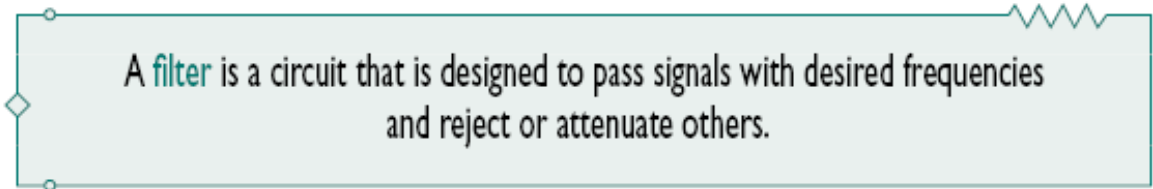


Passive filters



A **filter** is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies. Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment.

A filter is a *passive filter* if it consists of only passive elements R , L , and C . It is said to be an *active filter* if it consists of active elements (such as transistors and op amps) in addition to passive elements R , L , and C . We consider passive filters in this section and active filters in the next section. Besides the filters we study in these sections, there are other kinds of filters—such as digital filters, electromechanical filters, and microwave filters—which are beyond the level of the text.

As shown in Fig. 14.30, there are four types of filters whether passive or active:

1. A *lowpass filter* passes low frequencies and stops high frequencies, as shown ideally in Fig. 14.30(a).
2. A *highpass filter* passes high frequencies and rejects low frequencies, as shown ideally in Fig. 14.30(b).
3. A *bandpass filter* passes frequencies within a frequency band and blocks or attenuates frequencies outside the band, as shown ideally in Fig. 14.30(c).
4. A *bandstop filter* passes frequencies outside a frequency band and blocks or attenuates frequencies within the band, as shown ideally in Fig. 14.30(d).

Table 14.5 presents a summary of the characteristics of these filters. Be aware that the characteristics in Table 14.5 are only valid for first- or second-order filters—but one should not have the impression that only these kinds of filter exist. We now consider typical circuits for realizing the filters shown in Table 14.5.

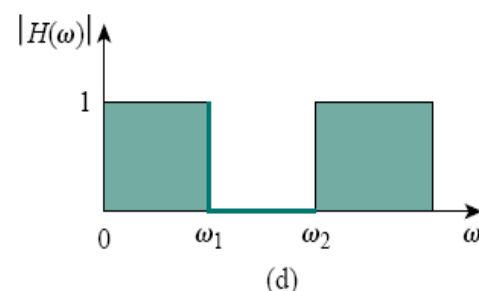
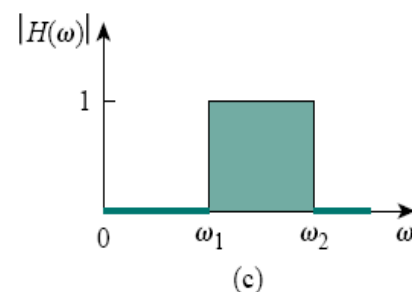
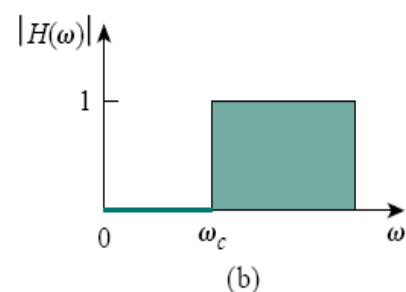
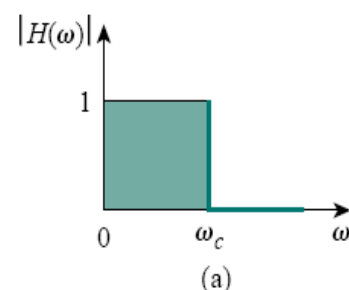


TABLE 14.5 Summary of the characteristics of filters.

Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

ω_c is the cutoff frequency for lowpass and highpass filters; ω_0 is the center frequency for bandpass and bandstop filters.

1 Lowpass Filter

14.7.1 Lowpass Filter

A typical lowpass filter is formed when the output of an RC circuit is taken off the capacitor as shown in Fig. 14.31. The transfer function (see also Example 14.1) is

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C} \\ \mathbf{H}(\omega) &= \frac{1}{1 + j\omega RC}\end{aligned}\quad (14.50)$$

Note that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 0$. Figure 14.32 shows the plot of $|H(\omega)|$, along with the ideal characteristic. The half-power frequency, which is equivalent to the corner frequency on the Bode plots but in the context of filters is usually known as the *cutoff frequency* ω_c , is obtained by setting the magnitude of $\mathbf{H}(\omega)$ equal to $1/\sqrt{2}$, thus

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

or

$$\omega_c = \frac{1}{RC} \quad (14.51)$$

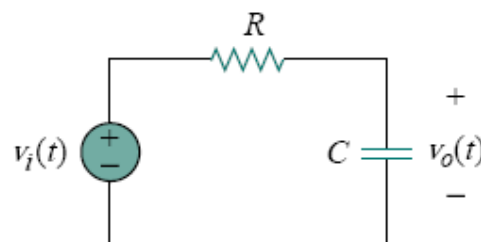


Figure 14.31 A lowpass filter.

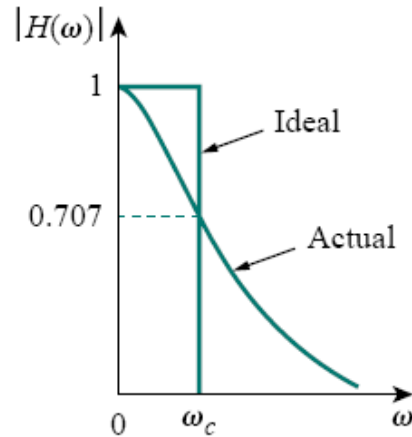


Figure 14.32 Ideal and actual frequency response of a lowpass filter.

The cutoff frequency is also called the *rolloff frequency*.

A **lowpass filter** is designed to pass only frequencies from dc up to the cutoff frequency ω_c .

A lowpass filter can also be formed when the output of an RL circuit is taken off the resistor. Of course, there are many other circuits for lowpass filters.

14.7.2 Highpass Filter

A highpass filter is formed when the output of an RC circuit is taken off the resistor as shown in Fig. 14.33. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$
$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} \quad (14.52)$$

Note that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 1$. Figure 14.34 shows the plot of $|H(\omega)|$. Again, the corner or cutoff frequency is

$$\omega_c = \frac{1}{RC} \quad (14.53)$$

A **highpass filter** is designed to pass all frequencies above its cutoff frequency ω_c .

A highpass filter can also be formed when the output of an RL circuit is taken off the inductor.

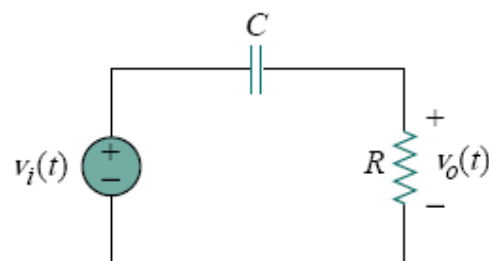


Figure 14.33 A highpass filter.

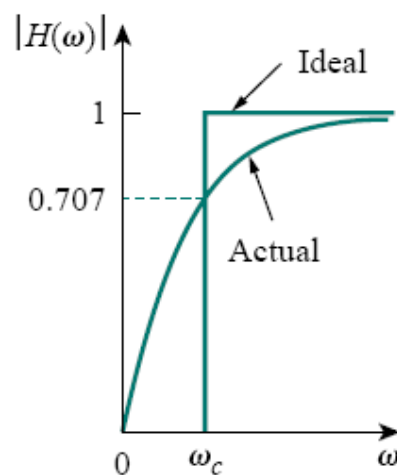


Figure 14.34 Ideal and actual frequency response of a highpass filter.

14.7.3 Bandpass Filter

The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig. 14.35. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)} \quad (14.54)$$

We observe that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 0$. Figure 14.36 shows the plot of $|H(\omega)|$. The bandpass filter passes a band of frequencies ($\omega_1 < \omega < \omega_2$) centered on ω_0 , the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (14.55)$$

A **bandpass filter** is designed to pass all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

Since the bandpass filter in Fig. 14.35 is a series resonant circuit, the half-power frequencies, the bandwidth, and the quality factor are determined as in Section 14.5. A bandpass filter can also be formed by cascading the lowpass filter (where $\omega_2 = \omega_c$) in Fig. 14.31 with the highpass filter (where $\omega_1 = \omega_c$) in Fig. 14.33.

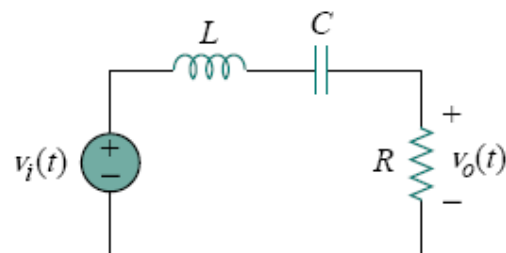


Figure 14.35 A bandpass filter.

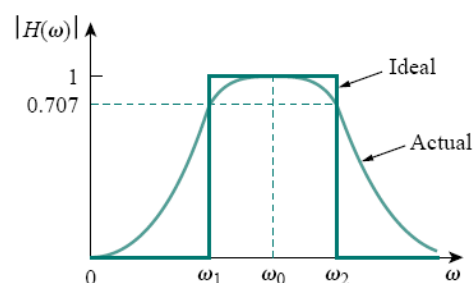


Figure 14.36 Ideal and actual frequency response of a bandpass filter.

14.7.4 Bandstop Filter

A filter that prevents a band of frequencies between two designated values (ω_1 and ω_2) from passing is variably known as a *bandstop*, *bandreject*, or *notch* filter. A bandstop filter is formed when the output *RLC* series resonant circuit is taken off the *LC* series combination as shown in Fig. 14.37. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)} \quad (14.56)$$

Notice that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 1$. Figure 14.38 shows the plot of $|H(\omega)|$. Again, the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (14.57)$$

while the half-power frequencies, the bandwidth, and the quality factor are calculated using the formulas in Section 14.5 for a series resonant circuit. Here, ω_0 is called the *frequency of rejection*, while the corresponding bandwidth ($B = \omega_2 - \omega_1$) is known as the *bandwidth of rejection*. Thus,

A **bandstop filter** is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega < \omega_2$.

Notice that adding the transfer functions of the bandpass and the bandstop gives unity at any frequency for the same values of R , L , and C . Of course, this is not true in general but true for the circuits treated here. This is due to the fact that the characteristic of one is the inverse of the other.

In concluding this section, we should note that:

1. From Eqs. (14.50), (14.52), (14.54), and (14.56), the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter as the next section shows.
2. There are other ways to get the types of filters treated in this section.
3. The filters treated here are the simple types. Many other filters have sharper and complex frequency responses.

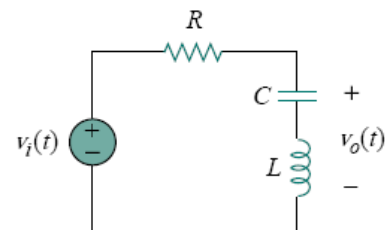
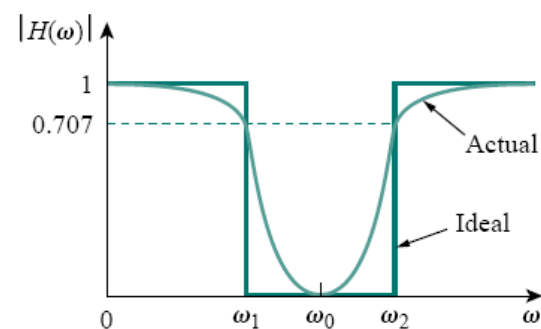


Figure 14.37 A bandstop filter.



Determine what type of filter is shown in Fig. 14.39. Calculate the corner or cutoff frequency. Take $R = 2 \text{ k}\Omega$, $L = 2 \text{ H}$, and $C = 2 \text{ }\mu\text{F}$.

Solution:

The transfer function is

$$\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R \parallel 1/sC}{sL + R \parallel 1/sC}, \quad s = j\omega \quad (14.10.1)$$

But

$$R \parallel \frac{1}{sC} = \frac{R/sC}{R + 1/sC} = \frac{R}{1 + sRC}$$

Substituting this into Eq. (14.10.1) gives

$$\mathbf{H}(s) = \frac{R/(1 + sRC)}{sL + R/(1 + sRC)} = \frac{R}{s^2RLC + sL + R}, \quad s = j\omega$$

or

$$\mathbf{H}(\omega) = \frac{R}{-\omega^2RLC + j\omega L + R} \quad (14.10.2)$$

Since $\mathbf{H}(0) = 1$ and $\mathbf{H}(\infty) = 0$, we conclude from Table 14.5 that the circuit in Fig. 14.39 is a second-order lowpass filter. The magnitude of $\mathbf{H}$ is

$$H = \frac{R}{\sqrt{(R - \omega^2RLC)^2 + \omega^2L^2}} \quad (14.10.3)$$

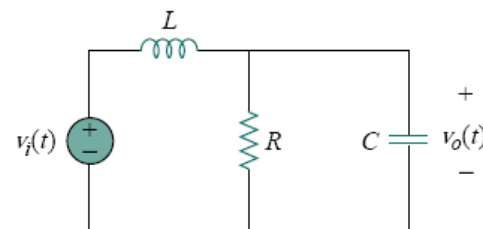


Figure 14.39 For Example 14.10.

The corner frequency is the same as the half-power frequency, i.e., where $\mathbf{H}$ is reduced by a factor of $1/\sqrt{2}$. Since the dc value of $H(\omega)$ is 1, at the corner frequency, Eq. (14.10.3) becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

or

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of R , L , and C , we obtain

$$2 = (1 - \omega_c^2 4 \times 10^{-6})^2 + (\omega_c 10^{-3})^2$$

Assuming that ω_c is in krad/s,

$$2 = (1 - 4\omega_c)^2 + \omega_c^2 \quad \text{or} \quad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in ω_c^2 , we get $\omega_c^2 = 0.5509$, or

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$

Series resonant circuit

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$B = \omega_2 - \omega_1$$

the full width at half

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

$$B = \frac{R}{L} = \frac{\omega_0}{Q}$$

we can also be approximated as

$$\omega_1 \approx \omega_0 - \frac{B}{2}, \quad \omega_2 \approx \omega_0 + \frac{B}{2}$$

Summary of the characteristics of resonant RLC circuits.

Characteristic	Series circuit	Parallel circuit
Resonant frequency, ω_0	$\frac{1}{\sqrt{LC}}$	$\frac{1}{\sqrt{LC}}$
Quality factor, Q	$\frac{\omega_0 L}{R}$ or $\frac{1}{\omega_0 RC}$	$\frac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, B	$\frac{\omega_0}{Q}$	$\frac{\omega_0}{Q}$
Half-power frequencies, ω_1, ω_2	$\omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$	$\omega_0 \sqrt{1 + \left(\frac{1}{2Q}\right)^2} \pm \frac{\omega_0}{2Q}$
For $Q \geq 10$, ω_1, ω_2	$\omega_0 \pm \frac{B}{2}$	$\omega_0 \pm \frac{B}{2}$

For the circuit in Fig. 14.40, obtain the transfer function $V_o(\omega)/V_i(\omega)$. Identify the type of filter the circuit represents and determine the corner frequency. Take $R_1 = 100\ \Omega = R_2$, $L = 2\text{ mH}$.

Answer: $\frac{R_2}{R_1 + R_2} \left(\frac{j\omega}{j\omega + \omega_c} \right)$, highpass filter

$$\omega_c = \frac{R_1 R_2}{(R_1 + R_2)L} = 25\text{ krad/s.}$$

Practice Problem 14.10

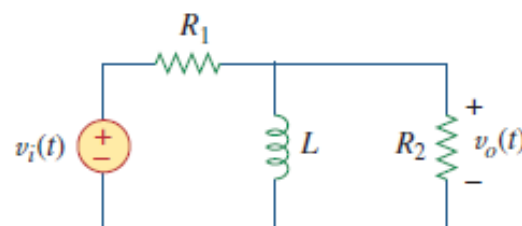


Figure 14.40
For Practice Prob. 14.10.

If the bandstop filter in Fig. 14.37 is to reject a 200-Hz sinusoid while passing other frequencies, calculate the values of L and C . Take $R = 150\ \Omega$ and the bandwidth as 100 Hz.

Solution:

We use the formulas for a series resonant circuit in Section 14.5.

$$B = 2\pi(100) = 200\pi\text{ rad/s}$$

But

$$B = \frac{R}{L} \quad \Rightarrow \quad L = \frac{R}{B} = \frac{150}{200\pi} = 0.2387\text{ H}$$

Rejection of the 200-Hz sinusoid means that f_0 is 200 Hz, so that ω_0 in Fig. 14.38 is

$$\omega_0 = 2\pi f_0 = 2\pi(200) = 400\pi$$

Since $\omega_0 = 1/\sqrt{LC}$,

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{(400\pi)^2(0.2387)} = 2.653\ \mu\text{F}$$

Design a bandpass filter of the form in Fig. 14.35 with a lower cutoff frequency of 20.1 kHz and an upper cutoff frequency of 20.3 kHz. Take $R = 20 \text{ k}\Omega$. Calculate L , C , and Q .

Answer: 15.915 H, 3.9 pF, 101.

6. Design of Filter

$$k = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

Solving these two equations,

$$L = \frac{k}{\pi f_c}$$

$$C = \frac{1}{\pi f_c k}$$

The constant- k , T and π section filters are shown in Fig. 15.8.

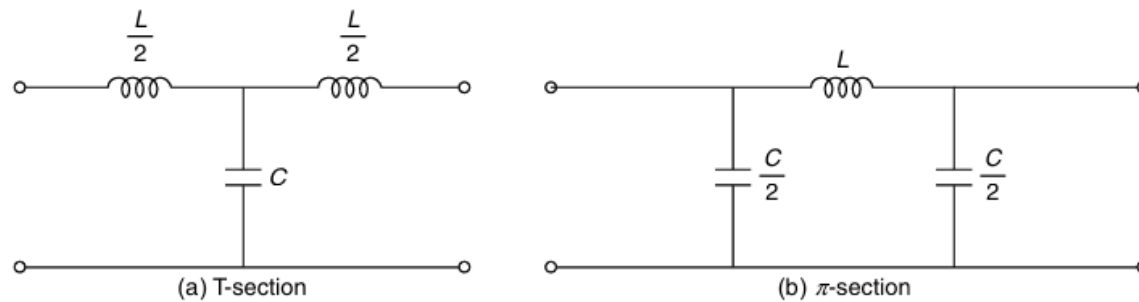


Fig. 15.8 Constant- k low-pass filter

Example 15.4

Each of the two series elements of a T-section low-pass filter consists of an inductor of 60 mH having negligible resistance and a shunt element having a capacitance of 0.2 μ F. Calculate (a) the cut-off frequency, (b) nominal impedance, and (c) characteristic impedance at frequencies of 1 kHz and 5 kHz.

Solution

$$L = 60 \text{ mH}, \quad C = 0.2 \text{ } \mu\text{F}$$

(a) Cut-off frequency

$$f_c = \frac{1}{\pi\sqrt{LC}} = \frac{1}{\pi\sqrt{60 \times 10^{-3} \times 0.2 \times 10^{-6}}} = 2.91 \text{ kHz}$$

(b) Nominal impedance

$$k = \sqrt{\frac{L}{C}} = \sqrt{\frac{60 \times 10^{-3}}{0.2 \times 10^{-6}}} = 547.72 \text{ } \Omega$$

(c) Characteristic impedance at 1 kHz

$$Z_0 = k\sqrt{1 - \left(\frac{f}{f_c}\right)^2} = 547.72\sqrt{1 - \left(\frac{1 \times 10^3}{2.91 \times 10^3}\right)^2} = 514.36 \text{ } \Omega$$

Characteristic impedance at 5 kHz

$$Z_0 = k\sqrt{1 - \left(\frac{f}{f_c}\right)^2} = 547.72\sqrt{1 - \left(\frac{5 \times 10^3}{2.91 \times 10^3}\right)^2} = j765.29 \text{ } \Omega$$

Formula used:-

1.) cut off frequency $f_c = \frac{1}{\pi \sqrt{LC}}$

2.) Nominal impedance $K = \sqrt{L/C}$

3.) Characteristic impedance

$$Z_0 = \frac{K}{\sqrt{\left(1 - \frac{f}{f_c}\right)^2}}$$

Do the calculation for both the frequencies given

4.) Attenuation

$$\alpha = 2 \cosh^{-1} \left(\frac{f}{f_c} \right)$$

Calculate for both frequencies

5.) phase shift

$$\beta = 2 \sin^{-1} \left(\frac{f}{f_c} \right)$$

Do calculation for both frequencies