REG.NO. :

VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

NAME OF THE SCHOOL: SAS
CONTINUOUS ASSESSMENT TEST - II
FALL SEMESTER 2024-2025

SLOT: C1+TC1+ T+TCC1

Programme Name & Branch	**: B. Tech**
Course Code and Course Name	**: BMAT201L (Complex variables and Linear Algebra)**
Faculty Name(s)	**: Common question paper for this slot**
Class Number(s)	**: Common question paper for this slot**
Date of Examination	**: 15-10-2024**
Exam Duration	**: 90 minutes** Maximum Marks: 50

General instruction(s):

- Answer All Questions
- M - Max mark; CO – Course Outcome; BL – Blooms Taxonomy Level (1 – Remember, 2 – Understand, 3 – Apply, 4 – Analyse, 5 – Evaluate, 6 – Create)
- Course Outcomes (Type the CO statements covered in this question paper. Use the CO number as per the syllabus copy)

Q. No	Question	M	CO	BL
1.	a) Discuss the nature of the singularity of the function $f(z)=\dfrac{z-\sin z}{z^2}$.	4	3	2
	b) Evaluate $\int_c \dfrac{2+3\sin \pi z}{z(z-1)^2}dz$ where C is the square bounded by the lines $x=\pm 3$ and $y=\pm 3i$ using Cauchy's integral formula.	6	3	3
2.	Evaluate $\int_{-\infty}^{\infty}\dfrac{\cos x}{4x^2+1}dx$ using Residue theorem.	10	3	3
3.	Let $A=\begin{pmatrix}3 & -2 & -1\\ 2 & 3 & 4\\ -2 & 0 & 5\end{pmatrix}$. Use Cayley-Hamilton theorem to find constants $a,\ b$ and c such that $A^4=aA^2+bA+cI$, where I is an identity matrix of order 3. Also find A^{-1}.	10	5	3
4.	On a particular day, four persons went to market. First person bought one apple, two oranges, one mango and three pineapples for 9 dollars, second person bought two apples, one orange, two mangoes and one pineapple for 10 dollars, third person bought one apple, one orange, one mango and two pineapples for 7 dollars and fourth person bought one apple, three oranges, one mango and one pineapple for 8 dollars. Find the cost of each fruit using Gauss Jordan method.	10	5	3
5.	a) Let $R^4(R)$ be a vector space over a field R and $W=\{(x, y, z, w)\in R^4 \mid x=2y, z=3w-y\}$. Verify whether W is a subspace of $R^4(R)$ or not.	5	5	2
	b) Let $R^3(R)$ be a vector space over a field R and S = {(1, 1, 0), (1, 0, 2), (1, 1, 1)} a linearly independent subset of R^3. Verify whether S is a basis of $R^3(R)$ or not.	5	5	2

Fall Sem. 2024-25

CAT2

Programme Name & Branch: B. Tech.

Course Name & Code: Complex Variables and Linear Algebra (BMAT201L)

KEY

1. (a) Clearly $z=0$ is a singular point of $f(z)=\dfrac{z-\sin z}{z^2}$.

And, $f(z)=\dfrac{z-\sin z}{z^2}=\dfrac{1}{z^2}\left\{z-\left(z-\dfrac{z^3}{3!}+\dfrac{z^5}{5!}-\dfrac{z^7}{7!}+...\right)\right\}=\dfrac{z}{3!}-\dfrac{z^3}{5!}+\dfrac{z^5}{7!}-...$ has no principal part. Therefore $z=0$ is a removable singularity of $f(z)=\dfrac{z-\sin z}{z^2}$.

(b) Let $f(z)=2+3\sin \pi z.$ Then, clearly $f(z)$ is analytic within and on C, where C is the square bounded by the lines $x=\pm 3$ and $y=\pm 3i$. And $z=0,\ 1$ are inside of C.

Therefore, by Cauchy's integral formula, we have

$$\begin{aligned}\int_c \frac{f(z)}{z(z-1)^2}dz &= \int_c \frac{f(z)}{z}dz-\int_c \frac{f(z)}{z-1}dz+\int_c \frac{f(z)}{(z-1)^2}dz\\ &=2\pi i\left\{f(0)-f(1)+f'(1)\right\}\\ &=2\pi i\left\{2-2-3\pi\right\}\\ &=-6\pi^2 i.\end{aligned}$$

2. Let $f(z)=\dfrac{e^{iz}}{4z^2+1}$. Then, clearly $f(z)$ is not analytic at $z=\dfrac{-i}{2},\ \dfrac{i}{2}$.

Consider $\displaystyle\int_C \frac{f(z)}{4z^2+1}\,dz$, where C is the closed contour consisting of semicircle $C_R:|z|=R$ together with the real axis from –R to R.

Here $z=\dfrac{i}{2}$ only lies inside the semicircle of the contour C.

Therefore, by the residue theorem, we have $\displaystyle\int_C f(z)\ dz=2\pi i(\mathrm{Re}\,s.f(z))_{z=-i/2}$

$\Rightarrow \displaystyle\int_{C_R} f(z)dz+\int_{-R}^{R} f(x)\ dx=\frac{\pi}{2\sqrt{e}}$. Taking, $R\to\infty$ we get $0+\displaystyle\int_{-\infty}^{\infty} f(x)\ dx=\frac{\pi}{2\sqrt{e}}$.

Hence $\displaystyle\int_{-\infty}^{\infty}\frac{\cos x}{4x^2+1}\ dx=\frac{\pi}{2\sqrt{e}}$.

3. The characteristic equation of A is $\lambda^3-11\lambda^2+41\lambda-75=0.$

By Cayley-Hamilton theorem, we have $A^3 - 11A^2 + 41A - 75I = 0$.

So,

$$A^3 = 11A^2 - 41A + 75I \Rightarrow A^4 = 11A^3 - 41A^2 + 75A$$

$$\Rightarrow A^4 = 11(11A^2 - 41A + 75I) - 41A^2 + 75A$$

$$\Rightarrow A^4 = 80A^2 - 376A + 825I$$

Therefore, $a = 80$, $b = -376$ and $c = 825$.

Also, $A^{-1} = \dfrac{1}{75}\begin{pmatrix} 15 & 10 & -5 \\ -18 & 13 & -14 \\ 10 & 4 & 13 \end{pmatrix}$.

4. Let us represent apple, orange, mango, banana with x, y, z, w respectively.

Then, from the given data, we have

$x + 2y + z + 3w = 9;\ 2x + y + 2z + w = 10;\ x + y + z + 2w = 7;\ x + 3y + z + w = 8$.

We can express the above system of linear equations as $\begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 3 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 9 \\ 10 \\ 7 \\ 8 \end{pmatrix}$

i.e., $AX = B$, where $A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 3 & 1 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$, $B = \begin{pmatrix} 9 \\ 10 \\ 7 \\ 8 \end{pmatrix}$.

The augmented matrix of A, B is $[A\ B]$ i.e., $\begin{pmatrix} 1 & 2 & 1 & 3 & 9 \\ 2 & 1 & 2 & 1 & 10 \\ 1 & 1 & 1 & 2 & 7 \\ 1 & 3 & 1 & 1 & 8 \end{pmatrix}$.

Using Gauss Jordan method, we can reduce this into $\begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$.

Therefore, the cost of each apple is 2 dollars, each orange is 1 dollar, each mango is 2 dollar and each banana is 1 dollar.

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5. (a) For any $a, b \in R$ and $\alpha = (x_1, y_1, z_1, w_1)$, $\beta = (x_2, y_2, z_2, w_2) \in W$, we have $a\alpha + b\beta \in W$. Therefore, W is a subspace of $R^4(R)$.

(b) For any $(x, y, z) \in R$ such that $(x, y, z) = a(1, 1, 0) + b(1, 0, 2) + c(1, 1, 1)$

which implies that $a = 2x - y - z$, $b = x - y$ and $c = z + 2y - 2x$.

Therefore, $L(S) = R^3$ and hence S is a basis of $R^3(R)$.