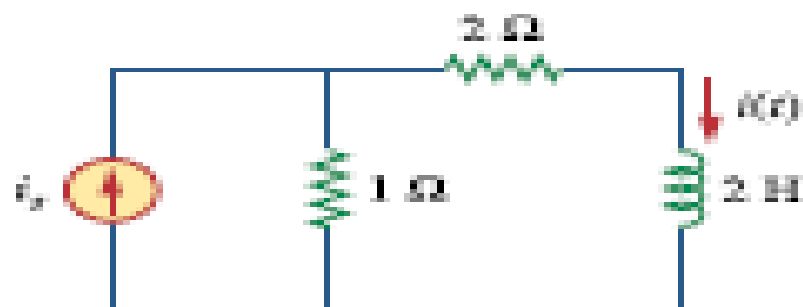


Fourier series problems

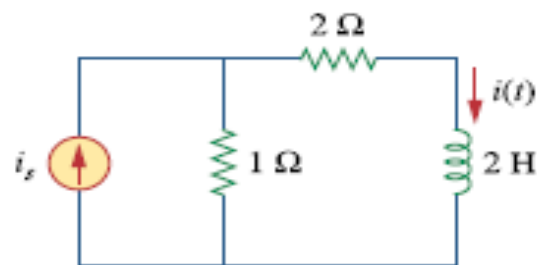
Find $i(t)$ in the circuit of Fig. 17.68 given that

$$i_s(t) = 1 + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos 3nt \text{ A}$$



$$i_s(t) = 1 + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos 3nt \text{ A}$$

When $i_s = 1$ (DC component)



$$i = 1/(1 + 2) = 1/3$$

For $n \geq 1$, $\omega_n = 3n$, $I_s = 1/n^2 \angle 0^\circ$

$$I = [1/(1 + 2 + j\omega_n^2)]I_s = I_s/(3 + j6n)$$

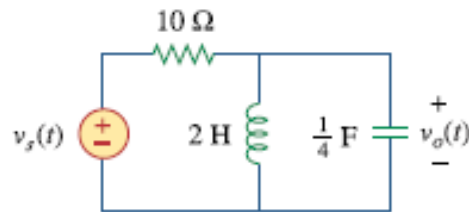
$$= \frac{\frac{1}{n^2} \angle 0^\circ}{3\sqrt{1 + 4n^2} \angle \tan^{-1}(6n/3)} = \frac{1}{3n^2\sqrt{1 + 4n^2}} \angle -\tan^{-1}(2n)$$

Thus,

$$i(t) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{3n^2\sqrt{1 + 4n^2}} \cos(3n - \tan^{-1}(2n))$$

$$v_s(t) = 3 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n\pi t)$$

Find $v_o(t)$.



For the DC case, the inductor acts like a short, $V_o = 0$.

For the AC case, we obtain the following:

$$\frac{V_o - V_s}{10} + \frac{V_o}{j2n\pi} + \frac{jn\pi V_o}{4} = 0$$

$$\left(1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)\right)V_o = V_s$$

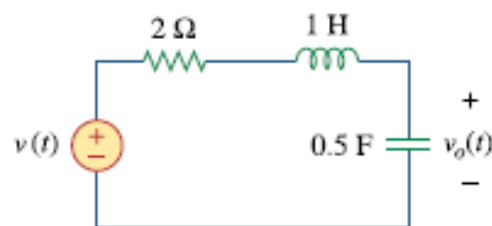
$$V_o = \frac{V_s}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)}$$

$$A_n \angle \Theta_n = \frac{4}{n\pi} \frac{1}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)} = \frac{4}{n\pi + j(2.5n^2\pi^2 - 5)}$$

$$A_n = \frac{4}{\sqrt{n^2\pi^2 + (2.5n^2\pi^2 - 5)^2}}; \quad \Theta_n = -\tan^{-1}\left(\frac{2.5n^2\pi^2 - 5}{n\pi}\right)$$

$$\underline{v_o(t) = \sum_{n=1}^{\infty} A_n \sin(n\pi t + \Theta_n) \text{ V}}$$

$$v(t) = \sum_{n=1}^{\infty} \frac{10}{n^2} \cos\left(nt + \frac{n\pi}{4}\right) \text{ V}$$



For any n , $V = [10/n^2] \angle (n\pi/4)$, $\omega = n$.

1 H becomes $j\omega_n L = jn$ and 0.5 F becomes $1/(j\omega_n C) = -j2/n$

$$V_o = \{-j(2/n)/[2 + jn - j(2/n)]\} V = \{-j2/[2n + j(n^2 - 2)]\} [(10/n^2) \angle (n\pi/4)]$$

$$= \frac{20 \angle ((n\pi/4) - \pi/2)}{n^2 \sqrt{4n^2 + (n^2 - 2)^2} \angle \tan^{-1}((n^2 - 2)/2n)}$$

$$= \frac{20}{n^2 \sqrt{n^2 + 4}} \angle [(n\pi/4) - (\pi/2) - \tan^{-1}((n^2 - 2)/2n)]$$

$$\underline{v_o(t) = \sum_{n=1}^{\infty} \frac{20}{n^2 \sqrt{n^2 + 4}} \cos\left(nt + \frac{n\pi}{4} - \frac{\pi}{2} - \tan^{-1} \frac{n^2 - 2}{2n}\right)}$$