0-1 Knapsack Problem using B&B

- Knapsack is a maximization problem as its objective is to maximize the profit value.
- Using B&B, we can solve only minimization problems. So, we have to convert knapsack problem into minimization problem by considering profits as negative values.
- The result is again converted into positive in order to give solution to the actual maximization problem.
- We construct state space tree.
- In state space tree, we compute Upper Bound (UB) and Cost (C) to each node.

$$\text{Upper Bound (UB)} = \sum_{i=1}^{n} p_i x_i \quad \text{(without fraction)}$$

$$\text{Cost (C)} = \sum_{i=1}^{n} p_i x_i \quad \text{(with fraction)}$$

Such that, sum of weights of items considered must be <= capacity of knapsack (m).

- Actual problem is 0-1 knapsack, which does not allow fractions. Only for solving the problem, we take fractions. But fractions will not be included in the final solution.
- **While constructing state space tree for 0-1 knapsack problem using B&B, remember the following:**

• If you get smaller UB for any node, then change global UB to that value.
• If you get cost greater than global UB for any node, then kill that node.

Example: 0-1 knapsack problem using FIFO-B&B

n = 4; m = 15; (p1, p2, p3, p4) = (10, 10, 12 18); (w1, w2, w3, w4) = (10, 10, 12 18)

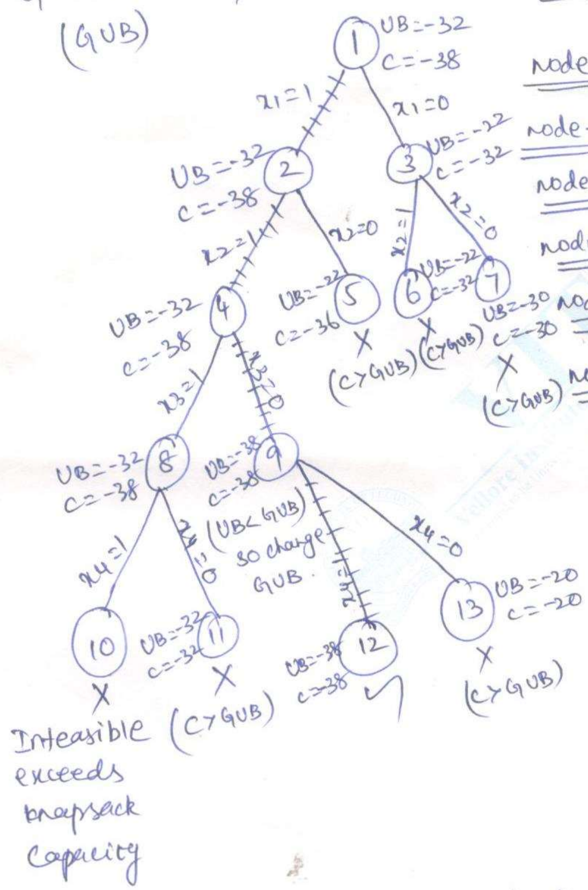
FIFO B&B

$n=4$
$m=15$

Item	1	2	3	4
profit	10	10	12	18
weight	2	4	6	9

Global UB $= \cancel{0}\ \cancel{-32}\ -38$
(GUB)

~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~8~~	~~9~~	~~12~~



Node-1: UB = 10+10+12 = 32, C = 10+10+12+$\frac{3}{9}$×18
= 10+10+12+6 = 38

node-2: UB = 10+10+12 = 32, C = 10+10+12+6 = 38

node-3: UB = 10+12 = 22, C = 10+12+10 = 32

node-4: UB = 10+10+12 = 32, C = 38

node-5: UB = 10+12 = 22, C = 10+12+14 = 3

node-6: UB = 10+12 = 22, C = 10+12+10 = 32

node-7: UB = 12+18 = 30, C = 12+18 = 30

node-8: UB = 10+10+12 = 32, C = 38

node-9: UB = 10+10+18 = 38, C = 38

node-10: UB = 10+10+12 = 32, C = 32

node-12: UB = 10+10+18 = 38, C = 38

node-13: UB = 20, C = 20.

Solution: $x_1 = 1$ $x_2 = 1$ $x_3 = 0$ $x_4 = 1$

Items selected = {1, 2, 4}

profit earned = 10+10+18 = 38

weight added = 2+4+9 = 15

Example: 0-1 knapsack problem using LIFO-B&B

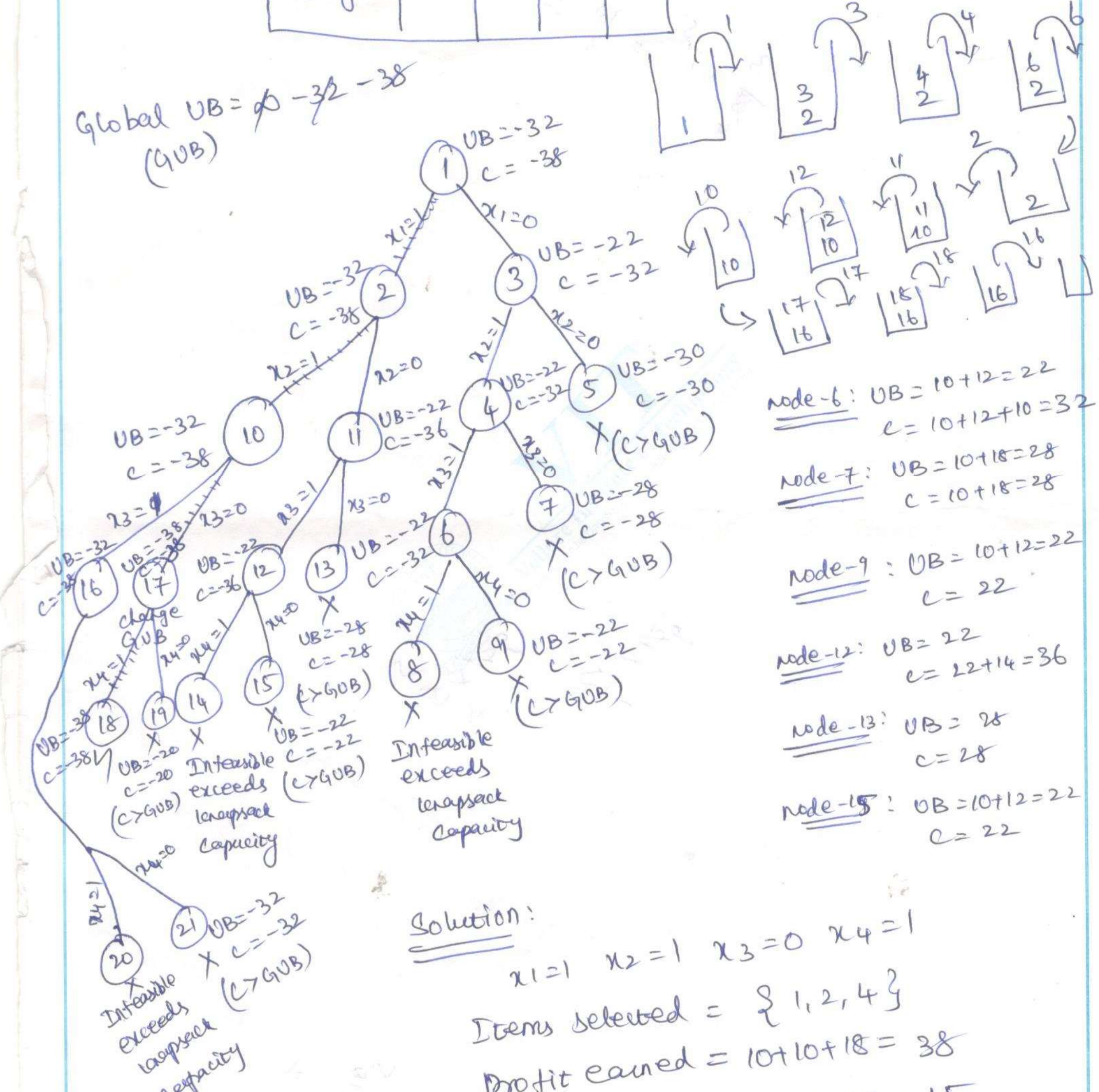
n = 4; m = 15; (p1, p2, p3, p4) = (10, 10, 12 18); (w1, w2, w3, w4) = (10, 10, 12 18)

LIFO B&B:

n = 4
m = 15

Item	1	2	3	4
Profit	10	10	12	18
weight	2	4	6	9

Global UB = ~~∅~~ −~~32~~ −38
(GUB)



node-6: UB = 10+12 = 22
c = 10+12+10 = 32

node-7: UB = 10+18 = 28
c = 10+18 = 28

node-9: UB = 10+12 = 22
c = 22

node-12: UB = 22
c = 22+14 = 36

node-13: UB = 28
c = 28

node-15: UB = 10+12 = 22
c = 22

Solution:

$x_1 = 1$ $x_2 = 1$ $x_3 = 0$ $x_4 = 1$

Items selected = {1, 2, 4}

Profit earned = 10+10+18 = 38

weight added = 2+4+9 = 15