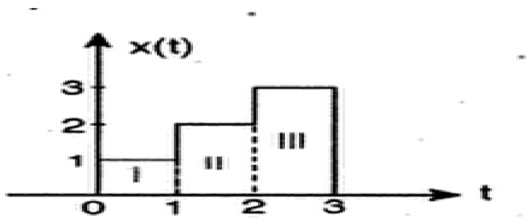
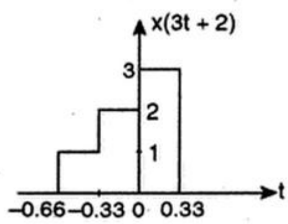
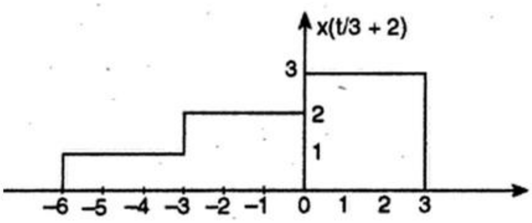
SCHOOL OF ELECTRONICS ENGINEERING
FALL SEMESTER _ 2023-24
CONTINUOUS ASSESSMENT TEST (CAT)-1
BECE202L– SIGNALS AND SYSTEMS

Course : **B.Tech (ECE)**
Class Nbrs : VL2023240102254, 56, 58, 60, 62.
Slot : **B2+TB2** **Course Type** : **ETH** **Course Mode** : **CBL**
Date : **11th, September 2023** **Marks** : **50** **Duration** : **90 Min**
Faculty: MALAYA KUMAR HOTA, S KALAIVANI, SUDHAKAR M S, YEPUGANTI KARUNA, ASHISH P

Each Question carries 10 marks

Q.No	Question	CO
1.	(a). Determine $x(3t+2)$ and $x\left(\frac{t}{3}+2\right)$ for the given signal $x(t)$  **ANSWER**   (b). Show that the functions $sin(n\omega_0 t)$ and $cos(m\omega_0 t)$ are orthogonal over interval $\left(t_0,\ t_0+\frac{2\pi}{\omega_0}\right)$ for any integer values of m, n. **ANSWER**	CO1

Let $x_1(t) = \sin(n\omega_0 t)$; $x_2(t) = \cos(m\omega_0 t)$

$x_1(t)$ & $x_2(t)$ are said to be orthogonal

if $$\int_{t_1}^{t_2} x_1(t)\cdot x_2(t)\,dt = 0.$$

$$\therefore \int_{t_0}^{t_0+2\pi/\omega_0} x_1(t)\,x_2(t)\,dt = \int_{t_0}^{t_0+2\pi/\omega_0} \sin(n\omega_0 t)\cdot\cos(m\omega_0 t)\,dt$$

$$= \int_{t_0}^{t_0+2\pi/\omega_0} \left[\frac{\sin(n+m)\omega_0 t + \sin(n-m)\omega_0 t}{2}\right] dt.$$

$$= \frac{1}{2(m+n)\omega_0}\Big[\cos(m+n)\omega_0 t\Big]_{t_0}^{t_0+2\pi/\omega_0} + \frac{1}{2(m-n)\omega_0}\Big[\cos(m-n)\omega_0 t\Big]_{t_0}^{t_0+2\pi/\omega_0}$$

$$= \frac{1}{2(m+n)\omega_0}\left[\cos\left\{(m+n)\omega_0\left(t_0+\frac{2\pi}{\omega_0}\right)\right\} - \cos(m+n)\omega_0 t_0\right] + \frac{1}{2(m-n)\omega_0}\left[\cos\left\{(m-n)\omega_0\left(t_0+\frac{2\pi}{\omega_0}\right)\right\} - \cos(m-n)\omega_0 t_0\right]$$

$$= \frac{1}{2(m+n)\omega_0}\left[\cos(m+n)\omega_0 t_0 - \cos(m+n)\omega_0 t_0\right] + \frac{1}{2(m-n)\omega_0}\left[\cos(m-n)\omega_0 t_0 - \cos(m-n)\omega_0 t_0\right]$$

$$= 0.$$

$\therefore$ Given two signals are orthogonal.

2. Classify the following signals: CO1

a. $x_1(t) = \left[2\,cos^2\left(\frac{\pi t}{2}\right) - 1\right]\sin\pi t\cos\pi t$ as periodic or non-periodic, if periodic find the fundamental period.

b. $x_2(t) = e^{t-1}u(-t)$ as energy or power signals and also calculate energy/power.

c. $x_3[n] = \begin{cases}\cos\pi n & ; \quad -4 \le n \le 4 \\ 0 & ; \quad otherwise\end{cases}$ as energy or power signals and also calculate energy/power.

d. $x_4[n] = \cos\left(\frac{\pi n}{7}\right)\sin\left(\frac{\pi n}{3}\right)$ as periodic or non-periodic, if periodic find the fundamental period.

ANSWER

$$x(t) = \left[2\cos^2\left(\frac{\pi t}{2}\right) - 1\right] \sin \pi t \cos \pi t$$

$$= [1 + \cos \pi t - 1] \sin \pi t \cos \pi t = \cos^2 \pi t \sin \pi t$$

$$= \frac{1 + \cos 2\pi t}{2} \sin \pi t = \frac{1}{2} \sin \pi t + \frac{1}{2} \cos 2\pi t \sin \pi t$$

$$= \frac{1}{2} \sin \pi t + \frac{1}{2} \cdot \frac{1}{2} \{\sin 3\pi t + \sin \pi t\}$$

$$= \frac{1}{2} \sin \pi t + \frac{1}{4} \sin 3\pi t + \frac{1}{4} \sin \pi t$$

$$= \frac{3}{4} \sin \pi t + \frac{1}{4} \sin 3\pi t = A_1 \sin 2\pi f_1 t + A_2 \sin 2\pi f_2 t$$

Here $f_1 = \frac{1}{2} \Rightarrow T_1 = \frac{1}{f_1} = 2$

and $f_2 = \frac{2}{3} \Rightarrow T_2 = \frac{1}{f_2} = \frac{3}{2}$

Here $\frac{T_1}{T_2} = \frac{2}{3/2} = \frac{4}{3}$, which is rational. Hence the signal is periodic. The least common multiple of T_1 and T_2 is 6. Hence, fundamental period, $T = 6$.

(ii).

$$x(t) = e^{t-1}u(-t) = e^{-1}e^{t}u(-t)$$

$$E = e^{-2} \int_{-\infty}^{0} e^{2t} dt = 0.5e^{-2} = 0.068 \text{ J}$$

(iii).

ii) $x(n) = \begin{cases} \cos(\pi n) & \text{for } -4 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$

$$= \{1 \quad 1 \quad 1 \quad 1 \quad \underset{\uparrow}{1} \quad 1 \quad 1 \quad 1 \quad 1\}$$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-4}^{4} |x(n)|^2 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 9$$

Since energy is finite, this is energy signal.

(iv).

Solution : i) $x(n) = \cos\left(\frac{\pi n}{7}\right)\sin\left(\frac{\pi n}{3}\right)$

$$= \frac{1}{2}\left\{\sin\left(\frac{\pi n}{7}+\frac{\pi n}{3}\right)+\sin\left(\frac{\pi n}{7}-\frac{\pi n}{3}\right)\right\}$$

$$= \frac{1}{2}\left\{\sin\frac{10\pi n}{21}-\sin\frac{4\pi n}{21}\right\} = \frac{1}{2}\{\sin 2\pi f_1 n+\sin 2\pi f_2 n\}$$

Here $f_1 = \frac{5}{21}$ and $f_2 = \frac{2}{21}$

Here both frequencies are rational. Hence signal is periodic. Here $N_1 = 21$ and $N_2 = 21$.

$\therefore$ $N = N_1 = N_2 = 21$

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3. (a). Check whether or not the given system is linear, time invariant, causal, memory less and stable. CO2

$$y(t) = x(t-2) + 2(x-t)$$

ANSWER

i.The system is not memoryless since the output depends on past and the future values of the input.

ii.The system is non-causal, since, the output depends on future values of input.

$$2x(t+\tau)$$

iii. The system is time variant

$$y(t,\tau) \neq y(t-\tau)$$
$$y(t-\tau 1) = x(t-2-\tau 1) + 2(x+\tau-\tau 1)$$

iv. The system is linear (satisifies superposition) and stable (satisifies BIBO criterion.

(b). Consider a system whose output,

$$\boldsymbol{y(n) = \sum_{k=-\infty}^{\infty} x(k)x(n+k)}$$

Determine whether or not the system is Linear, and Stable

ANSWER

a. Checking the system for homogeneity $x(n) = \delta(n); y(n) = \delta(n)$, if $x(n) = 2\,\delta(n)$ then $y(n) = 4\,\delta(n)$ which is non-linear.
b. Stability – assuming $x(n) = u(n)$ then $y(n) = \sum_{k=-\infty}^{\infty} u(k)u(n+k)$; $y(o) = \sum_{k=-\infty}^{\infty} u(k)u(k)$ produces unbounded result therefore, the system is unstable.

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4. Determine the response of the system with impulse response $h(n) = a^n\, u(n)$, $|a|<1$ to the input $x(n) = [u(n) - u(n-5)]$ in time domain. CO2

ANSWER

$$h(n) = a^n u(n) \qquad x(n) = [u(n) - u(n-5)]$$

$$
\begin{aligned}
y(n) &= x(n) * h(n) \\
&= [u(n) - u(n-5)] * h(n) \\
&= [u(n) * h(n)] - [u(n-5) * h(n)] \\
&= y_1(n) - y_1(n-5)
\end{aligned}
$$

$$
\begin{aligned}
y_1(n) &= u(n) * h(n) \\
&= \sum_{K=0}^{\infty} u(K)\, h(n-K)
\end{aligned}
$$

for $n \geq 0$,

$$
\begin{aligned}
y_1(n) &= \sum_{K=0}^{n} a^{n-K} = a^n \sum_{K=0}^{n} a^{-K} \\
&= a^n \left[\frac{1 - a^{-(n+1)}}{1 - a^{-1}} \right] = \frac{1 - a^{n+1}}{1 - a}
\end{aligned}
$$

$$y_1(n) = \frac{1 - a^{n+1}}{1 - a} u(n)$$

$$y(n) = \frac{1}{1-a} \left[(1 - a^{n+1}) u(n) - (1 - a^{n-4}) u(n-5) \right]$$

5. For the periodic signal $x(t)$ shown in Figure.2 CO3

i) Determine the exponential Fourier series.

ii) Sketch the graph of exponential Fourier spectra.

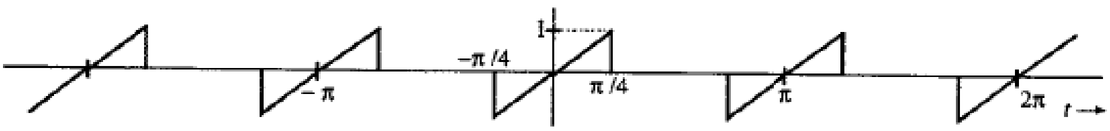

Figure.2

ANSWER

i) **Exponential Fourier series.**

$T_0 = \pi$, $\omega_0 = 2$ and $D_n = 0$

$$f(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2nt}, \qquad \text{where} \qquad D_n = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \frac{4t}{\pi} e^{-j2nt}\, dt = \frac{-j}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$

(ii).Exponential Fourier spectra: