Aim:

To write MATLAB codes to find the work done by a force $\vec{F}$ and visualize the force field with the path.

Mathematical form:

Let the given function be $\vec{F} = F_1(x,y,z)\hat{i} + F_2(x,y,z)\hat{j} + F_3(x,y,z)\hat{k}$, where (x,y,z) given in parametric form $\bar{r} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$, $a < t < b$. Then

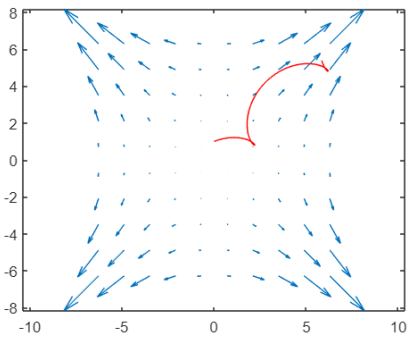
$$\int_a^b \bar{F} \cdot d\bar{r} = \int_a^b \left(\bar{F}(r(t)) \cdot \frac{d\bar{r}}{dt} \right) dt$$

Example 1. Finding the line integral $\int_C \bar{F} \cdot d\bar{r}$ along the given curve C given by $x(t) = t + \sin(\pi t/2)$, $y(t) = t + \cos(\pi t/2)$, $0 \le t \le 1$, where $\bar{F} = xy^2\hat{i} + x^2y\hat{j}$.

```
clc
clear all
syms x y t
f=input('Enter the components of 2D vector function [u,v] ');
r=input('Enter x,y in parametric form');
I=input('Enter the limits of integration for t in the form [a,b]');
a=I(1);b=I(2);
dr=diff(r,t);
F=subs(f,{x,y},r);
Fdr=sum(F.*dr);
I=int(Fdr,t,a,b)
P(x,y)=f(1);Q(x,y)=f(2);
x1=linspace(-2*pi,2*pi,10); y1=x1;
[X,Y] = meshgrid(x1,y1);
U=P(X,Y); V=Q(X,Y);
quiver(X,Y,U,V,1.5)
hold on
t1=linspace(0,2*pi);
x=subs(r(1),t1);y=subs(r(2),t1);
plot(x,y,'r')
```

Input

```
Enter the components of 2D vector function [u,v]:
[x*y^2 x^2*y]
Enter x(t) and y(t) in parametric form:
[t+sin((pi*t)/2) t+cos((pi*t)/2)]
Enter the limits of integration for t in the form
[a,b]:
[0,1]
```



Output

```
I =

2
```

Example 2. Evaluate $\int_C \bar{F} \cdot d\bar{r}$ along the given curve C given by $r:\left[t,t^2,t^3\right]$, $0 \le t \le 1$, where $\bar{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$.

```
clc
clear all
syms x y z t
f=input('Enter the components of 3D vector function [u,v,w] ');
r=input('Enter x,y,z in parametric form');
I=input('Enter the limits of integration for t in the form [a,b]');
a=I(1);b=I(2);
dr=diff(r,t);
F=subs(f,{x,y,z},r);
Fdr=sum(F.*dr);
I=int(Fdr,t,a,b)
P(x,y,z)=f(1);Q(x,y,z)=f(2); R(x,y,z)=f(3);
x1=linspace(0,1,10); y1=x1; z1=x1;
[X,Y,Z] = meshgrid(x1,y1,z1);
U=P(X,Y,Z); V=Q(X,Y,Z); W=R(X,Y,Z);
quiver3(X,Y,Z,U,V,W,1.5)
hold on
t1=linspace(0,1,10);
x=subs(r(1),t1);y=subs(r(2),t1);z=subs(r(3),t1);
plot3(x,y,z,'r')
```

Input

```
Enter the components of 2D vector function [u,v,w]
[x*y y*z z*x]
Enter r in parametric form [x(t) y(t) z(t)]
[t t^2 t^3]
Enter the limits of integration for t in the form [a,b]
[0,1]
```

Output

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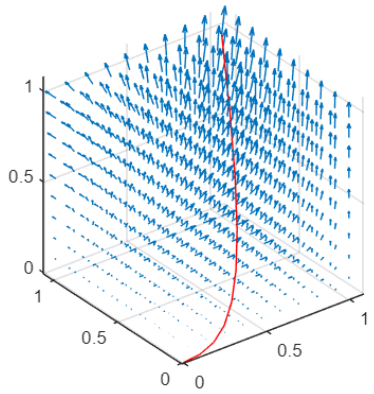
```



Exercise:

1) Find the work done for the force $\vec{F}$(x,y,z)= yz$\vec{\imath}$ + xz $\vec{\jmath}$ + (xy+2z) $\vec{k}$ along the line segment from (1,0,-2) to (4,6,3).
2) Find the work done for the force $\vec{F}$(x,y,)= $x^2\vec{\imath}$ + y^2 $\vec{\jmath}$ along the arc of the parabola y =$2x^2$ from (-1,2) to (2,8).