VIT
Vellore Institute of Technology

Final Assessment Test – June 2022

Course: BMAT102L Differential Equations and Transforms

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

PART – A (10 X 3 = 30 Marks)
Answer ALL Questions

1. Describe the motion governed by the differential equation $\frac{d^2x}{dt^2}+49x=0$ with the initial conditions $x(0)=-1, x'(0)=1$.

2. If $x(t)=Ae^t+Be^{2t}+C$ is the steady-state solution of $\frac{d^2x}{dt^2}+\frac{dx}{dt}-12x=e^t+e^{2t}-1$, then compute the constants A, B and C.

3. By eliminating the arbitrary function ψ, find the partial differential equation from the relation: $\psi(z-x-y)=xy$.

4. Find the Laplace transform of $f(t)=\frac{\sinh 5t}{t}$.

5. Find the inverse Laplace transform of $F(s)=\frac{s^2+6}{(s^2+1)(s^2+4)}$.

6. Employing the Laplace transform to the differential equation $\frac{d^2q}{dt^2}+3\frac{dq}{dt}+10000q=100\,\delta(t-2\pi)$ and using the initial conditions: $q(0)=1, q'(0)=0$, find the rational expression for the Laplace transform $Q(s)$ of $q(t)$ in its simplified form, where $\delta(t-a)$ denotes the Dirac delta function at $t=a$.

7. Find the half-range sine series of $f(t)=\begin{cases}\frac{1}{4}-t, & 0<t<\frac{1}{2}\\ t-\frac{3}{4}, & \frac{1}{2}<t<1,\end{cases}$ in $(0,1)$.

8. Given that $F(\omega)=\frac{1}{2}\sqrt{\frac{2}{\pi}}\cdot\frac{\sin\omega}{\omega}, \omega\neq 0$ is the complex Fourier transform of $f(x)=\begin{cases}\frac{1}{2}, & -1<t<1\\ 0, & \text{elsewhere},\end{cases}$ use the Fourier inversion formula to compute $\int_0^\infty\left\{\frac{\sin x}{x}\right\}dx$.

9. *Form a difference equation from the following relation:* $y_n=A(-1)^n+B.3^n$ *by eliminating the arbitrary constants.*

10. *Using the convolution property, find the inverse Z-transform of* $F(s)=\frac{z^2}{(z-3)(z-4)}$.

PART – B (7 X 10 = 70 Marks)
Answer ALL Questions

11. a) Consider an RCL circuit where $R=2, L=1, C=1$. The current $I(t)$ is driven by an electromotive force of $2\sin 3t$. The circuit equation for the voltage $V(t)$ across the capacitor is $\frac{d^2V}{dt^2}+2\frac{dV}{dt}+V=2\sin 2t$ with $V(0)=0, V'(0)=0$.
Describe the subsequent voltage oscillations, by using the method of undetermined coefficients. Also, interpret the results physically.

OR

11. b) *Using appropriate substitutions and the method of variation of parameters, solve*
$$t^2\frac{d^2x}{dt^2}+t\frac{dx}{dt}+9x=\log t.$$

12. a) Find the complete integral of $2p - 3q = pq$. Also, find the singular solution of $z = px + qy - 3pq$, where $= \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

OR

12 b) Find the general integral of $(x^2y - zx^2)p + (y^2z - xy^2)q = z^2x - yz^2$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

13. a) Find the Laplace transform of $f(t) = t^3 \sin 2t$, and hence evaluate

$$\int_0^\infty t^3 e^{-3t} \sin 2t\, dt.$$

OR

13. b) Using the convolution theorem, find the inverse Laplace transform of

$$F(s) = \frac{s}{s^4 + 37s^2 + 36}.$$

14. a) Using the Laplace transform, solve the following initial-value problem:

$\frac{d^2x}{dt^2} + x = f(t),\ x(0) = 1,\ x'(0) = 0$, where $f(t) = \begin{cases} 2, & \pi < t < 2\pi \\ 0, & \text{elsewhere.} \end{cases}$

OR

14. b) Using the Laplace transform, solve the following boundary-value problem:

$\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial t} = u$ for $0 < x < l, t > 0$ with $u(x,0) = 6e^{-3x}$, where $u(x,t)$ is bounded for all x and all t.

15. a) Find the Fourier series of the function $f(t) = (\pi - t)^2$, with period 2π, defined in $(0,2\pi)$. How should $f(t)$ be defined at the end points $t = 0, 2\pi$ in order that the Fourier series will converge to $f(t)$ on $0 \le t \le 2\pi$?

OR

15. b) Find the Fourier series of $f(t) = \begin{cases} 2, & -2 < t < 0 \\ t, & 0 < t < 2 \end{cases}$, $f(t + 4) = f(t)$. How should $f(t)$ be defined at $t = 0$ in order that the Fourier series will converge to $f(t)$ on $-2 < t < 2$?

16. a) Find the complex Fourier transform of $f(x) = \begin{cases} 1 - x^2, & -1 < x < 1 \\ 0, & \text{elsewhere.} \end{cases}$

OR

16. b) Find the sine transform of $f(x) = \frac{e^{-3x}}{x}$.

17. a) Find the Z-transform of $x_n = a^n$ and $y_n = n$. Use this information to find the inverse of the Z-transform of $X(z) = \frac{2z}{(z-2)(z-1)^2}$.

OR

17. b) Using the Z-transform, solve the difference equation:

$x_{n+2} - 8x_{n+1} + 15x_n = 1$ with $x_0 = 0, x_1 = 0$.

⇔⇔⇔