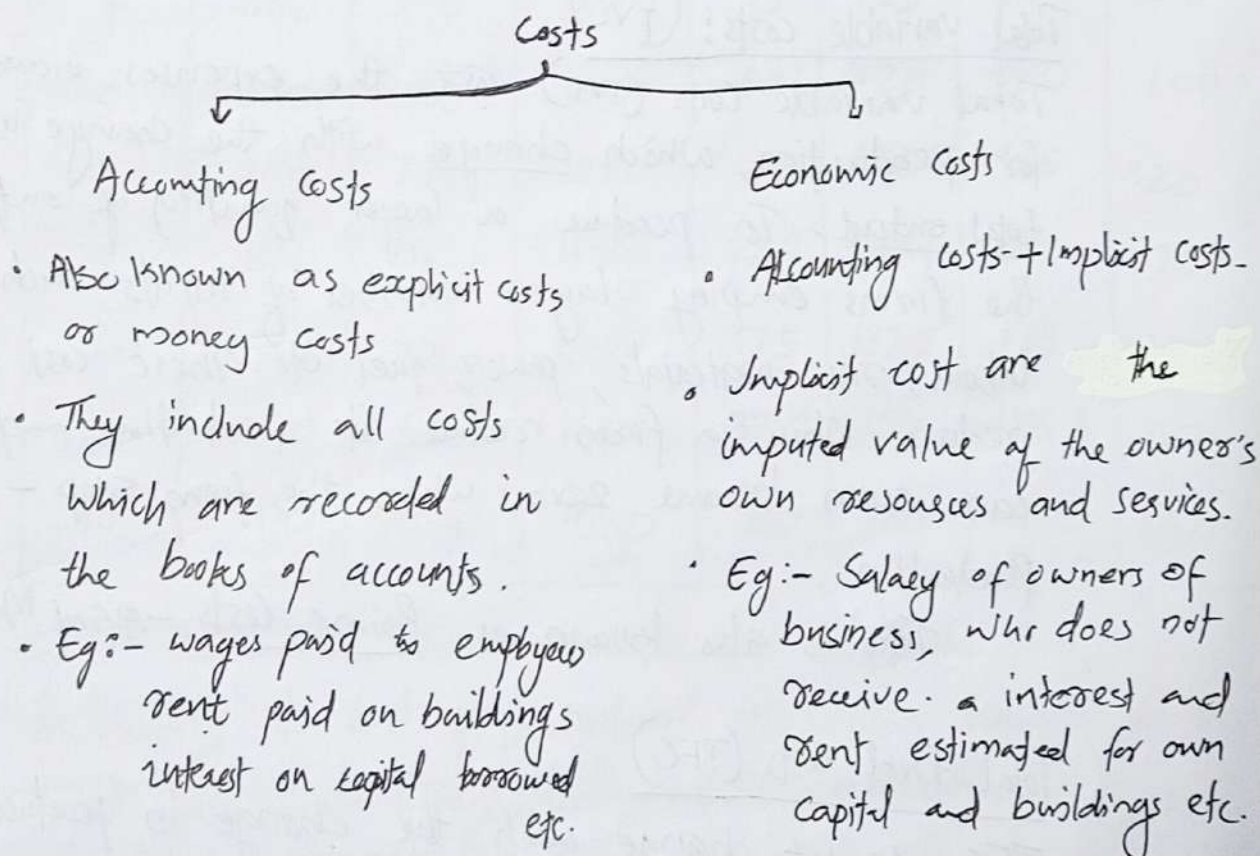


Module 5

Cost and Revenue Functions.

We learned that market price is determined by the demand and supply forces. For producers, their cost of production is considered as the key determinants of Supply of commodities.

Concept of Accounting and Economic costs



$$\text{Economic Costs} = \text{Accounting Costs} + \text{Implicit Costs.}$$

Production Costs

In the production process, a producer uses different factors of production as inputs. Some of the inputs may be fixed factors or variable factors. These factors of production are employed/paid at different prices. The expenditure incurred on these factors are the total costs of production of a firm. In other words, the total fixed and total variable costs contribute to the total costs.

Total variable costs: (TVC)

Total variable costs (TVC) are the expenses incurred for production, which change with the change in total output. To produce a larger quantity of output, the firms employ larger numbers of inputs such as labour, raw materials, power, fuel etc. These costs can reduce when a firm reduces its production and can even become zero, when the firm stops production.

This is also known as Prime Costs - Alfred Marshall.

Total Fixed Costs (TFC)

TFC do not change with the change in production. Such costs include rent payments, interest payments, depreciation etc. Fixed costs can occur even if there is no production by the firm.

TFC is also known as Supplementary Costs - A Marshall.

To study the cost nature of a production unit, we consider the following hypothetical cases of production level.

Table 5-1

Total output (Q)	TFC	TVC	Total Cost (TC)	AFC	AVC	ATC	MC
0	300	0	300	300	0	300	-
1	300	300	600	300	300	600	300
2	300	400	700	150	200	350	100
3	300	450	750	100	150	250	50
4	300	500	800	75	125	200	50
5	300	600	900	60	120	180	100
6	300	720	1020	50	120	170	120
7	300	890	1190	42.9	127.1	170	170
8	300	1100	1400	37.5	137.5	175	210
9	300	1350	1650	33.3	150	183.3	470
10	300	2000	2300	30	200	230	650

From the above table, we observe that the firm is increasing its unit of production. The firm has 300 Total fixed costs (TFC) throughout the production stages. As firm increases the production, the firm experiences increasing costs on variable factors of production.

[To increase more output, more labour, raw materials etc. are needed]. This is clear from the increasing of Total Variable Costs (TVC) on the table.

Additionally, Total costs was also measured

using TFC and TVC data. The table also presents the respective average cost of TFC (measured as AFC), TVC (measured as AVC) and TC (measured as ATC). Finally Marginal Costs (MC) increased for each additional stage of production is also presented.

Key lessons

- ↳ At the initial stage of increasing of production, the TVC increases minimally (also MC) compared to the later stage of increasing production (also MC). The first stage of increasing output with ~~min~~ relatively minimum increase in the costs is due to the advantage of economies of scale. On the contrary, the experience in the final stage of increasing output with relatively higher increase in the costs is due to the diseconomies of scale.
- ↳ It is to be noted that the TC is positive even there is no production. It is due to Fixed costs.
- ↳ This distinction between Fixed costs and variable costs is applicable only during short term period.
In the short run, firms may be selling products at loss. However, in the long run the firm expects to recover loss, otherwise, the firm may close down. In the long run, all costs are variable costs.

The above costs can be presented graphically as follows.

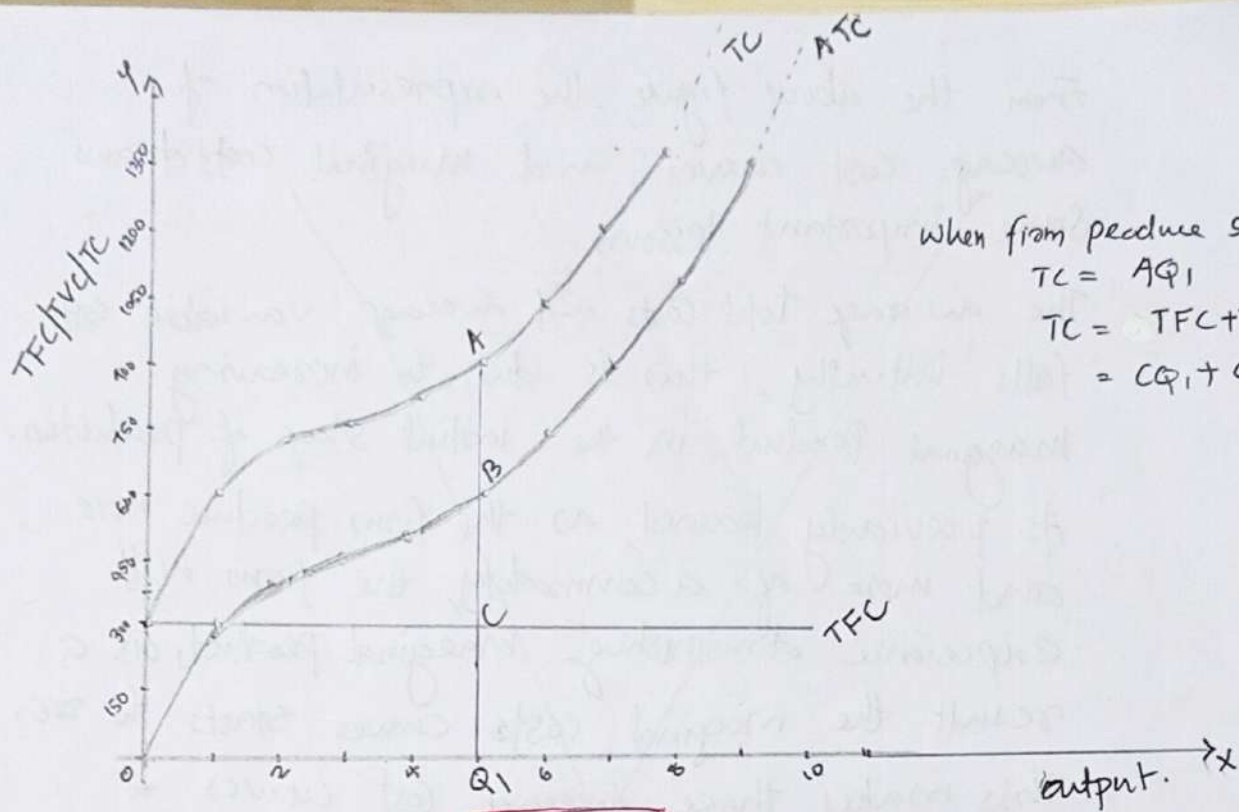


Figure 5.1

The Cost Function

The cost function expresses the functional relationship between total cost and factors that determine total cost. Usually, the factors that determine costs of production are the output level (Q), the level of technology used for production (T), the prices of factors (P_f) and fixed factors (F).

Symbolically, the function can be expressed as follows.

$$C = f(Q, T, P_f, F).$$

For simplification we assume the following.

- 1- A firm produces a single and homogenous good (q) with the help of certain factors of production.
- 2- Some of these factors are employed in fixed quantities, irrespective of output level in the short run. So they are assumed to be given.
- 3- The remaining factors are variable whose supply is assumed to be known and available at fixed market prices.
- 4- The technology used for production is assumed to be known and fixed.
- 5- The firm adjusts the output ' Q ' of commodity ' q ' to produce at the minimum cost ' c '.

Therefore, the total cost function may be expressed as

$$C = f(Q).$$

Here, we assume that all factors other than output is constant.

Short run Cost Curves

In the short run, firm can't change many factors of production such as plant, equipment, machines etc. in order to increase production. However, the firm may increase variable inputs such as labour, raw materials etc. in order to increase production.

As we have seen previously, in figure 5.1, the total cost (TC) is the sum total of TFC and TVC. We observed that, TFC was constant even the production was zero or positive. On the contrary TVC was zero when output was zero. TVC increased when more and more output is produced and as a result TC also increased. TC starts from the TFC when output is zero.

Now we consider Average costs and Marginal cost [Table 5.1].

Short-run Average Costs :

In the cost analysis, the average cost is very important. From table 5.1, we observed that average costs changed over time (different levels of production). For a firm, average costs will indicate the per unit cost of production, so the knowing average costs helps in pricing commodities. The firm will expect a price minimum to average costs. The three important average costs we derived are the following.

- i - Short-run variable costs (SAVC)
- ii - Average fixed costs (AFC)
- iii - Short run Average Total cost (SATC/ATC).

i-Short-run Average variable costs (SAVC)

$$SAVC = \frac{TVC}{Q}$$

TVC = Total variable cost
Q = Output.

ii- Average Fixed Costs (AFC)

$$AFC = \frac{TFC}{Q}$$

TFC = Total Fixed cost
Q = output.

iii- Short-run Average Total Cost (SATC/SAC)

$$SAC = \frac{TVC}{Q} + \frac{TFC}{Q} = SAVC + AFC$$

Finally, we also derived Short-run Marginal cost (SMC) in the same table 5.1. Knowing Marginal cost is important to determine the level of output of a firm. SMC is the addition to the total cost by producing an additional unit of output.

short

$$SMC = \frac{\Delta TC}{\Delta Q}$$

TC = Total costs
Q = output.

A combination of Average cost curves and Marginal cost is depicted in the following diagram for one further analysis.

output	TFC	TVC	TC	AFC	SAVC	SAC	MC
1	20	30	50	20	30	50	50
2	20	45	65	10	22.5	32.5	15
3	20	55	75	6.67	18.33	25	10
4	20	75	95	5	18.75	23.75	20
5	20	110	130	4	22	26	35
6	20	165	185	3.33	27.5	30.83	55

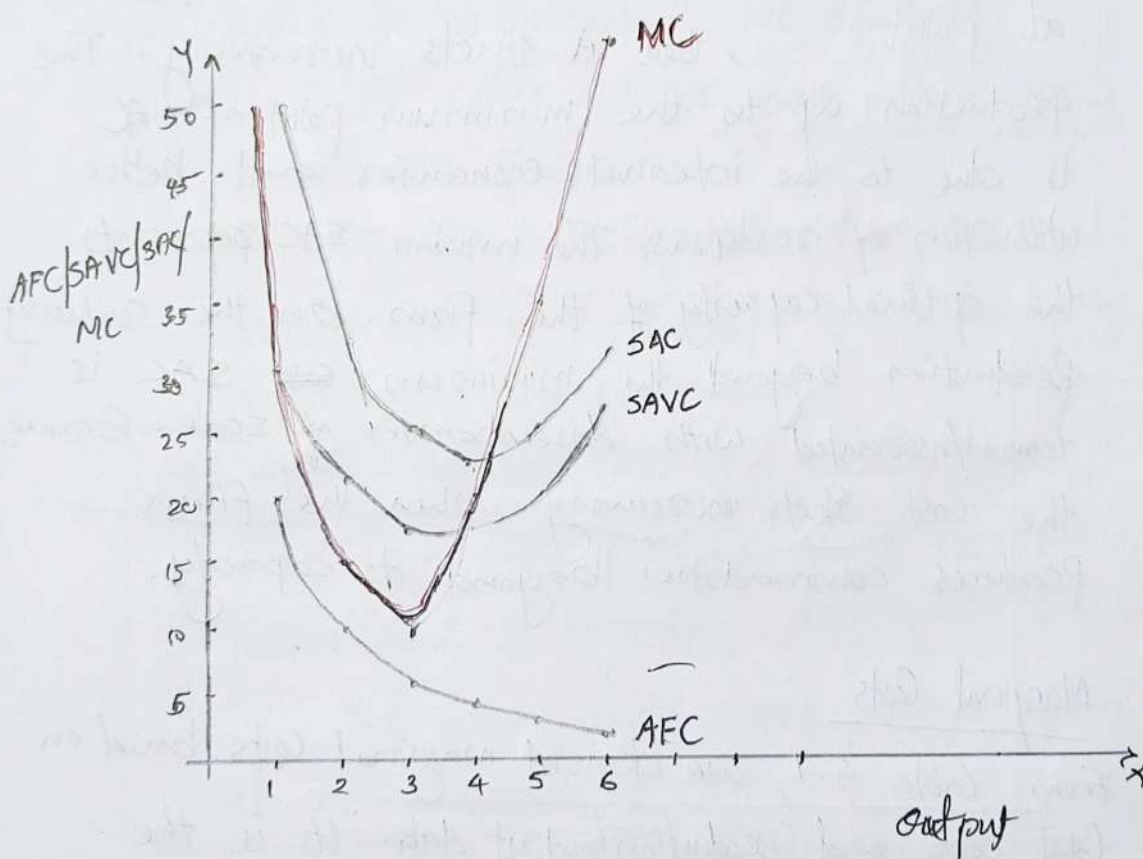


Figure 5.2

In the above figure AFC declines continuously and never touches x axis. AFC curve is a rectangular hyperbola.

Shape of SAC curve.

In the above figure, we observed that Average Cost (SAC) has a U Shape; meaning that the average cost was higher during the initial stage of production. This is because of the fact that the only a small level of output was produced using large quantity of inputs (fixed cost and variable costs). As production increased, the average cost also decreased as expected. We also observed that the SAC reached at the minimum at point , then it starts increasing. The production up to the minimum point of AC is due to the internal economies and better utilisation of resources. The minimum SAC represents the optimal capacity of the firm. On the contrary production beyond the minimum ~~cost~~ SAC is termed/associated with diseconomies of scale. Because the cost starts increasing when the firm produces commodities beyond its capacity.

Marginal Costs

From table 5.1, we derived marginal costs based on total costs and production/output data. It is the addition to the total cost, when the firm produce an additional unit of output.

$$MC = \frac{\Delta TC}{\Delta Q}$$

or

$$MC_n = TC_n - TC_{n-1}$$

Δ = change
 TC = Total Cost
 Q = output.

Similar to SAC curve, SMC curve is also U shaped one.

Relationship between Average Cost Curves and MC Curves.

From the above figure 5.2, we can interpret the association between shortrun average cost curves and MC curve.

i- Relation between SAC and MC Curves.

→ When SAC falls (initially), MC is less than SAC.

→ When AC is minimum, MC equals SAC.

MC curve cuts SAC curve from below at the minimum of SAC.

→ When SAC rises, MC is greater than SAC.

→ Nothing can be said about the direction of MC when SAC rises or falls.

ii- Relation between MC and SAVC. Curves.

→ When SAVC falls (initially), MC is less than SAVC.

→ When SAVC is minimum, MC equals SAVC.

MC curve cuts SAVC curve from below at the minimum of SAVC.

→ When SAVC rises, MC is greater than SAVC.

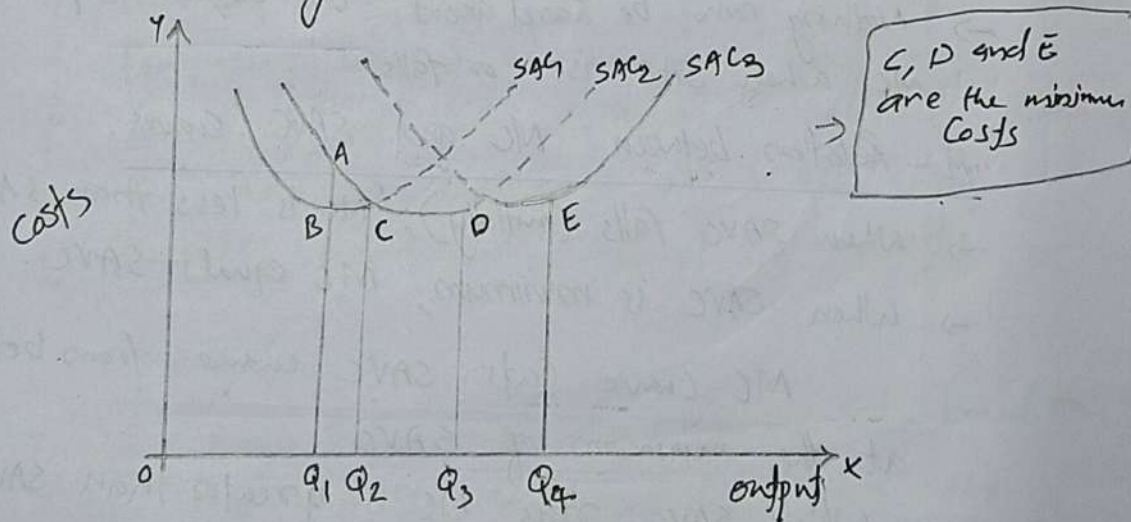
Summary Note: - Shortrun cost analysis can be examined using average cost curves and MC curves. However, AFC has little significance in the analysis since it does not affect the output level changes.

Long run Cost Curves

As learned previously, there are no fixed factors in the long run. hence no fixed costs. In the long run firms have only variable cost since only they are variable. We also learned that firms will produce when the cost is minimum.

"The long run average total cost (LAC) curve represents the minimum average cost of producing various levels of output from all possible short-run average cost curves (SAC)."

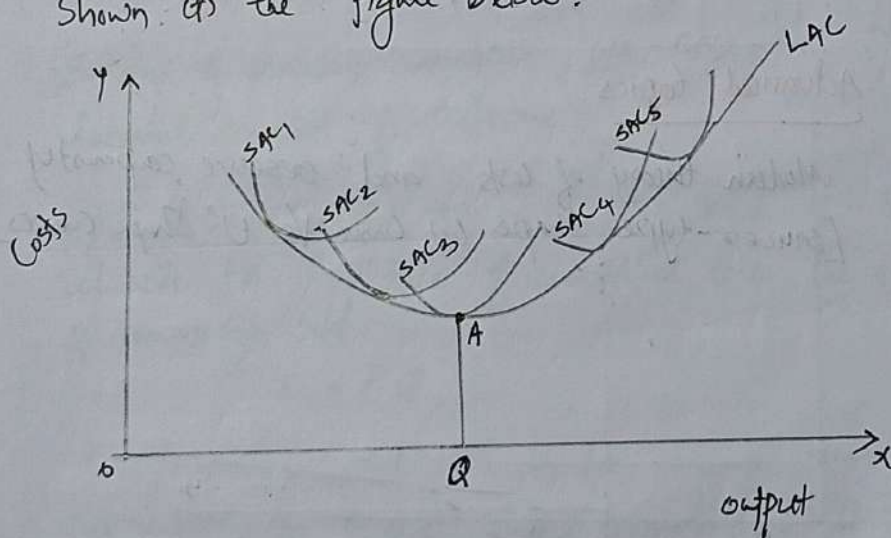
Let us consider the following figure for a better understanding.



In the above figure, different average cost curves (SAC) are presented based on different output possibilities of the firm. As the firm moves from SAC_1 to SAC_2 or SAC_2 to SAC_3 or SAC_1 to SAC_3 , the output level has increased.

If the firm want to produce OQ_1 level of output, the firm will prefer SAC_1 since the cost of producing OQ_1 level of output is lower on SAC_1 compared to SAC_2 . [i.e., $AQ_1 > BQ_1$]. When the firm want increase output level to OQ_2 the firm will move to point C on SAC_1 , since it is the minimum cost. Similarly if the firm decide to increase output level to OQ_3 and OQ_4 , the firms will move to the minimum points of SAC_2 (Point D) and SAC_3 (Point E), respectively.

The long run average curve (LAC) curve will be one that derive from the short run average cost curves as shown in the figure below.



It can be noticed that the LAC curve passes through the minimum point of SAC curves. The firm will produce commodities at the optimum when it produces OQ units of commodities. As the firms move out of this position it will be a non-optimum production.

level as cost is higher.

If the firm produce less than OQ output $\rightarrow$ under capacity

If the firm produce more than OQ output $\rightarrow$ over working

The LAC curve is known as the 'envelope' curve. — as it envelopes all the SAC curves.

Note:- The long-run Marginal Cost (LMC) curve of the firm intersects SAC₃ and LAC curves through their minimum point A. It can be presented in the same graph as well.

Advanced topics

Modern theory of costs and reserve capacity
[saucer-type Shape cost curve Vs 'U' Shape curves].

Revenue Functions

So far we analysed the costs and related aspects of a producer. To determine the profit of a producer, studying revenue is equally important.

The term 'revenue' refers to the receipts obtained by a firm from the sale of certain quantities of commodities at different prices.

The revenue function can be expressed as follows.

$$R = P \cdot Q$$

R = Revenue

P = Price of the commodity

Q = quantity sold.

Similar to previous analysis, we can also analyse different concepts of revenue;

Total Revenue: Total revenue is the value of total sales. It will be the price of commodities times the quantity of commodity sold.

$$TR = P \cdot Q$$

Average Revenue [AR]: AR is the average receipts from the sale of certain given commodities. It can be derived by dividing the total revenue with the units of commodities sold.

Marginal Revenue [MR]: Marginal Revenue will be the addition to total revenue, when the firm increases the quantity of sales. It can be expressed as follows.

$$MR_N = TR_N - TR_{N-1}$$

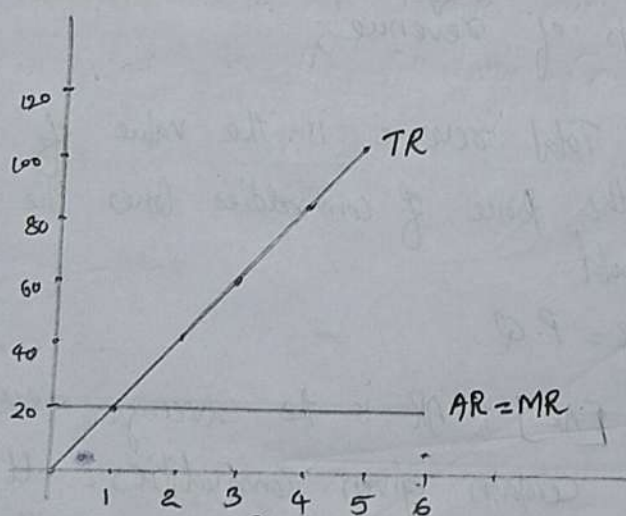
function

Graphical representation of Revenue Curves

Note: → This table and figure is based on perfect competitive market —

A hypothetical revenue data is provided in the table below. We assume that the price of commodity is 20.

Q	TR	AR	MR
1	20	20	20
2	40	20	20
3	60	20	20
4	80	20	20
5	100	20	20
6	120	20	20



We will not go for detailed analysis of AR, MR and TR curves under imperfect competitive market.

Moreover, here we are interested only in the "Total Revenue"

Average Revenue is the same as Demand curve?

Proof:

We know, $TR = P \times Q$

$$AR = \frac{TR}{Q}$$

$$= \frac{PQ}{Q}$$

$$AR = P$$

We already know, $Q = f(P)$

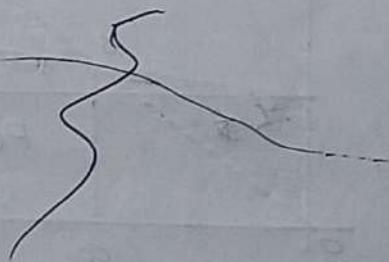
~~Thus,~~

$$AR = P$$

$$\text{demand} = f(P)$$

$$\underline{AR = \text{Demand}}$$

In Summary, in a perfectly competitive market, the total revenue curve will be a straight line with a positive slope, coming out of the origin.



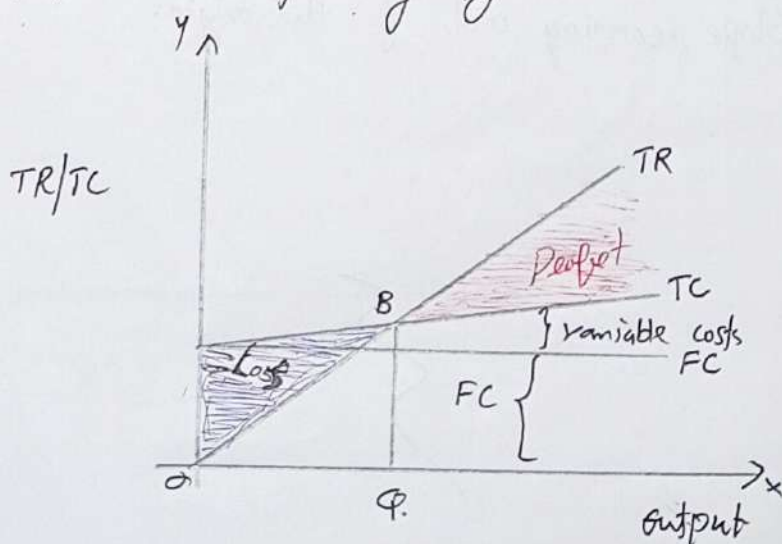
Break-even analysis

In the break-even analysis, we study the equilibrium condition of a firm based on its revenue and costs. For a firm to continue its operation the revenues should sufficiently cover the total costs.

Methods of Break even analysis

i- Graphical approach.

Under graphical approach we present the total costs, total revenue and level of output. The point where the total revenue equals the total cost is known as break-even point. It is presented in the following figure.



TR curve is presented on the diagram [assuming the commodities are sold at the same price]

The point of equality between TR and TC curves is the break-even point.

At point B, the firm earns no profit nor loss. If the firm expands its output beyond OQ_0 level, the firm will earn profit and if the firm reduces production below OQ_0 level, it incurs loss.

Margin of Safety (MS)

The break-even analysis is used to estimate the minimum quantity of sales that a firm has to sell in order to survive the production. This calculation is the calculation of the margin of safety.

Margin of safety is the Actual sales revenue minus the BE sales.

$$MS = \text{Actual sales revenue} - \text{BE sales}$$

Assumptions of BE analysis

- The cost and revenue functions are linear
- Total cost is divided into fixed and variable costs.
- Fixed cost is constant.
- variable costs change proportionately with output.
- Sales price is constant
- The number of units produced and sold of the product are identical.
- Factor prices are constant.
- Costs are affected only by quantity produced
- There is no change in technology and productivity.
- There is only one product.