Final Assessment Test – November/December 2023

Vellore Institute of Technology

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2069 /2071 / 2072 / 2073 / 2074 / 2075 / 2076 / 2077 / 2078 / 2279 / 2080 / 2081 /2082 /2083 / 2084 / 2085 / 2086 / 2087 / 2088 / 2089 / 2090

Slot: C2+TC2+TCC2

Time: Three Hours

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Show that $\psi = x^2 - y^2 - 3x - 2y + 2xy$ can represent the stream function of an incompressible fluid flow. Also find the corresponding velocity potential ϕ and hence the complex potential $f(z) = \varphi + i\psi$.

2. Find the analytic function $w = u + iv$, if $2u - 3v = 3y^2 - 4xy - 3x^2 + 3y - 2x$, and $f(0) = 0$. Hence find u.

3. Find the image of the triangular region in the z-plane bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ under the mapping (i) $w = 2z$ (ii) $w = e^{\frac{i\pi}{4}}z$.

4. Find the bilinear transformation that maps the points $z_1 = 0, z_2 = 1, z_3 = \infty$ into the points $w_1 = i$, $w_2 = -1$, $w_3 = -i$ and also find its invariant points.

5. Find the Laurent's series expansion of the function $f(z) = \frac{z}{(z-1)(z-3)}$ which are valid in the range (i) $0 < |z - 1| < 2$ (ii) $|z - 1| > 2$.

6. Evaluate $\int_0^{\infty} \frac{x \sin x}{(x^2+1)(x^2+4)} dx$, by contour integration.

7. Find the basis and dimension of row space, column space and null space of

$$A = \begin{bmatrix} 1 & -3 & 2 & -3 & 9 \\ 2 & 0 & 1 & 3 & 3 \\ -2 & -4 & 1 & -9 & 7 \\ 1 & 3 & -1 & 6 & -6 \end{bmatrix}.$$

8. Let $G : R^3 \to R^3$ be the linear mapping defined by

$$G(x; y; z) = (x - y + 2z;\ 2x + y;\ -x - 2y + 2z)$$

Find a basis and the dimension of (i) the image of G, (ii) the kernel of G.

9. Let $T : R^3 \to R^2$ be the linear transformation defined by

$T(x; y; z) = (3x + 2y - 4z;\ x - 5y + 3z)$

Find $[T]_\alpha^\beta$, for $\alpha = \{(1,1,1), (1,1,0), (1,0,0)\}$ and $\beta = \{(1,3), (2,5)\}$.

10. Apply the Gram–Schmidt orthogonalization process to find an orthogonal basis and for the subspace U of R^4 spanned by $v_1 = (2,1,3,-1)$, $v_2 = (7,4,3,-3)$, $v_3 = (5,7,7,8)$.

11. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, hence state the eigen values of A^{-1}, A^T and A^4.

12. Using Gauss-Jordan method, solve the system of equations

$$x + 2y + z = 8,\ 2x + 3y + 4z = 20,\ 4x + 3y + 2z = 16.$$

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