VIT

Vellore Institute of Technology

FALL SEMESTER 2023-2024

SCHOOL OF ADVANCED SCIENCES

DEPARTMENT OF MATHEMATICS

Continuous Assessment Test – I

Course Code : BMAT201L **Course Name:** Complex Variables and Linear Algebra
Slot : A2+TA2+TAA2 **Duration** : 90 Minutes
Date : **10/09/2023 Session 2 (14:00PM-15:30PM)**
Max. Marks : 50 Marks

Answer ALL the following questions.

Q. No	Question	Marks	BL
1.	(a) Verify that the families of curves $u = c_1$ and $v = c_2$ are cut orthogonally, when $w = u + iv = z^3$. (b) Prove that the real and imaginary parts of an analytic function $f(z) = u + iv$ satisfies the Laplace equation in two dimensions.	10	BL5
2.	Show that $\varphi = x^2 - y^2 + \frac{x}{x^2+y^2}$ can represent the velocity potential in an incompressible fluid flow. Also find the corresponding stream function and complex potential.	10	BL5
3.	Show that the transformation $w = \frac{z-i}{1-iz}$ maps (i) the interior of the circle $\|z\| = 1$ onto the lower half of the w-plane and (ii) the upper half of the z-plane onto the interior of the circle $\|w\| = 1$.	10	BL5
4.	Find the Laurent's series of $f(z) = \frac{z^2-1}{z^2+5z+6}$ valid in the region (i) $\|z\| < 2$, (ii) $\|z\| > 3$ (iii) $2 < \|z\| < 3$.	10	BL4
5.	Using Cauchy's residue theorem, evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$, where C is the circle (i) $\|z\| = 1$ and (ii) $\|z + 1 - i\| = 2$.	10	BL3