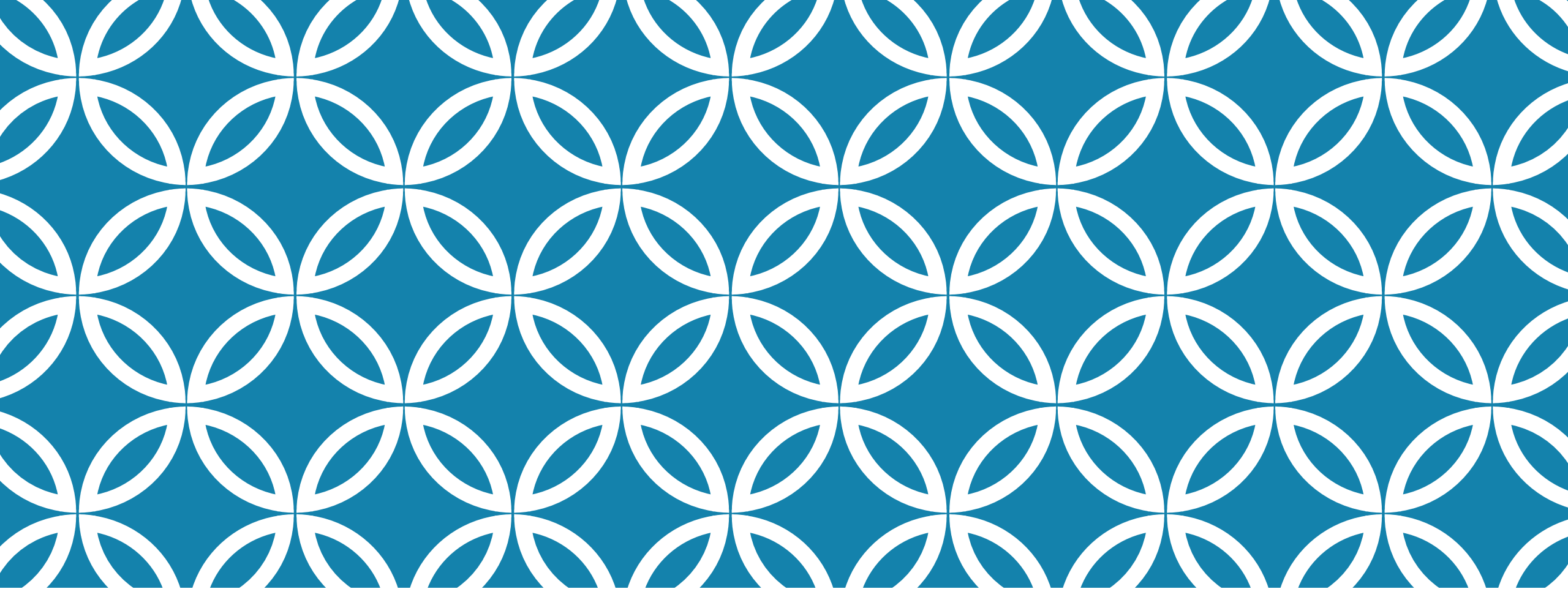




INTRODUCTION TO WAVES

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SYLLABUS & REFERENCE BOOKS

Waves on a string - Wave equation on a string (derivation) - Harmonic waves- reflection and transmission of waves at a boundary (Qualitative)
- Standing waves and their eigen frequencies.

1. H. D. Young and R. A. Freedman, University Physics with Modern Physics, 2020, 15th Edition, Pearson, USA
2. H. J. Pain, The Physics of Vibrations & Waves, 2013, 6th Edition, Wiley Publication India.

LECTURE PLAN

7 Lectures

- Introduction
- Wave on a string
- Wave equation for wave on a string (derivation)
- Harmonic waves
- Reflection and transmission of waves on a string at a boundary (Qualitative)
- Standing waves and their eigen frequencies (derivation)
- Numerical Problems

WAVES



Ocean Waves

Ocean waves travel thousand of kilometres across the open sea before breaking on the shore. How much water they move?

None

Kinds of waves:

Sound waves

Light or Electromagnetic waves

Seismic waves

Ultrasound waves

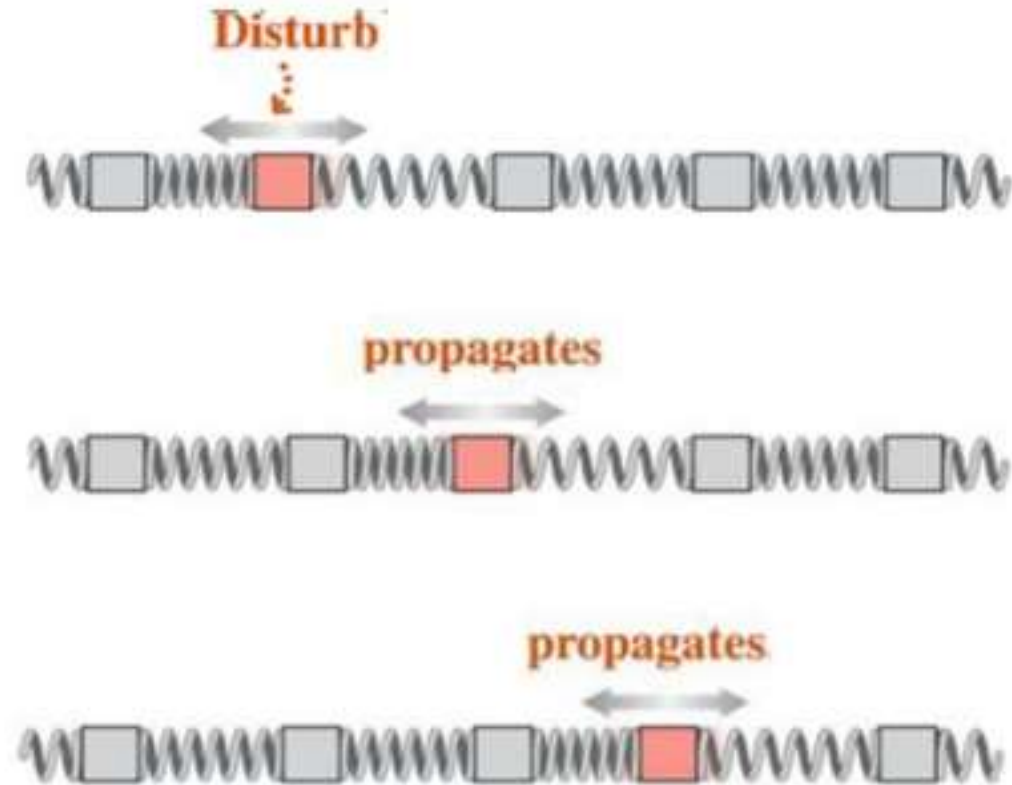
Many more.....

A wave is a travelling disturbance transporting energy not matter!

CONT'D...

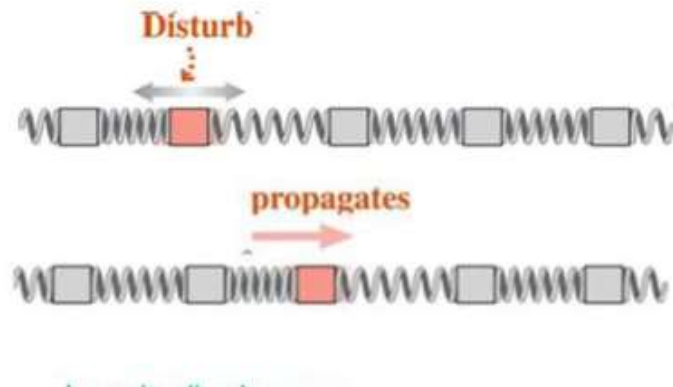
Mechanical waves: which require a material medium to propagate, e.g. sound waves, seismic waves etc.

Electromagnetic waves: which do not require medium, e.g. light waves, radio waves, microwaves etc.

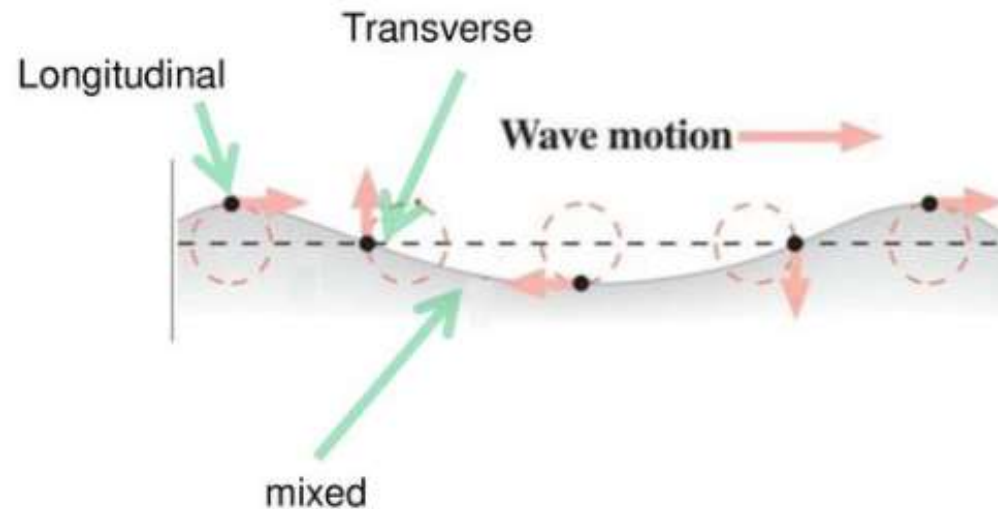
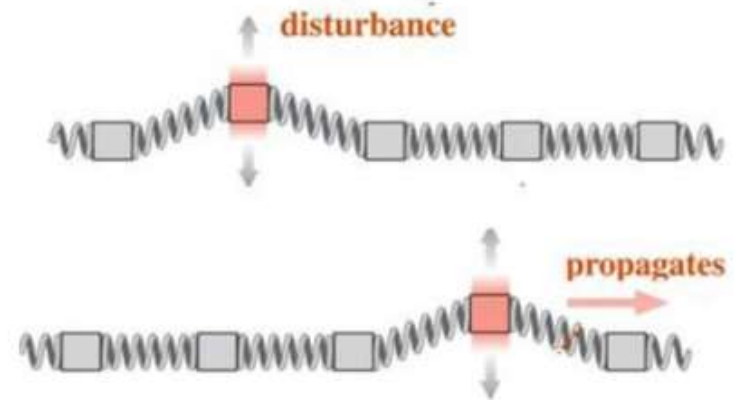


LONGITUDINAL AND TRANSVERSE WAVES

Longitudinal



Transverse



DIFFERENCE BETWEEN A LONGITUDINAL AND A TRANSVERSE WAVE

Longitudinal Wave	Transverse wave
The particles of the medium move in the direction of the wave propagation	The particles of the medium move perpendicular to the direction of wave propagation
It acts in one dimension	It acts in two dimensions
They cannot be polarized	The wave can be polarized using polarizers
This wave can be produced in any medium such as gas, liquid or solid	This wave can be produced in solid and liquid's surface
Examples: Seismic P-wave, sound waves, Vibrations of an elastic spring	Examples: Seismic S-wave, Electromagnetic waves, ripples on the surface of water
It is made of rarefactions and compressions	It is made of troughs and crests

WHAT IS A WAVE?

“A wave is a disturbance of a continuous medium that propagates with a fixed shape at constant velocity”.

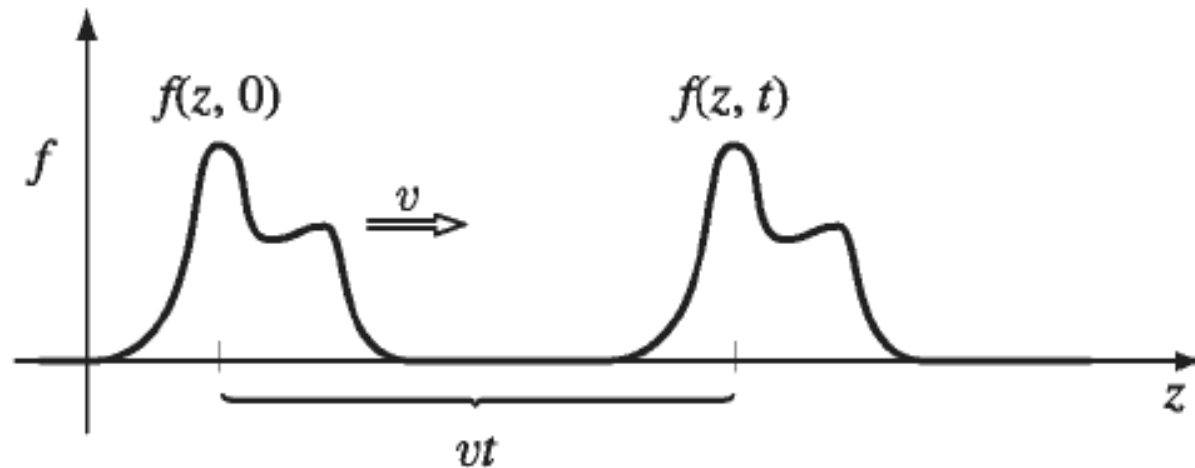
Qualifiers for the above definition:

- ❖ If the medium is dispersive, different frequencies travel at different speeds.
- ❖ In the presence of absorption, the wave will diminish in size as it moves.
- ❖ In two or three dimensions, as the wave spreads out its amplitude will decrease.
- ❖ Standing waves do not propagate at all.

CONT'D....

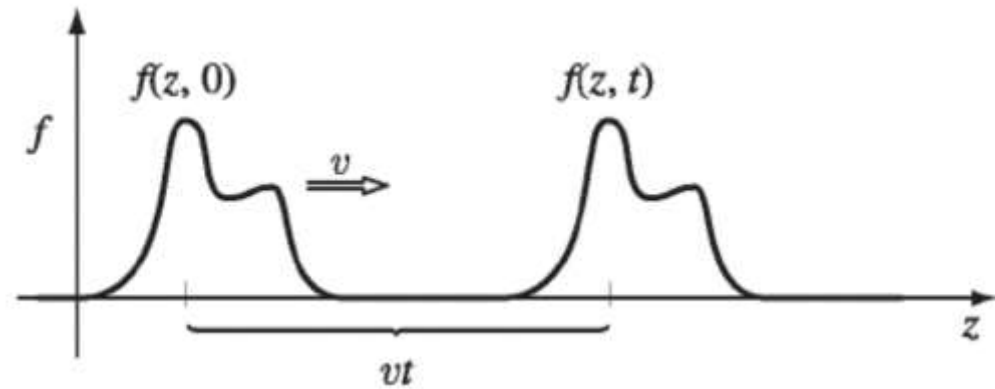
But these are refinements;

Let's start with the simple case: fixed shape, constant speed.



How can we represent such an object mathematically?

CONT'D...



- ❖ $f(z, t)$ represents the displacement of the string at the point z , at time t .
- ❖ Given the initial shape of the string, $g(z) = f(z, 0)$, what is the subsequent form, $f(z, t)$?
- ❖ The displacement at point z , at the later *time* t , is the same as the displacement a distance vt to the left (i.e. at $z - vt$), back at *time* $t = 0$:

$$f(z, t) = f(z - vt, 0) = g(z - vt)$$

That statement captures (mathematically) the essence of wave motion. It tells us that the function $f(z, t)$, which *might* have depended on z and t in *any* old way, *in fact* depends on them only in the very special combination $z - vt$;

when that is true, the function $f(z, t)$ represents a wave of fixed shape traveling in the z direction at speed v .

EXAMPLES

For example, if A and b are constants (with the appropriate units),

$$f_1(z, t) = Ae^{-b(z-vt)}, f_2(z, t) = A \sin[b(z-vt)], f_3(z, t) = A/[b(z-vt)^2 + 1]$$

all represent waves (with different shapes, of course), but

$$f_4(z, t) = Ae^{-b(bz^2+vt)} \text{ and } f_5(z, t) = A \sin(bz) \cos(bvt)^3$$

do *not*.

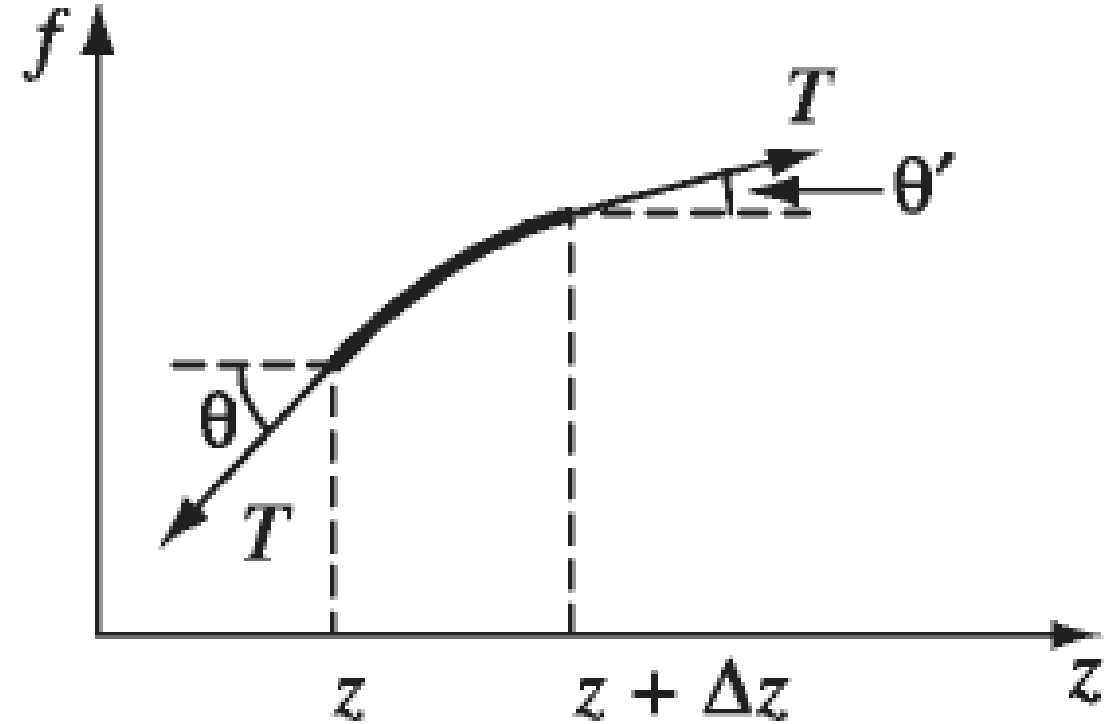
WAVES ON A STRING

- Imagine a very long string under tension T .
- If it is displaced from equilibrium, the net transverse force on the segment between z and $z + \Delta z$ is

$$\Delta F = T \sin\theta' - T \sin\theta \quad (1)$$

- Provided that the distortion of the string is not too great, these angles are small (the figure is exaggerated, obviously),
- we can replace the sine by the tangent:

$$\Delta F \cong T (\tan\theta' - \tan\theta) = T \left(\left. \frac{\partial f}{\partial z} \right|_{z+\Delta z} - \left. \frac{\partial f}{\partial z} \right|_z \right) \cong T \frac{\partial^2 f}{\partial z^2} \Delta z \quad (2)$$



CONT'D....

If the mass per unit length is μ , Newton's second law says

$$\Delta F = \mu(\Delta z) \frac{\partial^2 f}{\partial t^2} \quad (3)$$

and therefore

$$\frac{\partial^2 f}{\partial z^2} = \frac{\mu}{T} \frac{\partial^2 f}{\partial t^2} \quad (4)$$

Evidently, small disturbances on the string satisfy,

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} \quad (5)$$

where v (which, as we'll soon see, represents the speed of propagation) is

$$v = \sqrt{\frac{T}{\mu}} \quad (6)$$

Equation (5) is known as the **(classical) wave equation**, because it admits as solutions all functions of the form $f(z, t) = g(z - vt)$.

CONT'D....

(that is, all functions that depend on the variables z and t in the special combination

$u = z - vt$), Such functions represent waves propagating in the z direction with speed v . That means

$$\frac{\partial f}{\partial z} = \frac{dg}{du} \frac{\partial u}{\partial z} = \frac{dg}{du}, \quad \frac{\partial f}{\partial t} = \frac{dg}{du} \frac{\partial u}{\partial t} = -v \frac{dg}{du},$$

and

$$\frac{\partial^2 f}{\partial z^2} = \frac{\partial}{\partial z} \left(\frac{dg}{du} \right) = \frac{d^2 g}{du^2} \frac{\partial u}{\partial z} = \frac{d^2 g}{du^2},$$

$$\frac{\partial^2 f}{\partial t^2} = -v \frac{\partial}{\partial t} \left(\frac{dg}{du} \right) = -v \frac{d^2 g}{du^2} \frac{\partial u}{\partial t} = v^2 \frac{d^2 g}{du^2},$$

so

$$\frac{d^2 g}{du^2} = \frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}.$$

qed !

CONT'D....

- ❖ Note that $g(u)$ can be any (differentiable) function whatever.
- ❖ If the disturbance propagates without changing its shape, then it satisfies the wave equation.
- ❖ But functions of the form $g(z - vt)$ are not the only solutions.
- ❖ The wave equation involves the square of v , so we can generate another class of solutions by simply changing the sign of the velocity:

$$f(z, t) = h(z + vt)$$

- ❖ This, of course, represents a wave propagating in the negative z direction, and it is certainly reasonable (on physical grounds) that such solutions would be allowed.
- ❖ The most general solution to the wave equation is the sum of a wave to the right and a wave to the left:

$$f(z, t) = g(z - vt) + h(z + vt)$$

PROBLEM 1

By explicit differentiation, check that the functions f_1 , f_2 , and f_3 in the text satisfy the wave equation. Show that f_4 and f_5 do not.

For example, if A and b are constants (with the appropriate units),

$$f_1(z, t) = Ae^{-b(z-vt)}, f_2(z, t) = A \sin[b(z-vt)], f_3(z, t) = A/[b(z-vt)^2 + 1]$$

all represent waves (with different shapes, of course), but

$$f_4(z, t) = Ae^{-b(bz^2+vt)} \text{ and } f_5(z, t) = A \sin(bz) \cos(bvt)^3$$

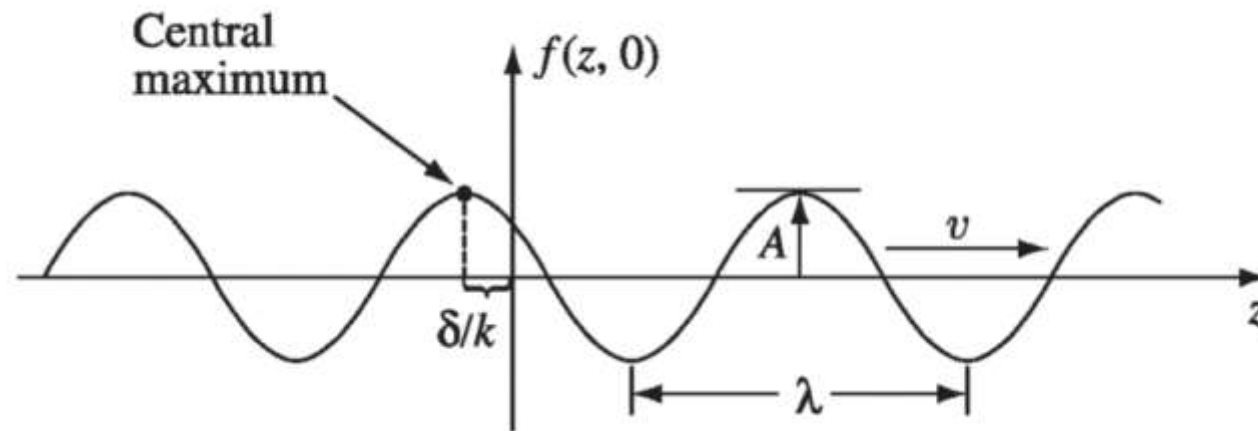
do not.

PROBLEM 2

Show that the standing wave $f(z, t) = A \sin(kz) \cos(kvt)$ satisfies the wave equation, and express it as the sum of a wave traveling to the left and a wave traveling to the right.

SINUSOIDAL WAVES

Terminology. Of all possible wave forms, the sinusoidal one $f(z, t) = A \cos[k(z - vt) + \delta]$ is the most familiar



- ❖ The argument of the cosine is called the phase, and δ is the phase constant.
- ❖ Here at $z = vt - \delta/k$, the phase is zero; This is called the "central maximum."
- ❖ If $\delta = 0$, the central maximum passes the origin at time $t = 0$; more generally, δ/k is the distance by which the central maximum (and therefore the entire wave) is "delayed."

CONT'D...

- ❖ k is the wave number; it is related to the wavelength by the equation,

$$\lambda = \frac{2\pi}{k}$$

for when z advances by $2\pi/k$, the cosine executes one complete cycle.

- ❖ As time passes, the entire wave train proceeds to the right, at speed v . At any fixed point z , the string vibrates up and down, undergoing one full cycle in a period,

$$T = \frac{2\pi}{kv}$$

- ❖ The frequency ν (number of oscillations per unit time) is,

$$\nu = \frac{1}{T} = \frac{kv}{2\pi} = \frac{v}{\lambda}$$

CONT'D....

A more convenient unit is the angular frequency ω , so-called because in the analogous case of uniform circular motion, it represents the number of radians swept out per unit time:

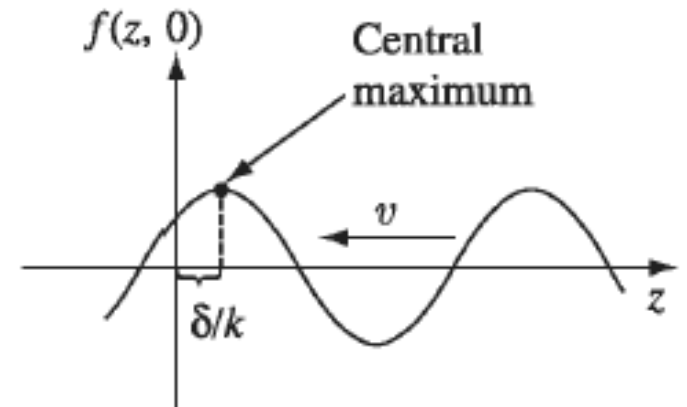
$$\omega = 2\pi\nu = kv$$

Ordinarily, it's nicer to write sinusoidal waves in terms of ω , rather than ν :

$$f(z, t) = A \cos(kz - \omega t + \delta)$$

A sinusoidal oscillation of wave number k and (angular) frequency ω traveling to the *left* would be written

$$f(z, t) = A \cos(kz + \omega t - \delta)$$



COMPLEX NOTATION

Euler's Formula: $e^{i\theta} = \cos\theta + i\sin\theta$

The sinusoidal wave function can be written as

$$f(z, t) = \text{Re}[Ae^{i(kz - \omega t + \delta)}]$$

where $\text{Re}(\xi)$ denotes the real part of the complex number ξ . This invites us to introduce the complex wave function

$$\tilde{f}(z, t) = \tilde{A}e^{i(kz - \omega t)}$$

with the complex amplitude $\tilde{A} = Ae^{i\delta}$ absorbing the phase constant. The actual wave function is the real part of $\tilde{f}$:

$$f(z, t) = \text{Re}[\tilde{f}(z, t)]$$

If you know $\tilde{f}$, it is a simple matter to find f ; the advantage of the complex notation is that exponentials are much easier to manipulate than sines and cosines.

LINEAR COMBINATION OF WAVE FUNCTIONS

Suppose you want to combine two sinusoidal waves:

$$f_3 = f_1 + f_2 = \text{Re}(\tilde{f}_1) + \text{Re}(\tilde{f}_2) = \text{Re}(\tilde{f}_1 + \tilde{f}_2) = \text{Re}(\tilde{f}_3),$$

with $\tilde{f}_3 = \tilde{f}_1 + \tilde{f}_2$. You simply add the corresponding *complex* wave functions, and then take the real part. In particular, if they have the same frequency and wave number,

$$\tilde{f}_3 = \tilde{A}_1 e^{i(kz - \omega t)} + \tilde{A}_2 e^{i(kz - \omega t)} = \tilde{A}_3 e^{i(kz - \omega t)},$$

where

$$\tilde{A}_3 = \tilde{A}_1 + \tilde{A}_2, \text{ or } A_3 e^{i\delta_3} = A_1 e^{i\delta_1} + A_2 e^{i\delta_2}.$$

In other words, you just add the (complex) amplitudes. The combined wave still has the same frequency and wavelength,

$$f_3(z, t) = A_3 \cos(kz - \omega t + \delta_3),$$

VELOCITIES IN WAVE MOTION

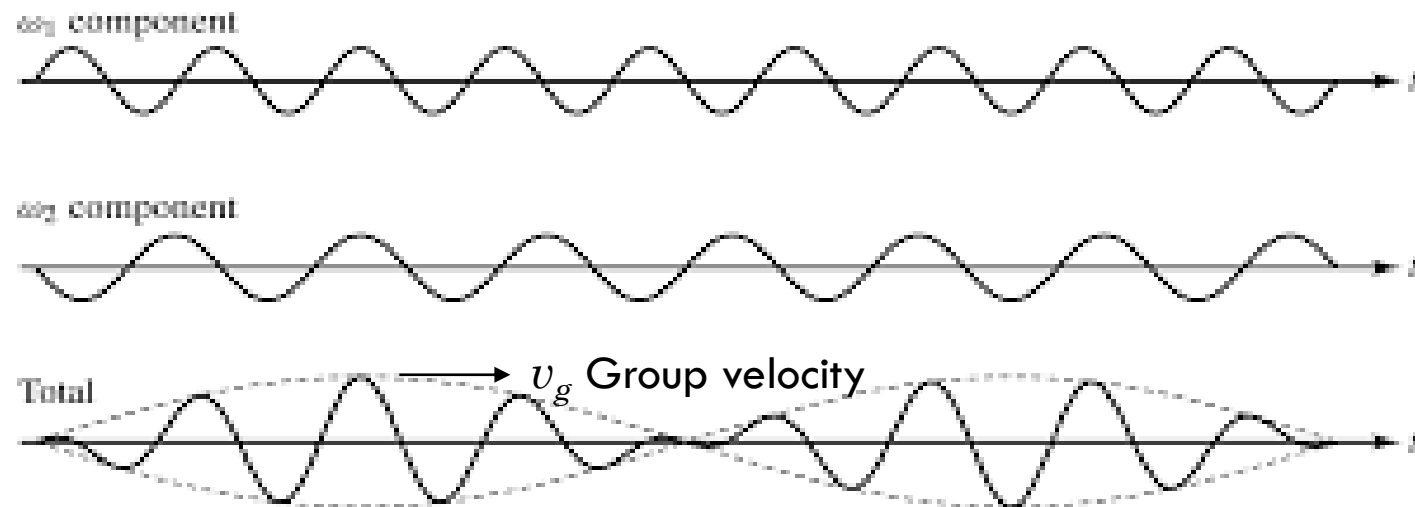
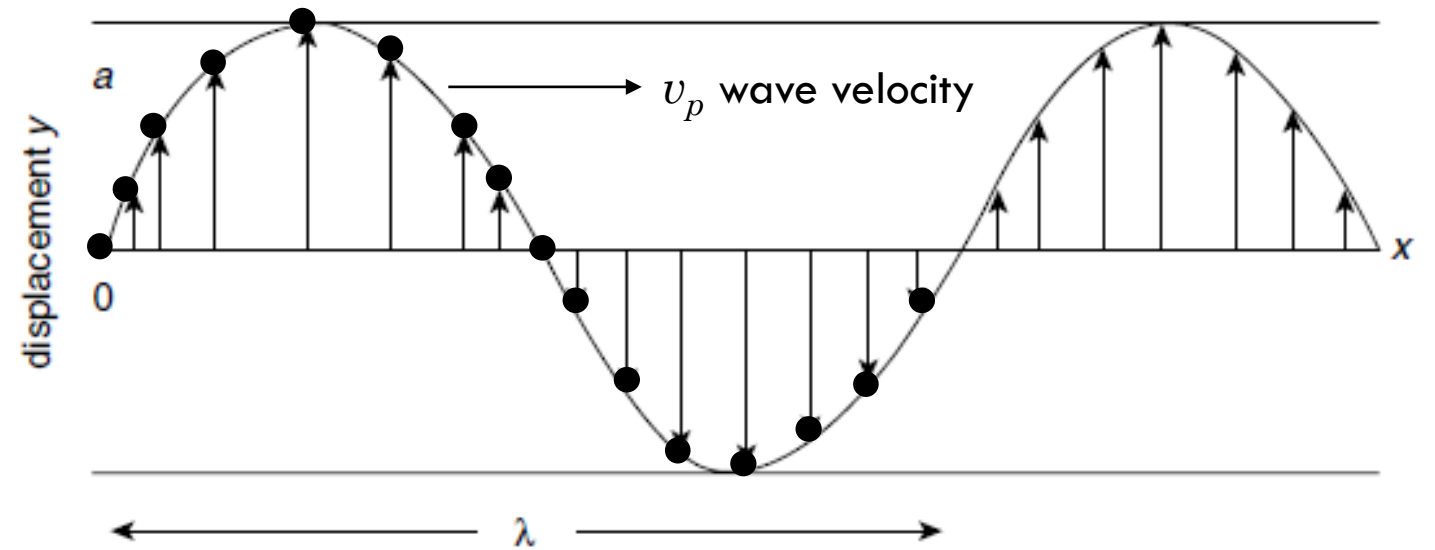
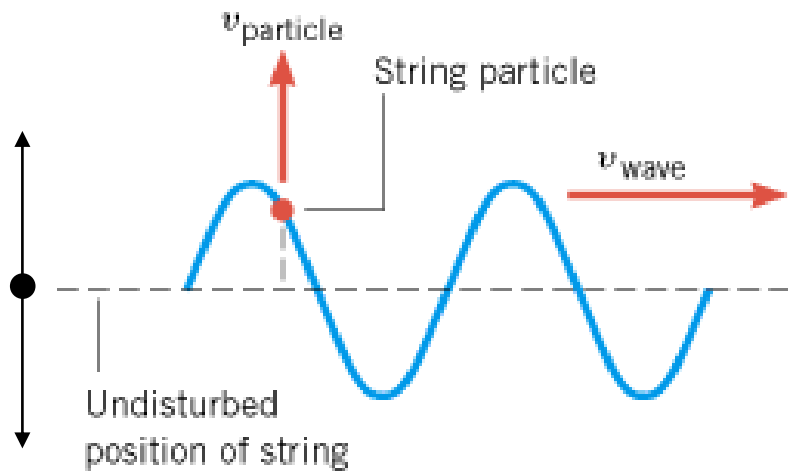
The particles or constituents of the medium which make up the medium do not progress through the medium with the waves.

Their motion is simple harmonic, limited to oscillations, transverse or longitudinal, about their equilibrium positions.

It is their phase relationships we observe as waves, not their progressive motion through the medium.

There are three velocities in wave motion which are quite distinct although they are connected mathematically. They are:

- ❖ The particle velocity, which is the simple harmonic velocity of the oscillator about its equilibrium position.
- ❖ The wave or phase velocity, the velocity with which planes of equal phase, crests or troughs, progress through the medium.
- ❖ The group velocity. A number of waves of different frequencies, wavelengths and velocities may be superposed to form a group.

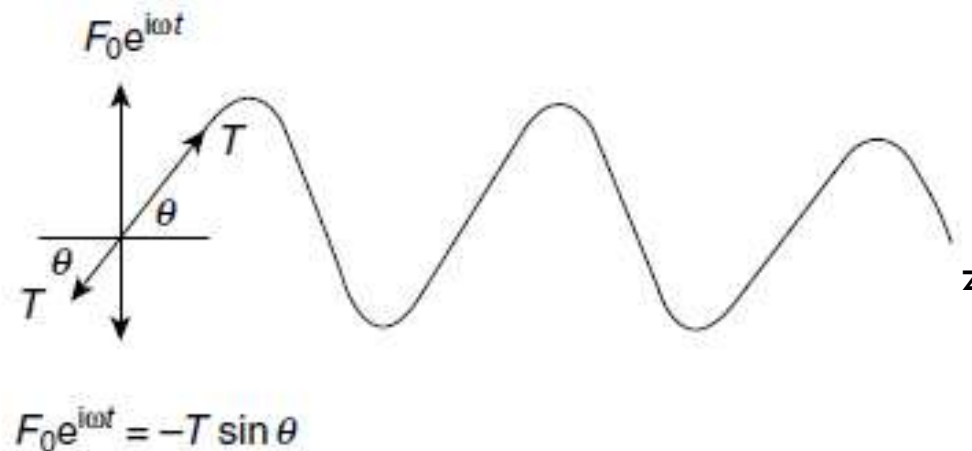


CHARACTERISTIC IMPEDANCE OF A STRING

Any medium through which waves propagate presents an impedance to those waves.

A string presents such an impedance to progressive waves and this is defined, as the transverse impedance for transverse waves

$$Z = \frac{\text{Transverse force on the string}}{\text{Transverse Velocity of the string}} = \frac{F}{\dot{f}}$$



CONT'D.....

- ❖ In Figure, we consider progressive waves on the string which are generated at one end by an oscillating force, $F = F_0 e^{i\omega t}$, which is restricted to the direction transverse to the string and operates only in the plane of the paper.
- ❖ The tension in the string has a constant value, T , and at the end of the string the balance of forces shows that the applied force is equal and opposite to $T \sin\theta$ at all time, so that

$$F_0 e^{i\omega t} = -T \sin\theta \approx -T \tan\theta = -T \left(\frac{\partial f}{\partial z} \right)$$

Where θ is small.

The displacement of the progressive wave may be represented as

$$f(z, t) = A e^{i(\omega t - kz)}$$

CONT'D....

Where the amplitude A may be complex. At the end of the string, where $z=0$,

$$F_o e^{i\omega t} = -T \left(\frac{\partial f}{\partial z} \right) \Big|_{z=0} = ikTAe^{i(\omega t - kz)} \quad \leftarrow \text{Transverse Force}$$

and $f = Ae^{i(\omega t - kz)}$

The transverse velocity of the string, $\dot{f} = \frac{\partial f}{\partial t} = i\omega Ae^{i(\omega t - kz)} \quad \leftarrow \text{Transverse Velocity}$

Thus transverse impedance is given by, $Z = \frac{ikTAe^{i(\omega t - kz)}}{i\omega Ae^{i(\omega t - kz)}} = \frac{kT}{\omega} = \frac{T}{v}$

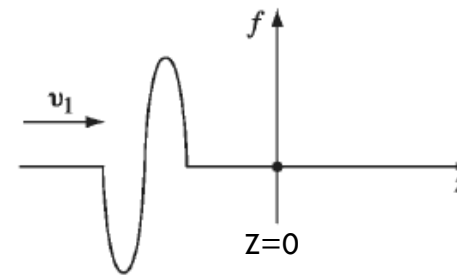
Which is called characteristic impedance of the string.

We know that wave velocity is given by, $v = \sqrt{\frac{T}{\mu}}$ which gives $T = \mu v^2$ and therefore, $Z = \mu v$

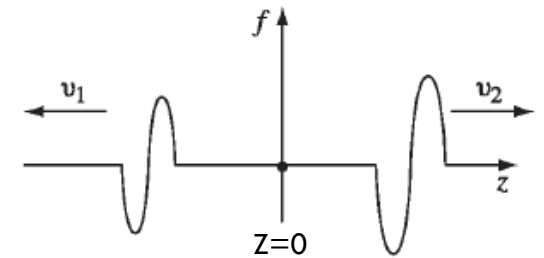
Since the wave velocity is determined by the inertia and the elasticity of the medium, the impedance is also governed by these properties.

BOUNDARY CONDITIONS: REFLECTION AND TRANSMISSION

- ❖ We would like to see what happens when a wave reaches the end of the string.
- ❖ Depending upon the end of the string is open or fixed, the wave on the string may be partially or completely reflected or transmitted.
- ❖ Suppose, for instance, that the string is simply tied onto a second string. The tension T is the same for both, but the mass per unit length μ presumably is not, and hence the wave velocities v_1 and v_2 are different (remember, $v = \sqrt{\frac{T}{\mu}}$).
- ❖ Let's say, for convenience, that the knot occurs at $z = 0$.



(a) Incident pulse



(b) Reflected and transmitted pulses

$$\tilde{f}_I(z, t) = \tilde{A}_I e^{i(k_1 z - \omega t)}, \quad (z < 0)$$

Incident wave

$$\tilde{f}_R(z, t) = \tilde{A}_R e^{i(-k_1 z - \omega t)}, \quad (z < 0)$$

Reflected wave

$$\tilde{f}_T(z, t) = \tilde{A}_T e^{i(k_2 z - \omega t)}, \quad (z > 0)$$

Transmitted wave

CONT'D....

For a sinusoidal incident wave, then, the net disturbance of the string is:

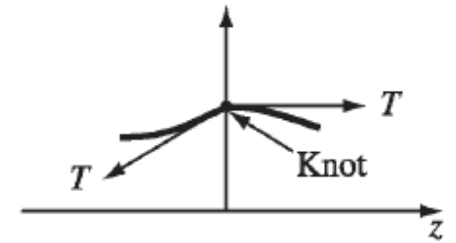
$$\tilde{f}(z, t) = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)}, & \text{for } z < 0, \\ \tilde{A}_T e^{i(k_2 z - \omega t)}, & \text{for } z > 0. \end{cases}$$

At the join ($z = 0$), the displacement just slightly to the left ($z = 0^-$) must equal the displacement slightly to the right ($z = 0^+$)—else there would be a break between the two strings. Mathematically, $f(z, t)$ is *continuous* at $z = 0$:

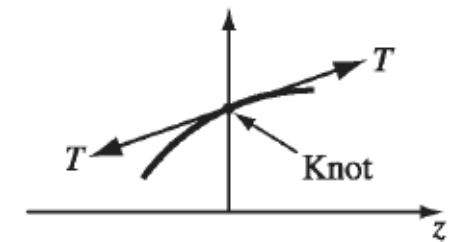
$$f(0^-, t) = f(0^+, t).$$

If the knot itself is of negligible mass, then the *derivative* of f must *also* be continuous:

$$\left. \frac{\partial f}{\partial z} \right|_{0^-} = \left. \frac{\partial f}{\partial z} \right|_{0^+}.$$



(a) Discontinuous slope; force on knot



(b) Continuous slope; no force on knot

CONT'D....

$$\tilde{f}(0^-, t) = \tilde{f}(0^+, t), \quad \left. \frac{\partial \tilde{f}}{\partial z} \right|_{0^-} = \left. \frac{\partial \tilde{f}}{\partial z} \right|_{0^+}$$

These boundary conditions determine the outgoing amplitudes ($\tilde{A}_R$ and $\tilde{A}_T$) in terms of the incoming one ($\tilde{A}_I$):

$$\tilde{A}_I + \tilde{A}_R = \tilde{A}_T, \quad k_1(\tilde{A}_I - \tilde{A}_R) = k_2 \tilde{A}_T$$

From which it follows that:

$$\tilde{A}_R = \left(\frac{k_1 - k_2}{k_1 + k_2} \right) \tilde{A}_I, \quad \tilde{A}_T = \left(\frac{2k_1}{k_1 + k_2} \right) \tilde{A}_I$$

In terms of velocities

$$\tilde{A}_R = \left(\frac{v_2 - v_1}{v_2 + v_1} \right) \tilde{A}_I, \quad \tilde{A}_T = \left(\frac{2v_2}{v_2 + v_1} \right) \tilde{A}_I$$

The real amplitudes and phases, then, are related by:

$$A_R e^{i\delta_R} = \left(\frac{v_2 - v_1}{v_2 + v_1} \right) A_I e^{i\delta_I}, \quad A_T e^{i\delta_T} = \left(\frac{2v_2}{v_2 + v_1} \right) A_I e^{i\delta_I}$$

CONT'D....

If the second string is lighter than the first ($\mu_2 < \mu_1$, so that $v_2 > v_1$), all three waves have the same phase angle ($\delta_R = \delta_T = \delta_I$), and the outgoing amplitudes are

$$A_R = \left(\frac{v_2 - v_1}{v_2 + v_1} \right) A_I, \quad A_T = \left(\frac{2v_2}{v_2 + v_1} \right) A_I$$

If the second string is heavier than the first ($v_2 < v_1$), the reflected wave is out of phase by 180° ($\delta_R + \pi = \delta_T = \delta_I$). In other words, since

$$\cos(-k_1 z - \omega t + \delta_I - \pi) = -\cos(-k_1 z - \omega t + \delta_I)$$

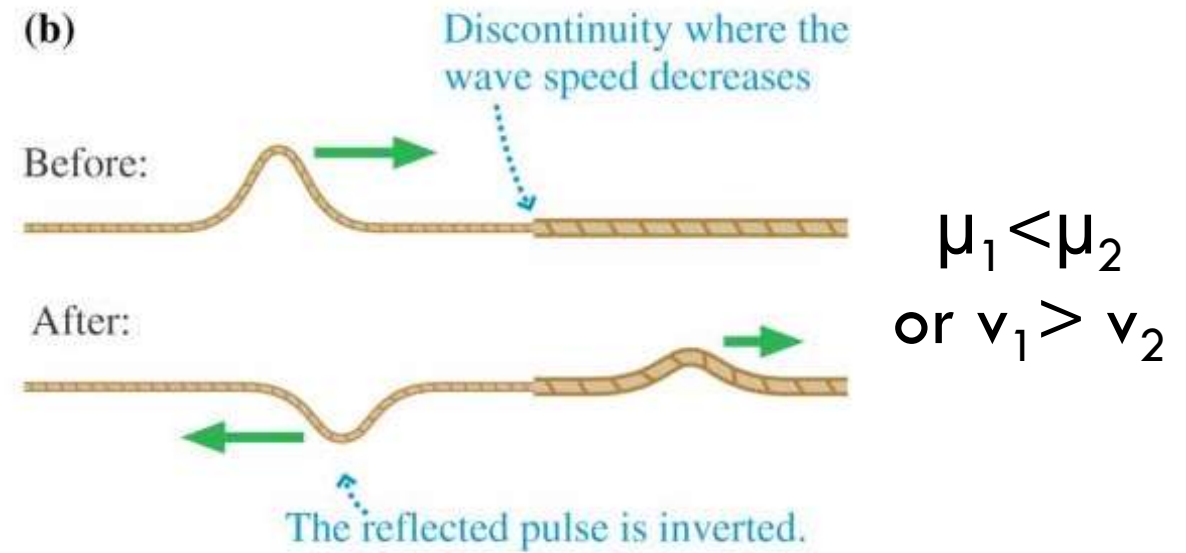
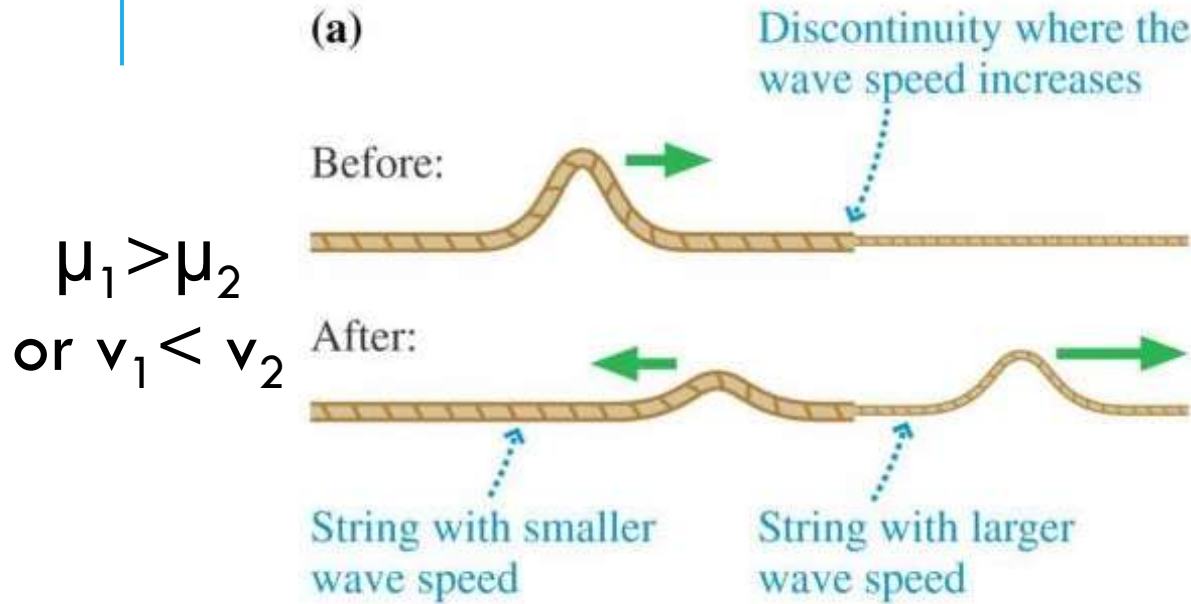
the reflected wave will be upside down. The amplitudes in this case are:

$$A_R = \left(\frac{v_1 - v_2}{v_2 + v_1} \right) A_I \quad \text{and} \quad A_T = \left(\frac{2v_2}{v_2 + v_1} \right) A_I$$

If the second string is infinitely massive, then $v_2=0$ and $A_R = A_I$ and $A_T = 0$

No transmitted wave.

BOUNDARY CONDITIONS: REFLECTION AND TRANSMISSION



$$\text{Reflection coefficient, } R = \frac{A_R}{A_I} = \frac{v_2 - v_1}{v_1 + v_2} = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$\text{Transmission coefficient, } T = \frac{A_T}{A_I} = \frac{2v_2}{v_1 + v_2} = \frac{2Z_1}{Z_1 + Z_2}$$

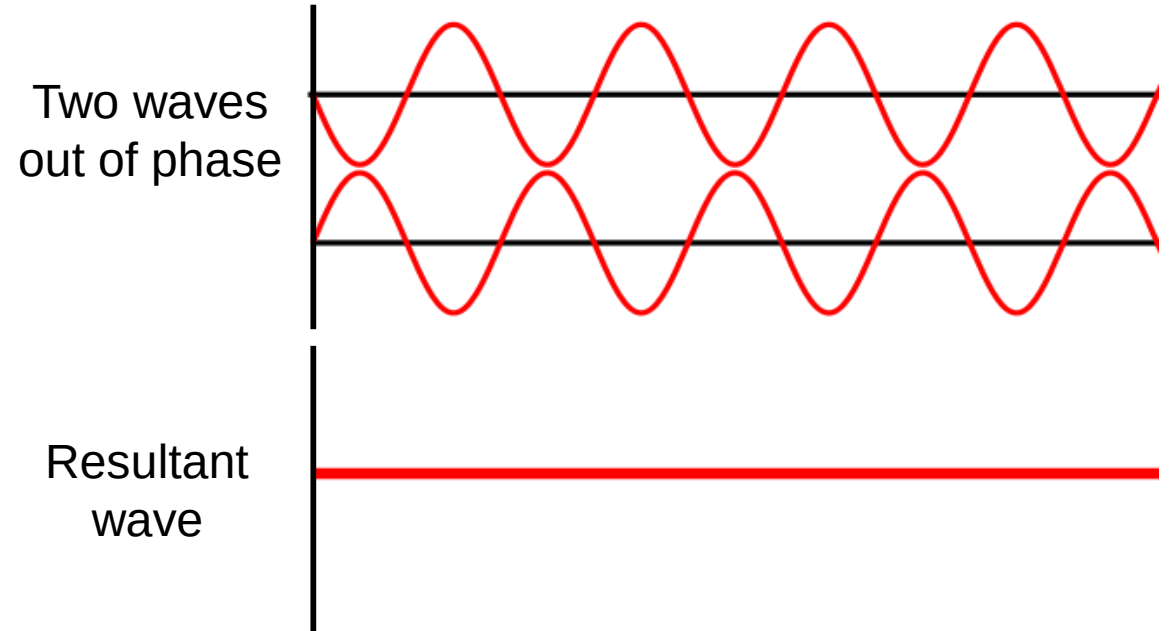
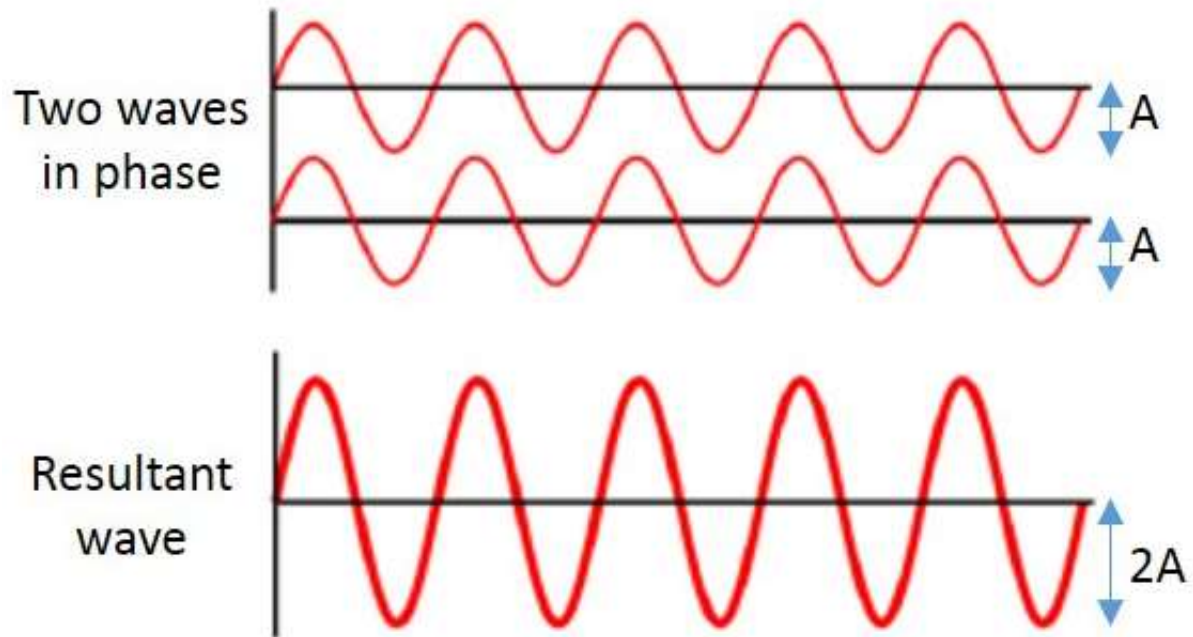
$$\text{Reflection coefficient, } R = \frac{A_R}{A_I} = \frac{v_1 - v_2}{v_1 + v_2} = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\text{Transmission coefficient, } T = \frac{A_T}{A_I} = \frac{2v_2}{v_1 + v_2} = \frac{2Z_1}{Z_1 + Z_2}$$

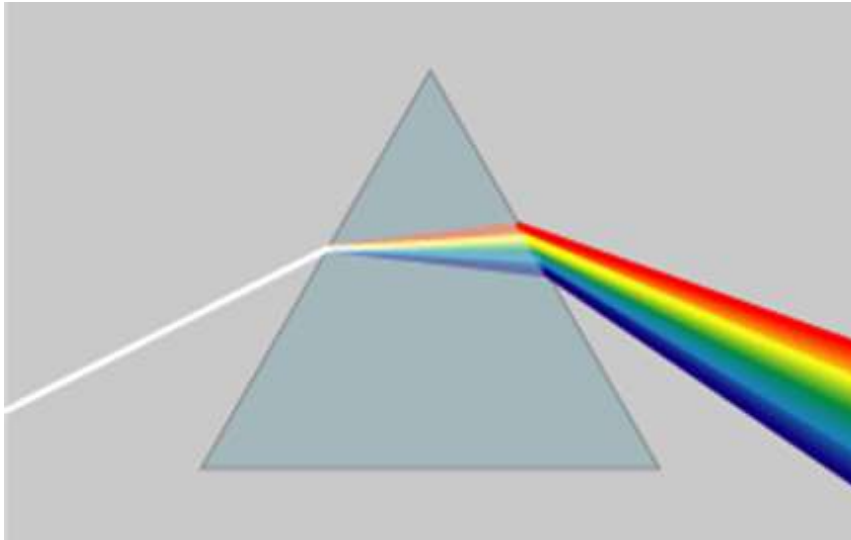
If $Z_2 = \infty$, this is equivalent to $z=0$ being a fixed end to the string because no transmitted wave exists.

If $Z_2 = 0$, so that $z=0$ is a free end of the string, then $A_R/A_I = 1$ and $A_T/A_I = 2$. This explains the 'flick' at the end of a whip or free ended string when a wave reaches it.

SUPERPOSITION OF WAVES



WAVES WITH DISPERSION

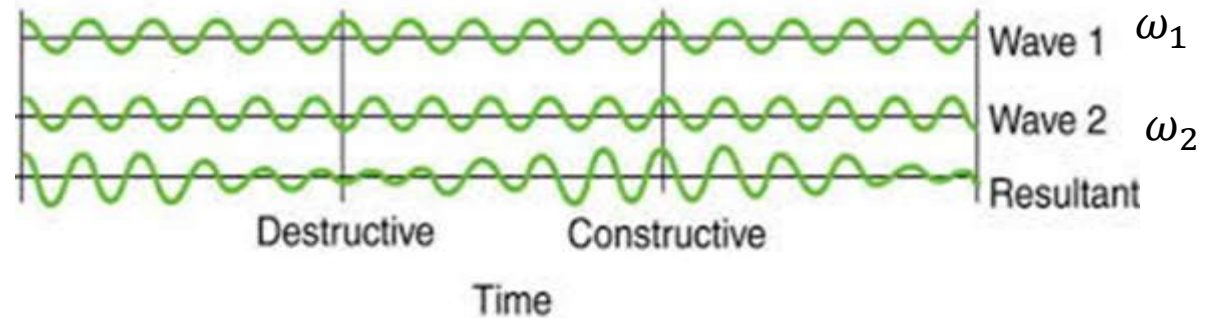


Dispersion causes the spatial separation of a white light into components of different wavelength (different colour)

- Waves occur as a mixture of a number or group of component frequencies e.g. white light is composed of visible wavelength spectrum of 400 nm to 700 nm.
- The behavior of such a group leads to the **group velocity**.

SUPERPOSITION OF TWO WAVES OF NEARLY EQUAL FREQUENCIES

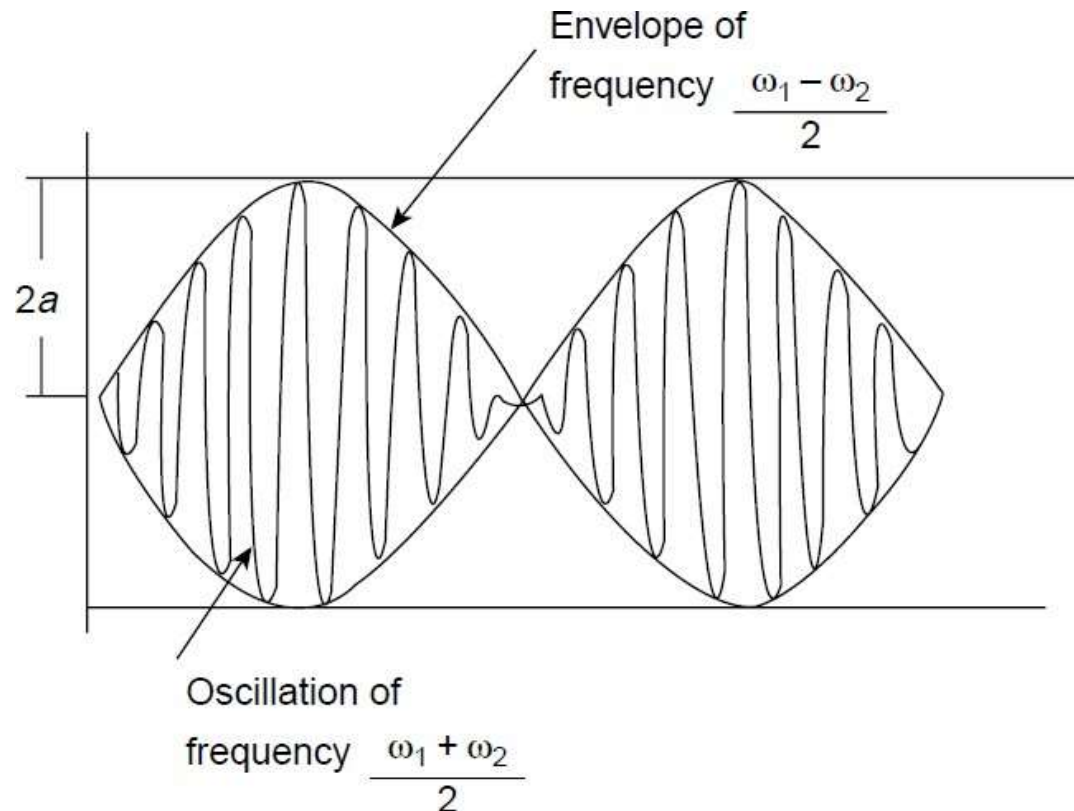
A group consists of two components of equal amplitude A but frequencies ω_1 and ω_2 which differ by a small amount.



- Their displacements:
- $f_1(z, t) = A \cos(k_1 z - \omega_1 t)$ and $f_2(z, t) = A \cos(k_2 z - \omega_2 t)$
- Superposition of amplitude and phase:
- $f = f_1 + f_2 = A \cos(k_1 z - \omega_1 t) + A \cos(k_2 z - \omega_2 t)$
- Using trigonometry, $\cos C + \cos D = 2 \cos((C - D)/2) \cos((C + D)/2)$
- We get $f = 2A \cos \left[\frac{((k_1 - k_2)z - (\omega_1 - \omega_2)t)}{2} \right] \cos \left[\frac{((k_1 + k_2)z - (\omega_1 + \omega_2)t)}{2} \right]$

WAVE GROUP/WAVE PACKET

A resultant wave with a frequency $(\omega_1 + \omega_2)/2$ which is very close to the frequency of either component but with a maximum amplitude of $2A$, modulated in space and time by a very slowly varying envelope of frequency $(\omega_1 - \omega_2)/2$ and wave number $(k_1 - k_2)/2$.



GROUP VELOCITY

- Two frequency components have different phase velocities so that $\omega_1/k_1 \neq \omega_2/k_2$.

$$\text{group velocity} = \frac{(\omega_1 - \omega_2)}{(k_1 - k_2)} = \frac{\Delta\omega}{\Delta k}$$

- The superposition of the two waves will no longer remain constant and the group profile will change with time.
- In a dispersive medium, the phase velocity is frequency dependent (i.e. ω/k not constant).
- If a group contains a number of components of frequencies which are nearly equal the original, expression for the **group velocity** is written:

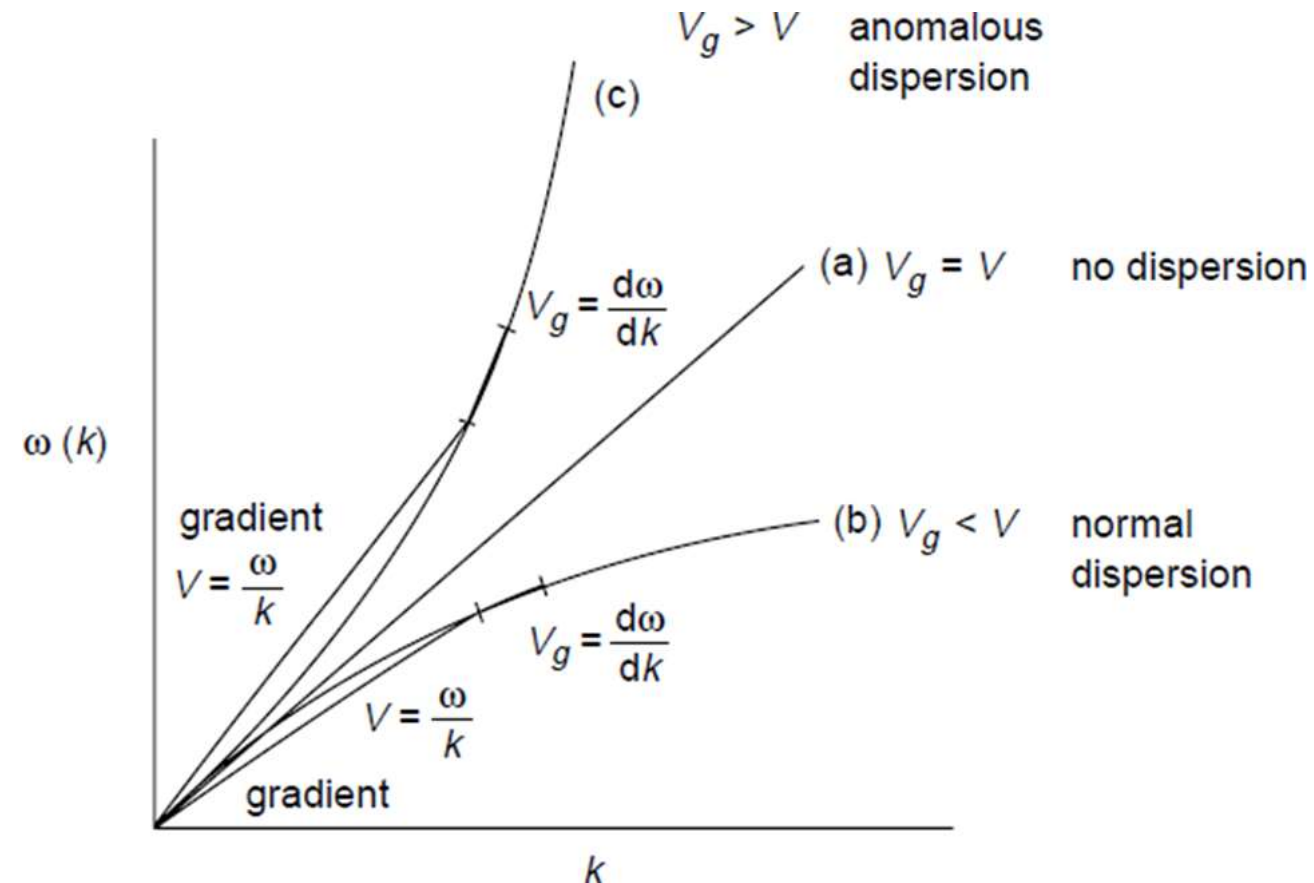
$$\text{group velocity, } v_g = \frac{\Delta\omega}{\Delta k} = \frac{d\omega}{dk}$$

RELATION BETWEEN GROUP AND PHASE VELOCITY

Since $\omega = kv$ (v is the phase velocity) and

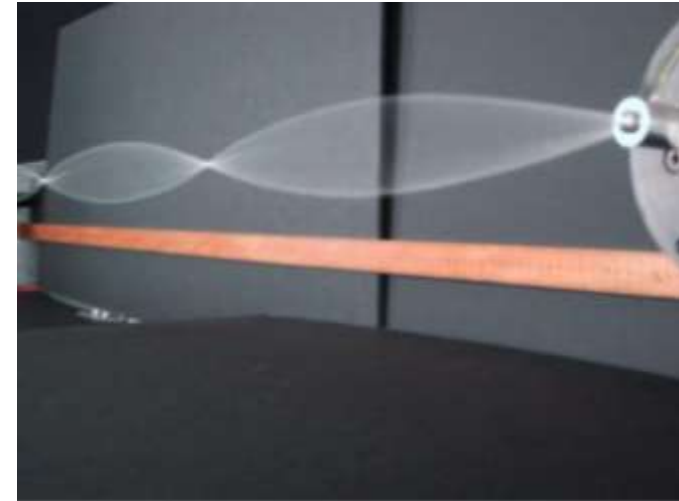
$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}(kv) = v + k \frac{dv}{dk} = v - \lambda \frac{dv}{d\lambda}$$

- A non-dispersive medium where ω/k is constant, so that $v_g = v$, for instance free space behaviour towards light waves
- A normal dispersion relation, $v_g < v$
- An anomalous dispersion relation, $v_g > v$



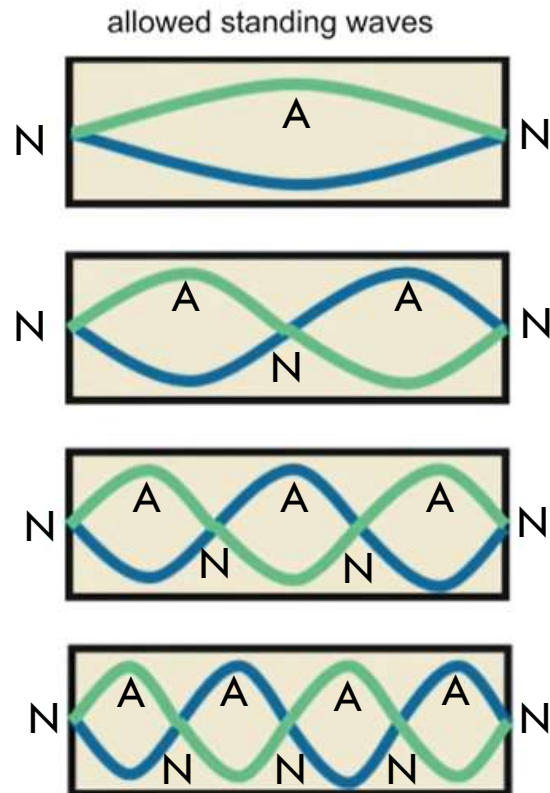
A STANDING WAVE IS SUPERPOSITION OF TWO WAVES TRAVELLING IN OPPOSITE DIRECTION

- When two identical waves of the same amplitude and frequency travel in opposite directions with the same speed along the same path superpose each other, the resultant wave does not travel in the either direction and is called a stationary or standing wave.
- It is called a standing wave because it does not appear to move.
- Standing wave can also be created when a single wave is reflected off a fixed boundary (string reflecting with one end of the string attached to the wall).
- In stationary or standing waves, the shape or profile of the wave stays fixed in a medium.
- An example of a stationary wave is the wave produced on the string of a string instrument.
- When the string is plucked, a wave is caused to travel up and down.
- Since both ends of the string are fixed, the waveform is reflected back up and down the string or along its path. This confines a wave to stay within it.



STANDING WAVES ON A STRING OF FIXED LENGTH (1-DIMENSIONAL STANDING WAVES)

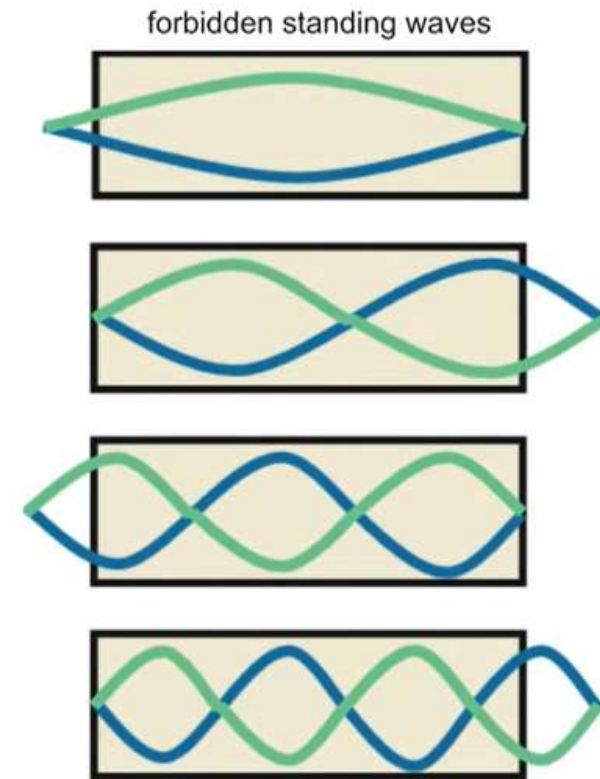
Allowed standing waves have integer no. of half wavelengths



Transverse standing waves can have only particular frequencies for a string of given length.

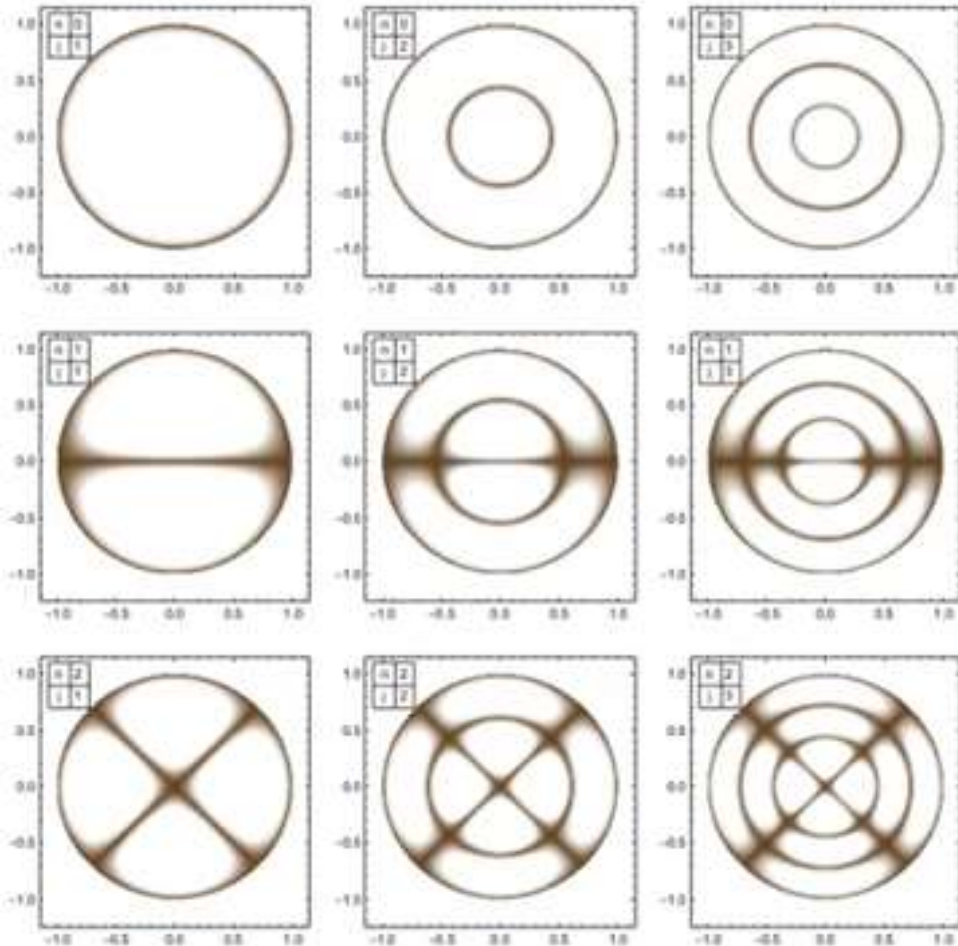
Nodes are points in 1-dimensional standing waves

Non-integer no. of half wavelengths are forbidden

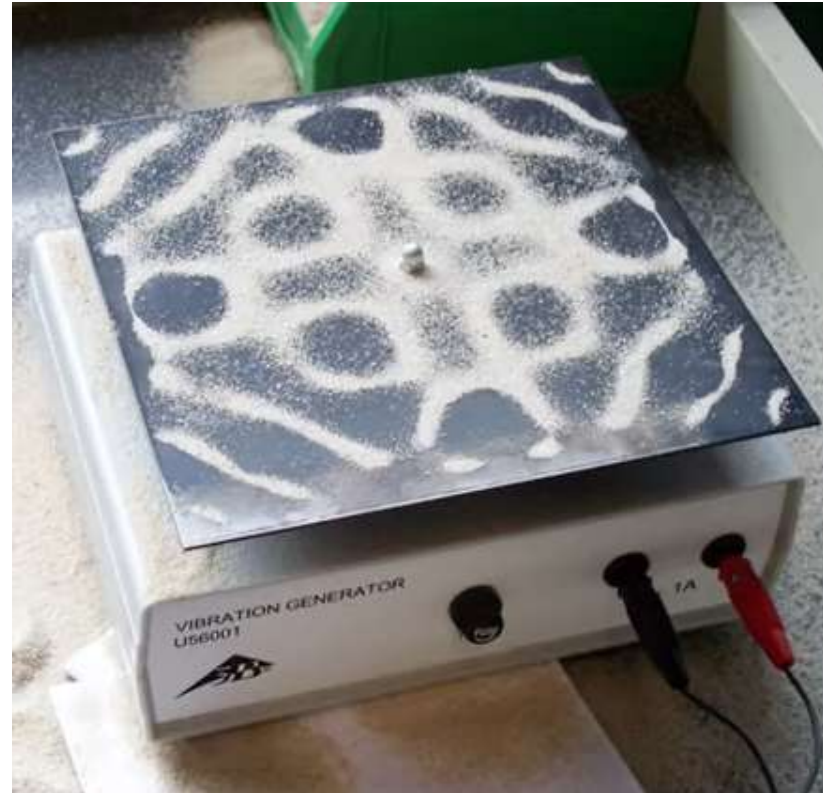


NODES IN 2-DIMENSIONAL STANDING WAVES

Radial and angular nodes



Sand on a vibrating surface



A 3-dimensional standing waves have nodes that are two dimensional

STANDING WAVES ON A STRING OF FIXED LENGTH (ANALYTICAL TREATMENT)

- ❖ We have already seen that a progressive wave is completely reflected at an infinite impedance with a π phase change in amplitude.
- ❖ *A standing wave, also known as a stationary wave, is a wave which oscillates in time but whose peak amplitude profile does not move in space.*
- ❖ *The peak amplitude of the wave oscillations at any point in space is constant with respect to time, and the oscillations at different points throughout the wave are in phase.*
- ❖ A string of fixed length l with both ends rigidly clamped presents an infinite impedance at each end.
- ❖ Let us consider the simplest case of a monochromatic wave of one frequency ω with an amplitude A , travelling in the positive z -direction and an amplitude B , travelling in the negative z -direction.
- ❖ The displacement on the string at any point would then be given by
$$f(z, t) = Ae^{i(\omega t - kz)} + Be^{i(\omega t + kz)}$$

with the boundary condition that $f = 0$ at $z = 0$ and $z = l$ at all times.

CONT'D....

The condition $f = 0$ at $z = 0$ gives $0 = (A + B)e^{i\omega t}$ for all t , so that $A = -B$.

This expresses physically the fact that a wave in either direction meeting the infinite impedance at either end is completely reflected with a π phase change in amplitude.

This is a general result for all wave shapes and frequencies.

$$\text{Thus, } f = Ae^{i\omega t}(e^{-ikz} - e^{ikz}) = -2iAe^{i\omega t} \sin kz$$

an expression for ' f ' which satisfies the standing wave time independent form of the wave equation.

$$\frac{\partial^2 f}{\partial z^2} + k^2 f = 0 \text{ because } \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2} = -\frac{\omega^2}{v^2} f = -k^2 f$$

CONT'D....

The condition that $f = 0$ at $z = l$ for all t requires

$$\sin kl = \sin \frac{\omega l}{v} = 0 \text{ or } \frac{\omega l}{v} = n\pi$$

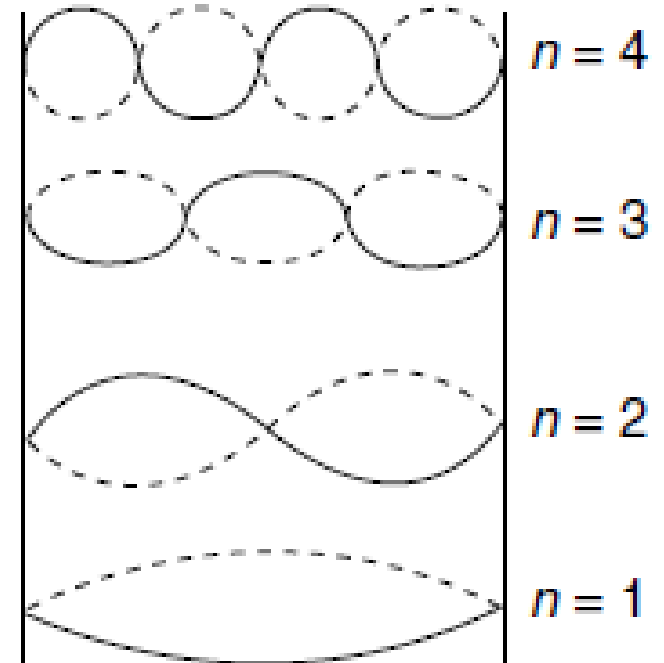
Limiting the value of allowed frequencies to

$$\omega_n = \frac{n\pi v}{l} \text{ or } v_n = \frac{nv}{2l} = \frac{v}{\lambda_n}$$

$$\text{That is } l = \frac{n\lambda_n}{2}$$

$$\text{giving } \sin \frac{\omega_n z}{v} = \sin \frac{n\pi z}{l}$$

These frequencies are the normal frequencies or modes of vibration. They are often called eigen frequencies, particularly in wave mechanics.



CONT'D....

Such allowed frequencies define the length of the string as an exact number of half wavelengths, and Figure shows the string displacement for the first four harmonics $n = 1, 2, 3, 4, \dots$. The value for $n = 1$ is called the **fundamental**.

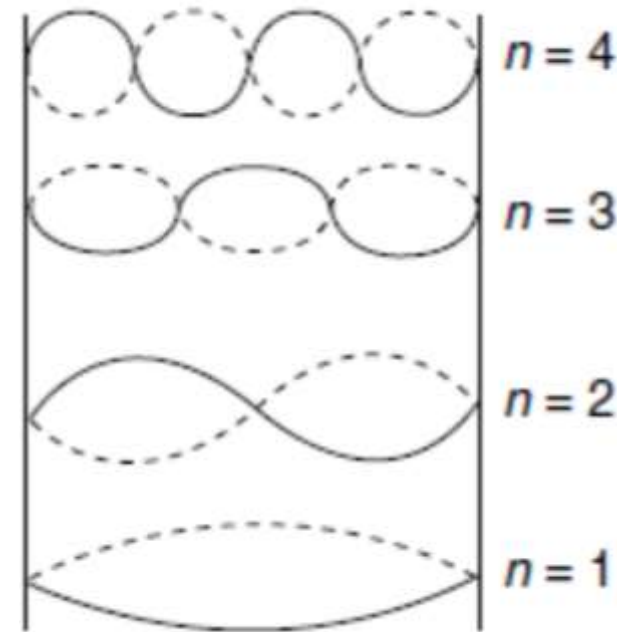
For each single harmonic $n > 1$ there will be a number of positions along the string which are always at rest. These points occur where

$$\sin \frac{\omega_n z}{v} = \sin \frac{n\pi z}{l} = 0$$

or $\frac{n\pi z}{l} = r\pi \quad (r = 0, 1, 2, 3, 4, \dots, n)$

The values $r = 0$ and $r = n$ give $z = 0$ and $z = l$, the ends of the string, but between the ends there are $(n-1)$ positions equally spaced along the string in the n^{th} harmonic where the displacement is always zero.

These positions are called **nodes or nodal points**, being the positions of zero motion in a system of standing waves.



CONT'D....

- ❖ Standing waves arise when a single mode is excited and the incident and reflected waves are superposed.
- ❖ If the amplitudes of these progressive waves are equal and opposite (resulting from complete reflection), nodal points will exist.
- ❖ Often however, the reflection is not quite complete and the waves in the opposite direction do not cancel each other to give complete nodal points.
- ❖ Whenever nodal points exist, however, we know that the waves travelling in opposite directions are exactly equal in all respects so that the energy carried in one direction is exactly equal to that carried in the other.
- ❖ This means that the total energy flux; that is, the energy carried across unit area per second in a standing wave system, is zero.

DIFFERENCES BETWEEN A STANDING WAVE AND A TRAVELING WAVE

Standing Wave: A standing wave, also known as a stationary wave, is a wave pattern that remains in a constant position. It occurs when two waves of equal amplitude and frequency traveling in opposite directions interfere with each other. Instead of propagating forward, the wave appears to oscillate in place.

Travelling Wave: A traveling wave is a wave that propagates through a medium, transferring energy from one point to another. It moves or travels from its source to other locations, carrying its energy along.

Differences between Standing Wave and Travelling Wave:

1. Nature of Wave: - Standing Wave: A standing wave exhibits a stationary pattern, oscillating in place without any net transfer of energy.

Travelling Wave: A travelling wave moves through space, transferring energy from one location to another.

2. Waveform: - Standing Wave: In a standing wave, the waveform appears to be fixed and does not change its shape over time.

Traveling Wave: A traveling wave exhibits a changing waveform as it propagates through space.

CONT'D...

3. **Interference:** - Standing Wave: A standing wave is formed by the interference of two waves traveling in opposite directions. This interference causes stationary points called nodes and points of maximum displacement called antinodes.

Travelling Wave: A travelling wave does not exhibit interference with another wave. It propagates through space without any interference pattern.

4. **Energy Transfer:** - Standing Wave: In a standing wave, there is no net transfer of energy. The wave appears to oscillate without any movement of energy from one point to another. –

Travelling Wave: A travelling wave transfers energy as it propagates through space. The energy is carried from the source to other locations.

5. **Examples:** - Standing Wave: Standing waves can be observed in musical instruments, such as guitar strings or organ pipes, where the waves reflect back and forth to create a stationary pattern.

Travelling Wave: Examples of travelling waves include ocean waves, sound waves, and light waves, which all propagate through their respective mediums.

EXAMPLE

A very long string fixed at both the ends has a linear density of 5.0 g/m and is stretched with a tension of 8.0 N. Waves of 100 Hz frequency and 2.0 mm amplitude are generated in the string.

- (i) What is the spacing between adjacent nodes in the resulting standing waves?
- (ii) What is the maximum displacement of the string?

Solution: (i) The speed of the transverse waves on the string is given by $v = \sqrt{\frac{T}{\mu}}$

$T = 8.0 \text{ N}$ and $\mu = 5.0 \text{ g/m} = 5.0 \times 10^{-3} \text{ kg/m}$, therefore $v = \sqrt{\frac{8.0}{5.0 \times 10^{-3}}} = 40 \text{ m/s}$

Thus the wavelength is $\lambda = \frac{v}{\nu} = \frac{40}{100} = 0.4 \text{ m} = 40 \text{ cm}$

Consequently, the spacing between adjacent nodes will be 20 cm.

(ii) The maximum displacement, at the antinodes where $\sin kz = 1$, is 2×amplitude, i.e. 4.0 mm

Thank You



Questions???

