

Engineering Mechanics

Faculty : Dr. Prashik Gaikwad
School of Mechanical Engineering

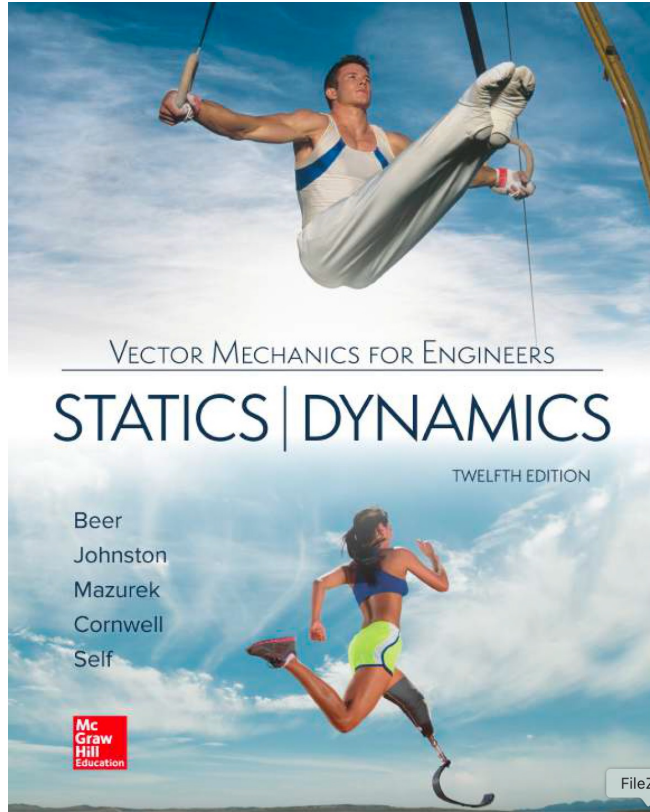
Contact details

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Cabin: CBMR 304-B

BMEE201L	Engineering Mechanics				L	T	P	C
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Pre-requisite	NIL				Syllabus version			
					1.0			
Course Objectives:								
1. To enable students to apply fundamental laws and basic concepts of rigid body mechanics to solve problems of bodies under rest or in motion.								
2. To enable the students to apply conditions of static equilibrium to analyse physical systems.								
3. To compute the properties of areas and bodies.								
Course Outcome:								
Upon successful completion of the course the students will be able to								
1. Compute the resultant and analyse equilibrium (without and with friction) of system of forces acting on particles and rigid bodies in plane and space.								
2. Predict the support-reactions and the internal forces of the members of trusses and frames.								
3. Apply transfer theorems to determine properties of various sections.								
4. Calculate motion parameters of particles and rigid bodies.								
Module:1 Statics of Particles					5 hours			
Fundamental concepts and principles - Resolution of a force -Resultant of forces in a plane- Equilibrium of a particle in a plane; Addition of concurrent forces in space- Equilibrium of a particle in space.								
Module:2 Statics of Rigid Bodies					7 hours			
Equivalent systems of forces- Principle of Transmissibility - Moment of a force about a point and an axis- Couples and force-couple systems- Equilibrium of rigid bodies in two and three dimensions- Types of beams, supports and reactions; Principle of virtual work – System of connected rigid bodies.								
Module:3 Analysis of Structures					5 hours			
Analysis of plane trusses - Method of joints and method of sections- Frames								
Module:4 Friction					5 hours			
The laws of dry friction – Coefficients of Friction- Angles of Friction- Types of Friction Problems - Wedges and Ladder friction- Belt friction.								
Module:5 Properties of Surfaces and Solids					7 hours			
First moments of areas and lines- Centroids of composite areas and lines- - Theorems of Pappus-Guldinus- Second moment of area- Parallel axis theorem- Rectangular and Polar Moments of inertia of composite areas- Radius of Gyration- Product of Inertia- Principal Axes and Principal Moments of Inertia- Mass moments of inertia of thin plates.								
Module:6 Dynamics of Particles					8 hours			
Kinematics of Particles: Displacement, Velocity and Acceleration – Rectilinear motion – Curvilinear motion – Tangential and Normal components – Radial and Transverse components.								
Kinetics of Particles: Newton's Second Law- Energy and Momentum Methods-Principle of Work and Energy-Principle of Impulse and Momentum- Direct Central Impact								
Module:7 Dynamics of Rigid Bodies					8 hours			
Kinematics of rigid bodies: Translation and fixed-axis rotation- General plane motion: velocity- Instantaneous centre of rotation- General plane motion: acceleration.								
Kinetics of rigid bodies:Equations of motion -Angular momentum- Plane motion of a rigid body- Principle of work and energy for rigid bodies- Principle of impulse and momentum for rigid bodies.								
Total Lecture hours:					45 hours			

Textbook for this course



Beer, Johnston, Cornwell, David Mazurek,
and Sanghi, Vector Mechanics for
Engineers: Statics and Dynamics, 12th
Edition, McGraw-Companies

Module 4

Friction

Module Overview

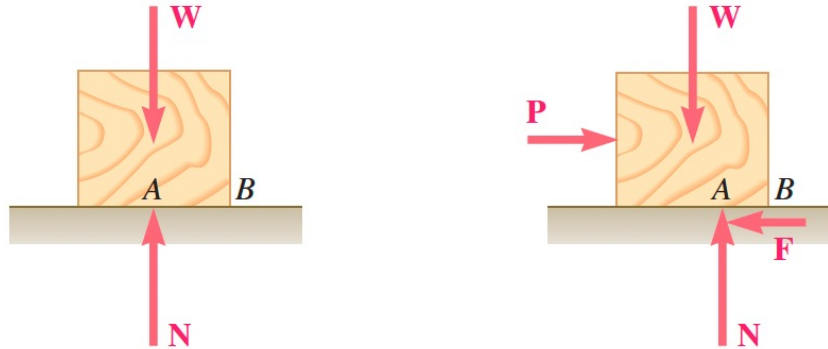
- Laws of dry friction
- Coefficients of Friction
- Angles of friction
- Types of friction problems
- Wedges friction

Introduction

- No perfectly frictionless surface exists.
- When two surfaces are in contact, tangential forces, called friction forces, always develop if you attempt to move one surface with respect to the other.
- However, these friction forces are limited in magnitude and do not prevent motion if you apply sufficiently large forces.
- Thus, the distinction between frictionless and rough surfaces is a matter of degree.
- Two types of friction
 - Dry friction also known as Coulomb friction
 - Fluid friction or viscosity

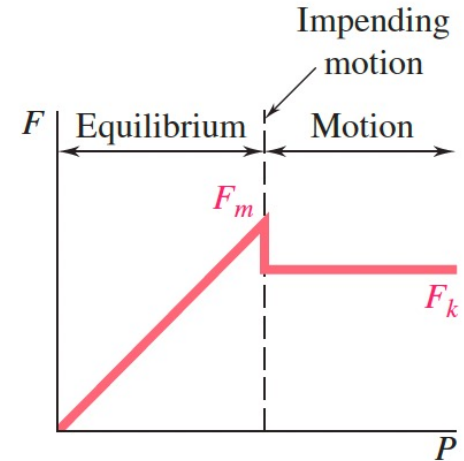
The law of dry friction

- Suppose that you apply a horizontal force P to the block.
- If P is small, the block does not move; some other horizontal force must therefore exist, which balances P .
- This other force is the static-friction force F , which is actually the resultant of a great number of forces acting over the entire surface of contact between the block and the plane



The law of dry friction

- If you increase the force P , the friction force F also increases, continuing to oppose P , until its magnitude reaches a certain maximum value F_m .
- If P is further increased, the friction force cannot balance it anymore, and the block starts sliding. As soon as the block has started in motion, the magnitude of F drops from F_m to a lower value F_k .
- This happens because less interpenetration occurs between the irregularities of the surfaces in contact when these surfaces move with respect to each other.
- From then on, the block keeps sliding with increasing velocity while the friction force, denoted by F_k and called the kinetic-friction force, remains approximately constant.



Coefficient of friction

Static friction

$$F_m = \mu_s N$$

μ_s is a constant called the **coefficient of static friction**.

Kinetic friction

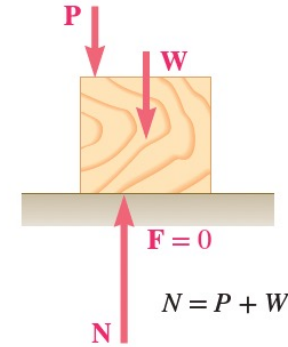
$$F_k = \mu_k N$$

μ_k is a constant called the **coefficient of kinetic friction**.

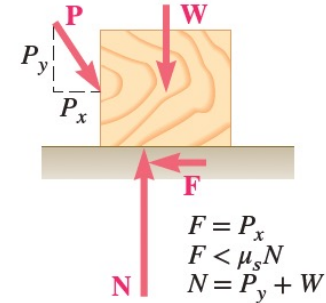
The coefficients of friction μ_s and μ_k do not depend upon the area of the surfaces in contact. Both coefficients, however, depend strongly on the nature of the surfaces in contact.

Four different situation

1. The forces applied to the body do not tend to move it along the surface of contact; there is no friction force
2. The applied forces tend to move the body along the surface of contact, but are not large enough to set it in motion. We can find the static-friction force F that has developed by solving the equations of equilibrium for the body. Because there is no evidence that F has reached its maximum value, the equation $F_m = \mu_s N$ cannot be used to determine the friction force



(a) No friction ($P_x = 0$)

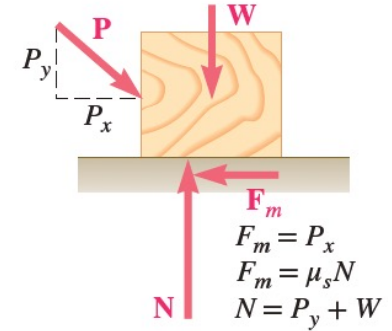


(b) No motion ($P_x < F_m$)

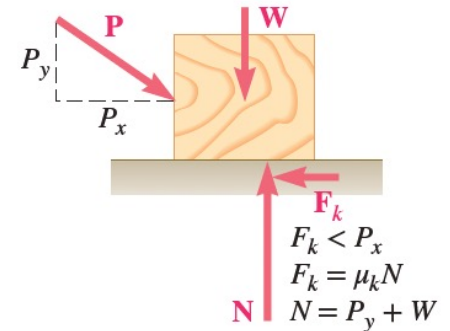
Four different situation

3. The applied forces are such that the body is just about to slide. We say that motion is impending. The friction force F has reached its maximum value F_m and, together with the normal force N , balances the applied forces. Both the equations of equilibrium and the equation $F_m = \mu_s N$ can be used. Note that the friction force has a sense opposite to the sense of impending motion.

4. The body is sliding under the action of the applied forces, and the equations of equilibrium no longer apply. However, F is now equal to F_k , and we can use the equation $F_k = \mu_k N$. The sense of F_k is opposite to the sense of motion.



(c) Motion impending $\longrightarrow (P_x = F_m)$



(d) Motion $\longrightarrow (P_x > F_k)$

Angle of Friction

Angle of static friction

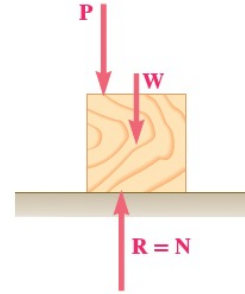
$$\tan \phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan \phi_s = \mu_s$$

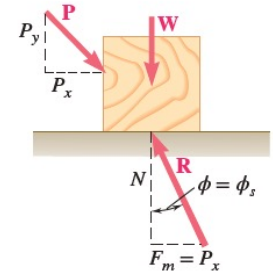
Angle of kinetic friction

$$\tan \phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

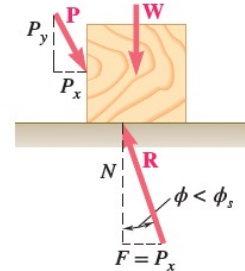
$$\tan \phi_k = \mu_k$$



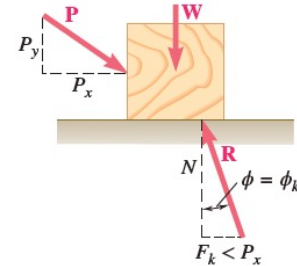
(a) No friction



(c) Motion impending $\longrightarrow$



(b) No motion



(d) Motion $\longrightarrow$

Numerical

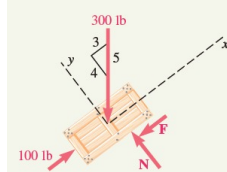
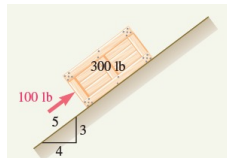


Fig. 1 Free-body diagram of crate showing assumed direction of friction force.

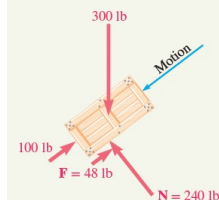


Fig. 2 Free-body diagram of crate showing actual friction force.

A 100-lb force acts as shown on a 300-lb crate placed on an inclined plane. The coefficients of friction between the crate and the plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the crate is in equilibrium, and find the value of the friction force.

STRATEGY: This is a friction problem of the first type: You know the forces and the friction coefficients and want to determine if the crate moves. You also want to find the friction force.

MODELING and ANALYSIS:

Force Required for Equilibrium. First determine the value of the friction force *required to maintain equilibrium*. Assuming that $\mathbf{F}$ is directed down and to the left, draw the free-body diagram of the crate (Fig. 1) and solve the equilibrium equations:

$$+\nearrow \Sigma F_x = 0: \quad 100 \text{ lb} - \frac{3}{5}(300 \text{ lb}) - F = 0 \\ F = -80 \text{ lb} \quad \mathbf{F} = 80 \text{ lb} \nearrow$$

$$+\searrow \Sigma F_y = 0: \quad N - \frac{4}{5}(300 \text{ lb}) = 0 \\ N = +240 \text{ lb} \quad \mathbf{N} = 240 \text{ lb} \searrow$$

The force $\mathbf{F}$ required to maintain equilibrium is an 80-lb force directed up and to the right; the tendency of the crate is thus to move down the plane.

Maximum Friction Force. The magnitude of the maximum friction force that may be developed between the crate and the plane is

$$F_m = \mu_s N \quad F_m = 0.25(240 \text{ lb}) = 60 \text{ lb}$$

Because the value of the force required to maintain equilibrium (80 lb) is larger than the maximum value that may be obtained (60 lb), equilibrium is not maintained and *the crate will slide down the plane*.

Actual Value of Friction Force. The magnitude of the actual friction force is

$$F_{\text{actual}} = F_k = \mu_k N = 0.20(240 \text{ lb}) = 48 \text{ lb}$$

The sense of this force is opposite to the sense of motion; the force is thus directed up and to the right (Fig. 2):

$$\mathbf{F}_{\text{actual}} = 48 \text{ lb} \nearrow \swarrow$$

Note that the forces acting on the crate are not balanced. Their resultant is

$$\frac{3}{5}(300 \text{ lb}) - 100 \text{ lb} - 48 \text{ lb} = 32 \text{ lb} \swarrow$$

Numerical

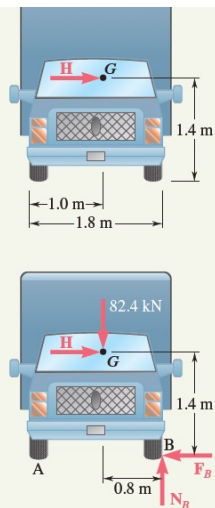


Fig. 1 Free-body diagram of the truck.

An 8400-kg truck is traveling on a level horizontal curve, resulting in an effective lateral force $\mathbf{H}$ (applied at the center of gravity G of the truck). Treating the truck as a rigid system with the center of gravity shown, and knowing that the distance between the outer edges of the tires is 1.8 m, determine (a) the maximum force $\mathbf{H}$ before tipping of the truck occurs, (b) the minimum coefficient of static friction between the tires and roadway such that slipping does not occur before tipping.

STRATEGY: For the direction of $\mathbf{H}$ shown, the truck would tip about the outer edge of the tire shown on the right in the diagram. At the verge of tip, the normal force and friction force are zero at the tire on the left, and the normal force at the right tire is at the outer edge. You can apply equilibrium to determine the value of $\mathbf{H}$ necessary for tip and the required friction force such that slipping does not occur.

MODELING: Draw the free-body diagram of the truck (Fig. 1), which reflects impending tip about point B . Obtain the weight of the truck by multiplying its mass of 8400 kg by $g = 9.81 \text{ m/s}^2$; that is, $W = 82\,400 \text{ N}$ or 82.4 kN.

ANALYSIS:

Free Body: Truck (Fig. 1).

$$+\circlearrowleft \Sigma M_B = 0: \quad (82.4 \text{ kN})(0.8 \text{ m}) - H(1.4 \text{ m}) = 0$$

$$H = +47.1 \text{ kN}$$

$$\mathbf{H} = 47.1 \text{ kN} \rightarrow$$

$$+\rightarrow \Sigma F_x = 0: \quad 47.1 \text{ kN} - F_B = 0$$

$$F_B = +47.1 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad N_B - 82.4 \text{ kN} = 0$$

$$N_B = +82.4 \text{ kN}$$

Minimum Coefficient of Static Friction. The magnitude of the maximum friction force that can be developed is

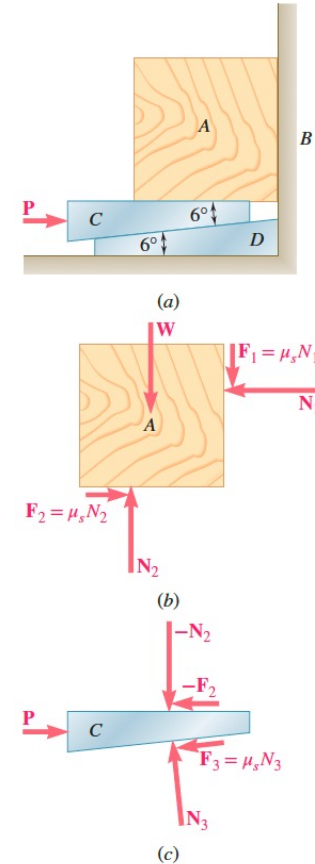
$$F_m = \mu_s N_B = \mu_s (82.4 \text{ kN})$$

Setting this equal to the friction force required, $F_B = 47.1 \text{ kN}$, gives

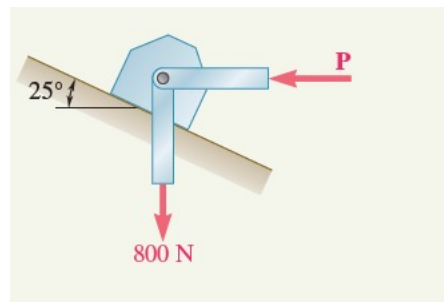
$$\mu_s (82.4 \text{ kN}) = 47.1 \text{ kN} \quad \mu_s = 0.572$$

Wedges

- Wedges are simple machines used to raise large stone blocks and other heavy loads. These loads are raised by applying to the wedge a force usually considerably smaller than the weight of the load.
- In addition, because of the friction between the surfaces in contact, a properly shaped wedge remains in place after being forced under the load. In this way, you can use a wedge advantageously to make small adjustments in the position of heavy pieces of machinery.



Practice problem



A support block is acted upon by two forces as shown. Knowing that the coefficients of friction between the block and the incline are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the force $\mathbf{P}$ required to (a) start the block moving up the incline, (b) keep it moving up, (c) prevent it from sliding down.