G/L/TX

Reg. No: 24BEE0230

Final Assessment Test – November 2024

VIT
Vellore Institute of Technology

Course: BMAT101L - Calculus

Time: Three Hours

Max. Marks: 100

- **KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE**
- **DON'T WRITE ANYTHING ON THE QUESTION PAPER**

Answer ALL Questions

(10 X 10 = 100 Marks)

1. For the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 4$ **[10]**
   i) find the local maxima and local minima.
   ii) find the intervals where the function is increasing and the intervals where it is decreasing.
   iii) identify the intervals where the function is concave up and concave down, hence find the points of inflection.

2. i) Find the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$. **[10]**
   ii) Find the volume of the solid generated by revolving the region bounded by the line $x + y = 2$ and the curve $x^2 = 4 - y$ about the x-axis.

3. i) Examine whether $u = y + z, v = x + 2z^2, w = x - 4yz - 2y^2$ are functionally dependent. If so find the relation between them. **[10]**
   ii) If $u = x^2y^2 + x^3y$, where $x = 2t^2$ and $y = 4t$, then find $\frac{du}{dt}$.

4. Expand $e^x\log(1 + y)$ in a Taylor's series about $(0,0)$ up to terms of third degree. **[10]**

5. A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction. **[10]**

6. Change the order of the integration and hence evaluate it. **[10]**

$$\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx.$$

7. Evaluate $\iiint(x^2 + y^2 + z^2)dx\,dy\,dz$ taken over the volume enclosed by the sphere $x^2 + y^2 + z^2 = 1$ by transforming into spherical polar coordinates. **[10]**

8. i) Evaluate $\int_0^1 \frac{1}{\sqrt{\log\frac{1}{x}}}\,dx$. **[10]**
   ii) Find the value of the integral $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}}$.

9.a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1,1,1)$ in the direction of $\vec{i} + \vec{j} - \vec{k}$. Also find curl(grad ϕ). **[10]**

OR

9.b) Show that $\vec{F} = (y^2 + 2xz^2)\,\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and find its scalar potential. **[10]**

10.a) Evaluate $\iint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and S is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$. **[10]**

OR

10.b) Verify Green's theorem in the xy plane for $\int_C \{(xy + y^2)dx + x^2 dy\}$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **[10]**

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