

Engineering Mechanics

Faculty : Dr. Prashik Gaikwad
School of Mechanical Engineering

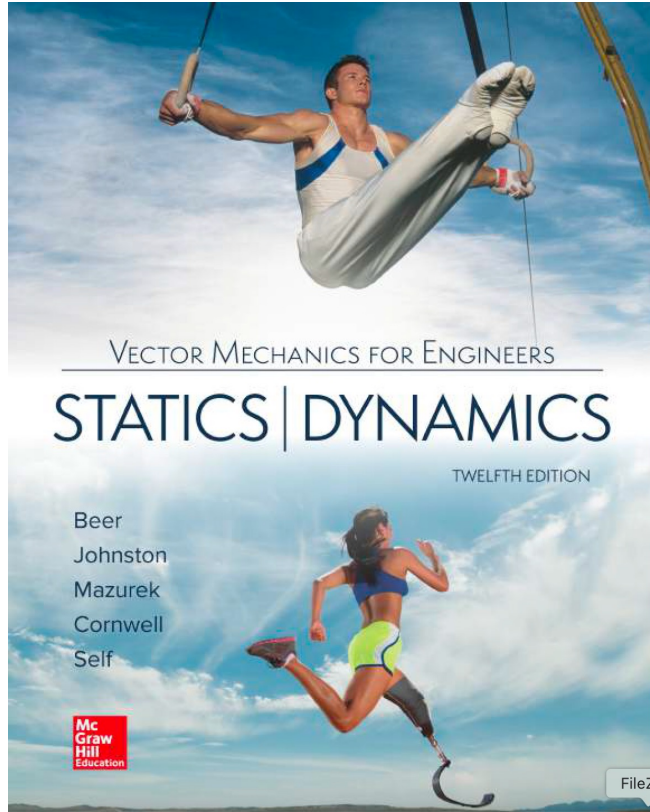
Contact details

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Cabin: CBMR 304-B

BMEE201L	Engineering Mechanics				L	T	P	C
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Pre-requisite	NIL				Syllabus version			
					1.0			
Course Objectives:								
1. To enable students to apply fundamental laws and basic concepts of rigid body mechanics to solve problems of bodies under rest or in motion.								
2. To enable the students to apply conditions of static equilibrium to analyse physical systems.								
3. To compute the properties of areas and bodies.								
Course Outcome:								
Upon successful completion of the course the students will be able to								
1. Compute the resultant and analyse equilibrium (without and with friction) of system of forces acting on particles and rigid bodies in plane and space.								
2. Predict the support-reactions and the internal forces of the members of trusses and frames.								
3. Apply transfer theorems to determine properties of various sections.								
4. Calculate motion parameters of particles and rigid bodies.								
Module:1 Statics of Particles					5 hours			
Fundamental concepts and principles - Resolution of a force -Resultant of forces in a plane- Equilibrium of a particle in a plane; Addition of concurrent forces in space- Equilibrium of a particle in space.								
Module:2 Statics of Rigid Bodies					7 hours			
Equivalent systems of forces- Principle of Transmissibility - Moment of a force about a point and an axis- Couples and force-couple systems- Equilibrium of rigid bodies in two and three dimensions- Types of beams, supports and reactions; Principle of virtual work – System of connected rigid bodies.								
Module:3 Analysis of Structures					5 hours			
Analysis of plane trusses - Method of joints and method of sections- Frames								
Module:4 Friction					5 hours			
The laws of dry friction – Coefficients of Friction- Angles of Friction- Types of Friction Problems - Wedges and Ladder friction- Belt friction.								
Module:5 Properties of Surfaces and Solids					7 hours			
First moments of areas and lines- Centroids of composite areas and lines- - Theorems of Pappus-Guldinus- Second moment of area- Parallel axis theorem- Rectangular and Polar Moments of inertia of composite areas- Radius of Gyration- Product of Inertia- Principal Axes and Principal Moments of Inertia- Mass moments of inertia of thin plates.								
Module:6 Dynamics of Particles					8 hours			
Kinematics of Particles: Displacement, Velocity and Acceleration – Rectilinear motion – Curvilinear motion – Tangential and Normal components – Radial and Transverse components.								
Kinetics of Particles: Newton's Second Law- Energy and Momentum Methods-Principle of Work and Energy-Principle of Impulse and Momentum- Direct Central Impact								
Module:7 Dynamics of Rigid Bodies					8 hours			
Kinematics of rigid bodies: Translation and fixed-axis rotation- General plane motion: velocity- Instantaneous centre of rotation- General plane motion: acceleration.								
Kinetics of rigid bodies:Equations of motion -Angular momentum- Plane motion of a rigid body- Principle of work and energy for rigid bodies- Principle of impulse and momentum for rigid bodies.								
Total Lecture hours:					45 hours			

Textbook for this course



Beer, Johnston, Cornwell, David Mazurek,
and Sanghi, Vector Mechanics for
Engineers: Statics and Dynamics, 12th
Edition, McGraw-Companies

Module 3

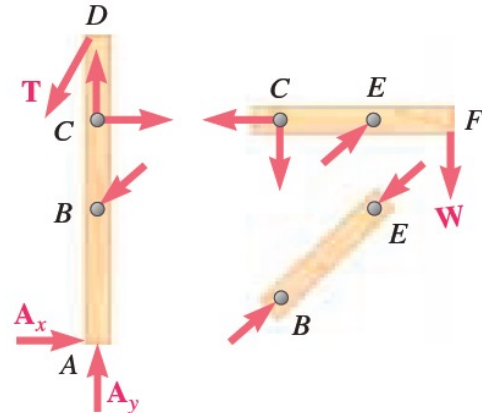
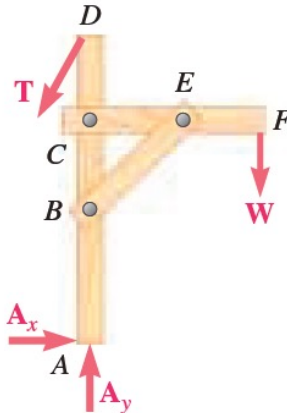
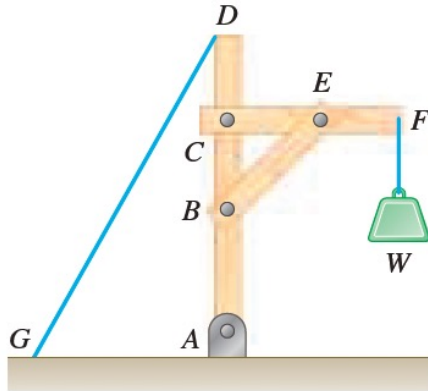
Analysis of Structures

Module Overview

- Analysis of Plane Trusses
- Method of joints
- Method of section
- Frames

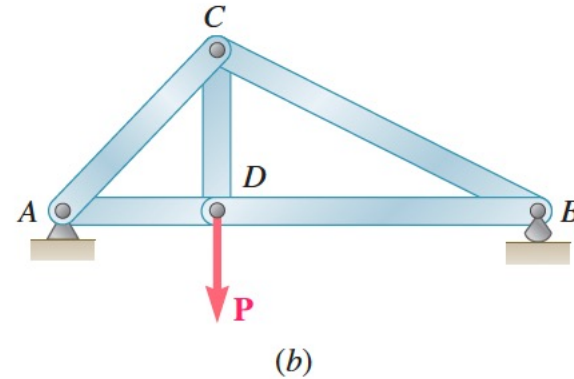
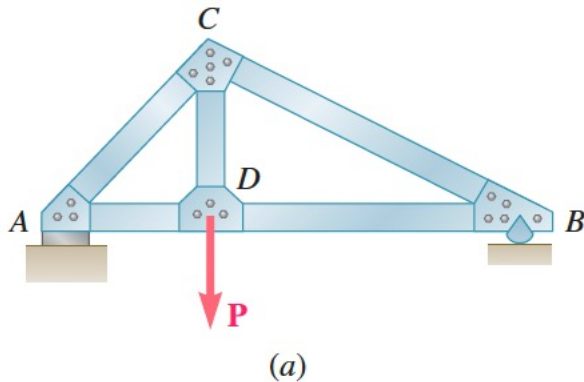
Introduction

- We studied the equilibrium of a single rigid body, where all forces involved were external to the rigid body.
- We now consider the equilibrium of structures made of several connected parts.
- This situation calls for determining not only the external forces acting on the structure, but also the forces that hold together the various parts of the structure.
- From the point of view of the structure as a whole, these forces are **internal forces**

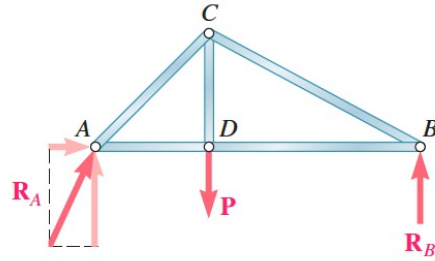


Truss

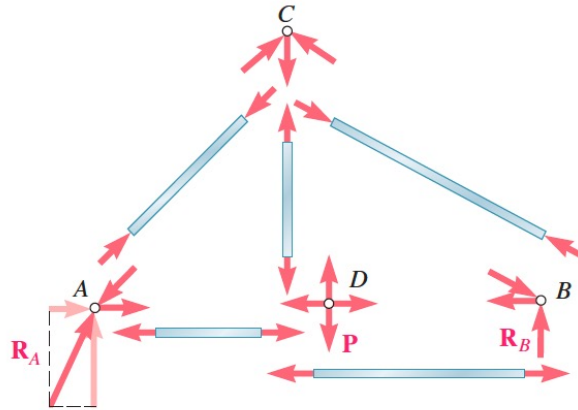
- Trusses, which are designed to support loads and are usually stationary, fully constrained structures.
- Trusses consist exclusively of straight members connected at joints located at the ends of each member.
- Members of a truss, therefore, are two-force members, i.e., members acted upon by two equal and opposite forces directed along the member.
- Truss members are connected at their extremities only; no member is continuous through a joint



Method of joints



(a)



Numerical

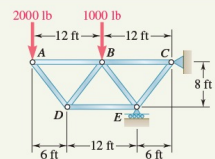


Fig. 1 Free-body diagram of the entire truss.

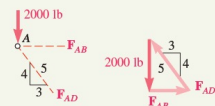


Fig. 2 Free-body diagram of joint A.

Using the method of joints, determine the force in each member of the truss shown.

STRATEGY: To use the method of joints, you start with an analysis of the free-body diagram of the entire truss. Then, look for a joint connecting only two members as a starting point for the calculations. In this example, we start at joint A and proceed through joints D, B, E, and C, but you could also start at joint C and proceed through joints E, B, D, and A.

MODELING and ANALYSIS: You can combine these steps for each joint of the truss in turn. Draw a free-body diagram; draw a force polygon or write the equilibrium equations; and solve for the unknown forces.

Entire Truss. Draw a free-body diagram of the entire truss (Fig. 1); external forces acting on this free body are the applied loads and the reactions at C and E. Write the equilibrium equations, taking moments about C.

$$\begin{aligned} +\circlearrowleft \Sigma M_C = 0: & (2000 \text{ lb})(24 \text{ ft}) + (1000 \text{ lb})(12 \text{ ft}) - E(6 \text{ ft}) = 0 \\ & E = +10,000 \text{ lb} \\ +\rightarrow \Sigma F_x = 0: & C_x = 0 \\ +\uparrow \Sigma F_y = 0: & -2000 \text{ lb} - 1000 \text{ lb} + 10,000 \text{ lb} + C_y = 0 \\ & C_y = +7000 \text{ lb} \end{aligned}$$

Joint A. This joint is subject to only two unknown forces: the forces exerted by AB and those by AD. Use a force triangle to determine F_{AB} and F_{AD} (Fig. 2). Note that member AB pulls on the joint so AB is in tension, and member AD pushes on the joint so AD is in compression. Obtain the magnitudes of the two forces from the proportion

$$\frac{2000 \text{ lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$F_{AB} = 1500 \text{ lb } T \quad \leftarrow$$

$$F_{AD} = 2500 \text{ lb } C \quad \leftarrow$$

As an alternative to the force triangle approach, remember that a more general analytic solution can also be used. This alternate method is especially conducive to joint equilibrium problems that involve more than three forces, and is illustrated later in this sample problem for the analysis of joints B, E, and C.

Joint D. Because you have already determined the force exerted by member AD, only two unknown forces are now involved at this joint. Again, use a force triangle to determine the unknown forces in members DB and DE (Fig. 3).



Fig. 3 Free-body diagram of joint D.

$$\begin{aligned} F_{DB} &= F_{DA} \\ F_{DE} &= 2\left(\frac{3}{5}\right)F_{DA} \end{aligned}$$

$$F_{DB} = 2500 \text{ lb } T \quad \leftarrow$$

$$F_{DE} = 3000 \text{ lb } C \quad \leftarrow$$

Joint B. Because more than three forces act at this joint (Fig. 4), determine the two unknown forces F_{BC} and F_{BE} by solving the equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Suppose you arbitrarily assume that both unknown forces act away from the joint, i.e., that the members are in tension. The positive value obtained for F_{BC} indicates that this assumption is correct; member BC is in tension. The negative value of F_{BE} indicates that the second assumption is wrong; member BE is in compression.

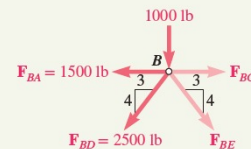


Fig. 4 Free-body diagram of joint B.

$$\begin{aligned} +\uparrow \Sigma F_y = 0: & -1000 - \frac{4}{5}(2500) - \frac{4}{5}F_{BE} = 0 \\ & F_{BE} = -3750 \text{ lb} \\ & F_{BE} = 3750 \text{ lb } C \quad \leftarrow \\ +\rightarrow \Sigma F_x = 0: & F_{BC} - 1500 - \frac{3}{5}(2500) - \frac{3}{5}(3750) = 0 \\ & F_{BC} = +5250 \text{ lb} \\ & F_{BC} = 5250 \text{ lb } T \quad \leftarrow \end{aligned}$$

Joint E. Assume the unknown force F_{EC} acts away from the joint (Fig. 5). Summing x components, you obtain

$$\begin{aligned} +\rightarrow \Sigma F_x = 0: & \frac{3}{5}F_{EC} + 3000 + \frac{3}{5}(3750) = 0 \\ & F_{EC} = -8750 \text{ lb} \\ & F_{EC} = 8750 \text{ lb } C \quad \leftarrow \end{aligned}$$

Summing y components, you obtain a check of your computations:

$$\begin{aligned} +\uparrow \Sigma F_y = 10,000 - \frac{4}{5}(3750) - \frac{4}{5}(8750) \\ = 10,000 - 3000 - 7000 = 0 \end{aligned} \quad (\text{checks})$$

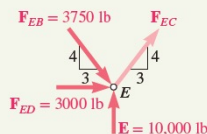
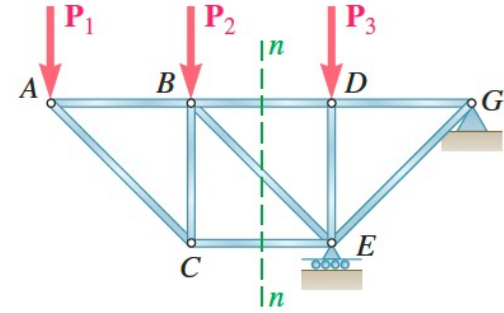


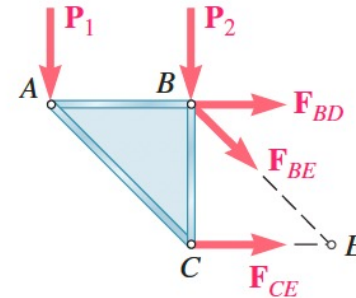
Fig. 5 Free-body diagram of joint E.

Method of section

- Assume, for example, that we want to determine the force in member BD of the truss shown in Fig.
- We have passed the section nn through members BD, BE, and CE, and we have chosen the portion ABC of the truss as the free body (Fig. b).
- The forces acting on this free body are the loads P_1 and P_2 at points A and B and the three unknown forces F_{BD} , F_{BE} , and F_{CE} .



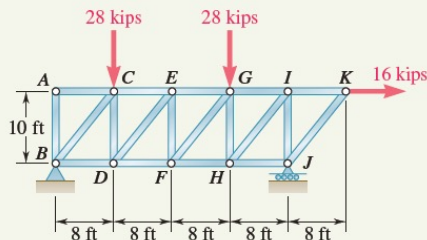
(a)



(b)

Rigid bodies

Determine the forces in members EF and GI of the truss shown.



Free-Body: Entire Truss. Draw a free-body diagram of the entire truss. External forces acting on this free body consist of the applied loads and the reactions at B and J (Fig. 1). Write and solve the following equilibrium equations.

$$+\circlearrowleft \Sigma M_B = 0:$$

$$-(28 \text{ kips})(8 \text{ ft}) - (28 \text{ kips})(24 \text{ ft}) - (16 \text{ kips})(10 \text{ ft}) + J(32 \text{ ft}) = 0$$

$$J = +33 \text{ kips} \quad \mathbf{J = 33 \text{ kips} \uparrow}$$

$$\pm \rightarrow \Sigma F_x = 0: \quad B_x + 16 \text{ kips} = 0$$

$$B_x = -16 \text{ kips} \quad \mathbf{B_x = 16 \text{ kips} \leftarrow}$$

$$+\circlearrowleft \Sigma M_J = 0:$$

$$(28 \text{ kips})(24 \text{ ft}) + (28 \text{ kips})(8 \text{ ft}) - (16 \text{ kips})(10 \text{ ft}) - B_y(32 \text{ ft}) = 0$$

$$B_y = +23 \text{ kips} \quad \mathbf{B_y = 23 \text{ kips} \uparrow}$$

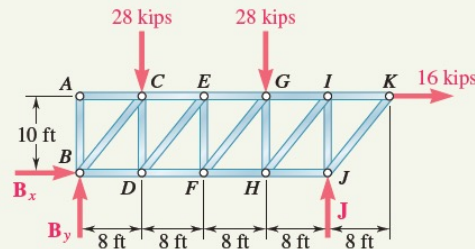


Fig. 1 Free-body diagram of the entire truss.

Force in Member EF . Pass section nn through the truss diagonally so that it intersects member EF and only two additional members (Fig. 2). Remove the intersected members and choose the left-hand portion of the truss as a free body

Rigid bodies

(Fig. 3). Three unknowns are involved; to eliminate the two horizontal forces, we write

$$+\uparrow \Sigma F_y = 0: \quad +23 \text{ kips} - 28 \text{ kips} - F_{EF} = 0$$

$$F_{EF} = -5 \text{ kips}$$

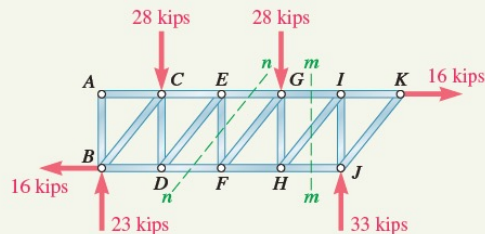


Fig. 2 Sections nn and mm that will be used to analyze members EF and GI .

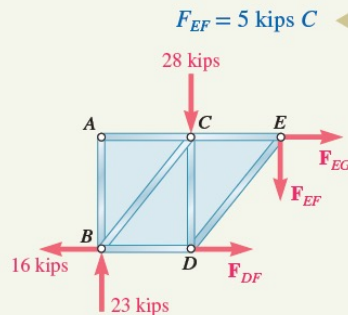


Fig. 3 Free-body diagram to analyze member EF .

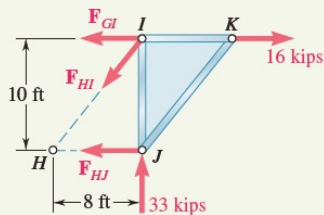


Fig. 4 Free-body diagram to analyze member GI .

The sense of F_{EF} was chosen assuming member EF to be in tension; the negative sign indicates that the member is in compression.

Force in Member GI . Pass section mm through the truss vertically so that it intersects member GI and only two additional members (Fig. 2). Remove the intersected members and choose the right-hand portion of the truss as a free body (Fig. 4). Again, three unknown forces are involved; to eliminate the two forces passing through point H , sum the moments about that point.

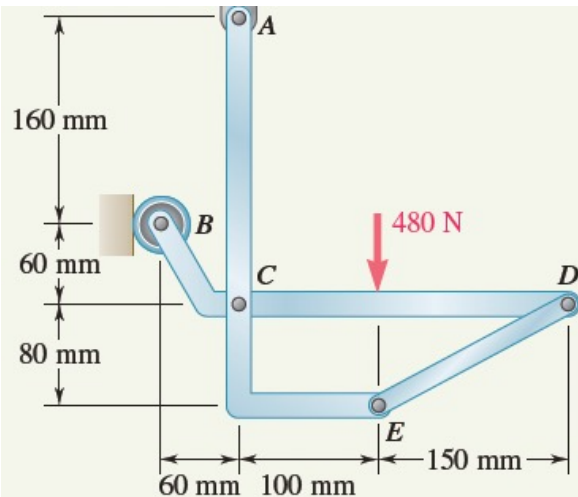
$$+\circlearrowleft \Sigma M_H = 0: \quad (33 \text{ kips})(8 \text{ ft}) - (16 \text{ kips})(10 \text{ ft}) + F_{GI}(10 \text{ ft}) = 0$$

$$F_{GI} = -10.4 \text{ kips} \quad F_{GI} = 10.4 \text{ kips C}$$

Frames

- When we study trusses, we are looking at structures consisting entirely of pins and straight two-force members.
- The forces acting on the two-force members are directed along the members themselves.
- We now consider structures in which at least one of the members is a multi-force member, i.e., a member acted upon by three or more forces.
- These forces are generally not directed along the members on which they act; their directions are unknown; therefore, we need to represent them by two unknown components.
- Frames are designed to support loads and are usually stationary, fully constrained structures.

Numerical problem



In the frame shown, members *ACE* and *BCD* are connected by a pin at *C* and by the link *DE*. For the loading shown, determine the force in link *DE* and the components of the force exerted at *C* on member *BCD*.

STRATEGY: Follow the general procedure discussed in this section. First treat the entire frame as a free body, which will enable you to find the reactions at *A* and *B*. Then, dismember the frame and treat each member as a free body, which will give you the equations needed to find the force at *C*.

MODELING and ANALYSIS: Because the external reactions involve only three unknowns, compute the reactions by considering the free-body diagram of the entire frame (Fig. 1).

$$+\uparrow \Sigma F_y = 0: \quad A_y - 480 \text{ N} = 0 \quad A_y = +480 \text{ N} \quad A_y = 480 \text{ N} \uparrow$$

Numerical problem

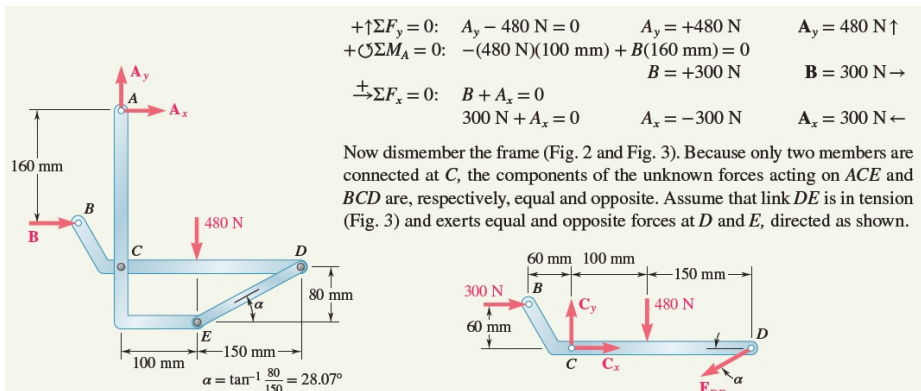
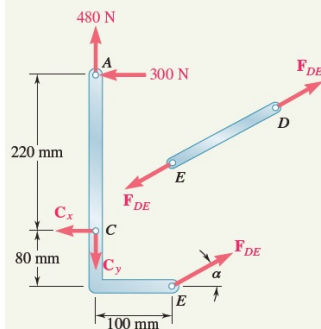


Fig. 1 Free-body diagram of the entire frame.

Fig. 2 Free-body diagram of member BCD.



Free Body: Member BCD. Using the free body BCD (Fig. 2), you can write and solve three equilibrium equations:

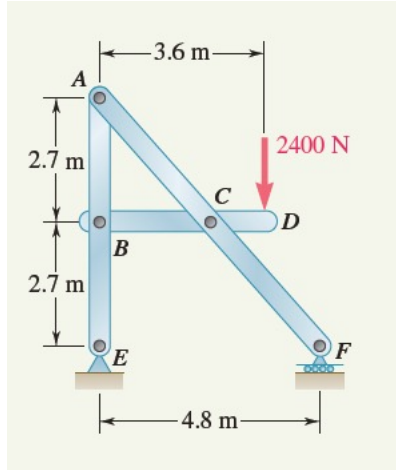
$$\begin{aligned}
 +\circlearrowleft \Sigma M_C = 0: & \quad (F_{DE} \sin \alpha)(250 \text{ mm}) + (300 \text{ N})(60 \text{ mm}) + (480 \text{ N})(100 \text{ mm}) = 0 \\
 & \quad F_{DE} = -561 \text{ N} & \quad F_{DE} = 561 \text{ N} \leftarrow \\
 \rightarrow \Sigma F_x = 0: & \quad C_x - F_{DE} \cos \alpha + 300 \text{ N} = 0 \\
 & \quad C_x - (-561 \text{ N}) \cos 28.07^\circ + 300 \text{ N} = 0 & \quad C_x = -795 \text{ N} \\
 +\uparrow \Sigma F_y = 0: & \quad C_y - F_{DE} \sin \alpha - 480 \text{ N} = 0 \\
 & \quad C_y - (-561 \text{ N}) \sin 28.07^\circ - 480 \text{ N} = 0 & \quad C_y = +216 \text{ N}
 \end{aligned}$$

From the signs obtained for \$C_x\$ and \$C_y\$, the force components \$C_x\$ and \$C_y\$ exerted on member BCD are directed to the left and up, respectively. Thus, you have

$$C_x = 795 \text{ N} \leftarrow, C_y = 216 \text{ N} \uparrow$$

REFLECT and THINK: Check the computations by considering the free body ACE (Fig. 3). For example,

Practice problem



Determine the components of the forces acting on each member of the frame shown.

Practice problems

Determine the forces in members FH , GH , and GI of the roof truss shown.

