\pi}\int_0^{\pi} t\,dt$$

$$= \frac{1}{2\pi}\left(\frac{t^2}{2}\bigg|_0^{\pi} = \frac{\pi}{4}\right)$$

a0=pi/4

The a_n coefficient is

$$a_n = \frac{2}{2\pi}\int_0^{\pi} t\cos nt\,dt$$

Using a table of integrals, we find that

$$a_n = \frac{1}{\pi}\left[\frac{1}{n^2}\cos nt + \frac{1}{n}t\sin nt\right]_0^{\pi}$$

The second term is zero at $t = \pi$ and 0 and the first term can be written as

$$a_n = \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

since the cosine term will be +1 or −1 depending upon the value of n. Thus

$$a_n = \frac{(-1)^n - 1}{\pi n^2}$$

In addition,

$$b_n = \frac{2}{2\pi}\int_0^{\pi} t\sin t\,dt$$

Once again, using a set of integral tables we find that

$$b_n = \frac{1}{\pi}\left[\frac{1}{n^2}\sin nt - \frac{1}{n}t\cos nt\right]_0^{\pi}$$

The first term will be zero at each limit, but the second term is nonzero at the uppe and thus

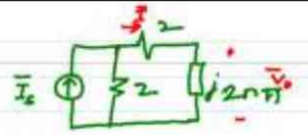
$$b_n = \frac{-\pi}{n\pi}(-1)^n$$

$$= \frac{-(-1)^n}{n}$$

Therefore, the Fourier series expansion is

$$v(t) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{\pi n^2}\right]\cos nt - \frac{(-1)^n}{n}\sin nt$$

Question 3



$$\bar{I} = \frac{\bar{I}_s(2)}{4+j2n\pi} \qquad \bar{V}_o = j2n\pi\,\bar{I}$$

$$\bar{V}_o = \frac{j2n\pi\,\bar{I}_s(2)}{4+j2n\pi} = \frac{j4n\pi\,\bar{I}_s}{4+j2n\pi} \rightarrow 2(2+jn\pi)$$

DC $n=0$ $\bar{I}_s = 2$

$\bar{V}_o = 0$

n^{th} harmonic, $n \geq 1$

$$\bar{I}_s = \frac{4}{n\pi}\angle -90^\circ$$

$$\bar{V}_o = \frac{\bar{I}_s\, 4n\pi\angle 90^\circ}{2\sqrt{4+n^2\pi^2}\;\angle \tan^{-1}\left(\frac{n\pi}{2}\right)}$$

$$\bar{V}_o = \frac{\left(\frac{4}{\cancel{n\pi}}\cancel{\angle -90^\circ}\right)\overset{2}{\cancel{4}}\cancel{n\pi}\cancel{\angle 90^\circ}}{\cancel{2}\sqrt{4+n^2\pi^2}\;\angle \tan^{-1}\left(\frac{n\pi}{2}\right)}$$

$$= \frac{8}{\sqrt{4+n^2\pi^2}}\angle -\tan^{-1}\left(\frac{n\pi}{2}\right)$$

$$\boxed{V_o(t) = \sum_{k=1}^{\infty} \frac{8}{\sqrt{4+n^2\pi^2}} \cos\left(n\pi t - \tan^{-1}\left(\frac{n\pi}{2}\right)\right), \quad n = 2k-1}$$

Question 4

The Fourier transform of the driving source is

$$I_g(\omega) = \mathcal{F}\{20\,\mathrm{sgn}(t)\}$$

$$= 20\left(\frac{2}{j\omega}\right)$$

$$= \frac{40}{j\omega}.$$

The transfer function of the circuit is the ratio of $\mathbf{I}_o$ to $\mathbf{I}_g$; so

$$H(\omega) = \frac{\mathbf{I}_o}{\mathbf{I}_g} = \frac{1}{4 + j\omega}.$$

The Fourier transform of $i_o(t)$ is

$$I_o(\omega) = I_g(\omega)H(\omega)$$

$$= \frac{40}{j\omega(4 + j\omega)}.$$

Expanding $I_o(\omega)$ into a sum of partial fractions yields

$$I_o(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{4 + j\omega}.$$

Evaluating K_1 and K_2 gives

$$K_1 = \frac{40}{4} = 10,$$

$$K_2 = \frac{40}{-4} = -10.$$

Therefore

$$I_o(\omega) = \frac{10}{j\omega} - \frac{10}{4 + j\omega}.$$

The response is

$$i_o(t) = \mathcal{F}^{-1}[I_o(\omega)]$$

$$= 5\,\mathrm{sgn}(t) - 10e^{-4t}u(t).$$

Question 5

Solution At $t = 0^-$, the capacitor is uncharged.

$$v_c(0^-) = 0$$

Since the voltage across the capacitor cannot change instantaneously,

$$v_c(0^+) = 0$$

For $t > 0$, the transformed network is shown in Fig. 7.35.

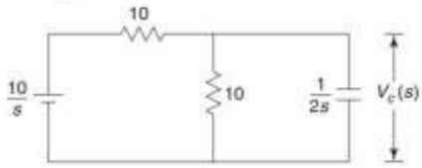


Fig. 7.35

Applying KCL at the node for $t > 0$,

$$\frac{V_c(s) - \frac{10}{s}}{10} + \frac{V_c(s)}{10} + \frac{V_c(s)}{\frac{1}{2s}} = 0$$

$$2sV_c(s) + 0.2V_c(s) = \frac{1}{s}$$

$$V_c(s) = \frac{1}{s(2s+0.2)} = \frac{0.5}{s(s+0.1)}$$

By partial-fraction expansion,

$$V_c(s) = \frac{A}{s} + \frac{B}{s+0.1}$$

$$A = sV_c(s)\Big|_{s=0} = \frac{0.5}{s+0.1}\Big|_{s=0} = \frac{0.5}{0.1} = 5$$

$$B = (s+0.1)V_c(s)\Big|_{s=-0.1} = \frac{0.5}{s}\Big|_{s=-0.1} = -\frac{0.5}{0.1} = -5$$

$$V_c(s) = \frac{5}{s} - \frac{5}{s+0.1}$$

Taking inverse Laplace transform,

$$v_c(t) = 5 - 5e^{-0.1t} \qquad \text{for } t > 0$$