

## †2.7 WYE-DELTA TRANSFORMATIONS

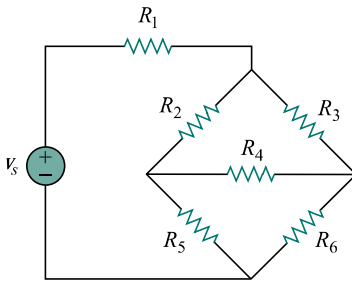


Figure 2.46 The bridge network.

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in Fig. 2.46. How do we combine resistors  $R_1$  through  $R_6$  when the resistors are neither in series nor in parallel? Many circuits of the type shown in Fig. 2.46 can be simplified by using three-terminal equivalent networks. These are the wye (Y) or tee (T) network shown in Fig. 2.47 and the delta ( $\Delta$ ) or pi ( $\Pi$ ) network shown in Fig. 2.48. These networks occur by themselves or as part of a larger network. They are used in three-phase networks, electrical filters, and matching networks. Our main interest here is in how to identify them when they occur as part of a network and how to apply wye-delta transformation in the analysis of that network.

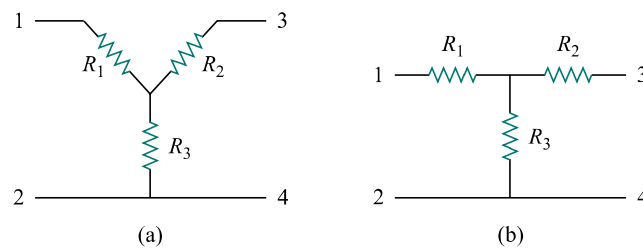


Figure 2.47 Two forms of the same network: (a) Y, (b) T.

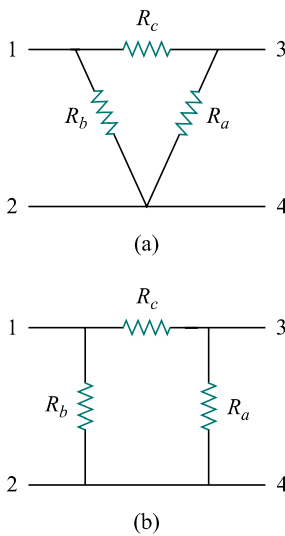


Figure 2.48 Two forms of the same network: (a)  $\Delta$ , (b)  $\Pi$ .

### Delta to Wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. We superimpose a wye network on the existing delta network and find the equivalent resistances in the wye network. To obtain the equivalent resistances in the wye network, we compare the two networks and make sure that the resistance between each pair of nodes in the  $\Delta$  (or  $\Pi$ ) network is the same as the resistance between the same pair of nodes in the Y (or T) network. For terminals 1 and 2 in Figs. 2.47 and 2.48, for example,

$$\begin{aligned} R_{12}(Y) &= R_1 + R_3 \\ R_{12}(\Delta) &= R_b \parallel (R_a + R_c) \end{aligned} \quad (2.46)$$

Setting  $R_{12}(Y) = R_{12}(\Delta)$  gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \quad (2.47a)$$

Similarly,

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \quad (2.47b)$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad (2.47c)$$

Subtracting Eq. (2.47c) from Eq. (2.47a), we get

$$R_1 - R_2 = \frac{R_c(R_b - R_a)}{R_a + R_b + R_c} \quad (2.48)$$

Adding Eqs. (2.47b) and (2.48) gives

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad (2.49)$$

and subtracting Eq. (2.48) from Eq. (2.47b) yields

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad (2.50)$$

Subtracting Eq. (2.49) from Eq. (2.47a), we obtain

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \quad (2.51)$$

We do not need to memorize Eqs. (2.49) to (2.51). To transform a  $\Delta$  network to Y, we create an extra node  $n$  as shown in Fig. 2.49 and follow this conversion rule:

Each resistor in the Y network is the product of the resistors in the two adjacent  $\Delta$  branches, divided by the sum of the three  $\Delta$  resistors.

### Wye to Delta Conversion

To obtain the conversion formulas for transforming a wye network to an equivalent delta network, we note from Eqs. (2.49) to (2.51) that

$$\begin{aligned} R_1 R_2 + R_2 R_3 + R_3 R_1 &= \frac{R_a R_b R_c (R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \\ &= \frac{R_a R_b R_c}{R_a + R_b + R_c} \end{aligned} \quad (2.52)$$

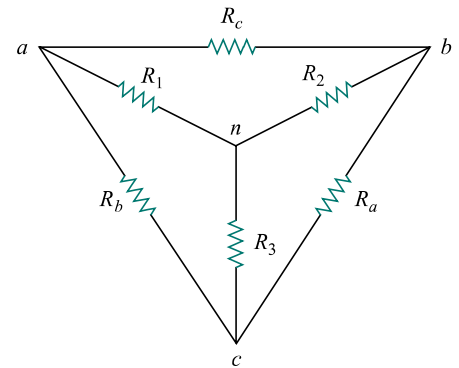
Dividing Eq. (2.52) by each of Eqs. (2.49) to (2.51) leads to the following equations:

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \quad (2.53)$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \quad (2.54)$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \quad (2.55)$$

From Eqs. (2.53) to (2.55) and Fig. 2.49, the conversion rule for Y to  $\Delta$  is as follows:



**Figure 2.49** Superposition of Y and  $\Delta$  networks as an aid in transforming one to the other.

Each resistor in the  $\Delta$  network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

The Y and  $\Delta$  networks are said to be *balanced* when

$$R_1 = R_2 = R_3 = R_Y, \quad R_a = R_b = R_c = R_\Delta \quad (2.56)$$

Under these conditions, conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y \quad (2.57)$$

One may wonder why  $R_Y$  is less than  $R_\Delta$ . Well, we notice that the Y-connection is like a “series” connection while the  $\Delta$ -connection is like a “parallel” connection.

Note that in making the transformation, we do not take anything out of the circuit or put in anything new. We are merely substituting different but mathematically equivalent three-terminal network patterns to create a circuit in which resistors are either in series or in parallel, allowing us to calculate  $R_{eq}$  if necessary.

### EXAMPLE 2.14

Convert the  $\Delta$  network in Fig. 2.50(a) to an equivalent Y network.

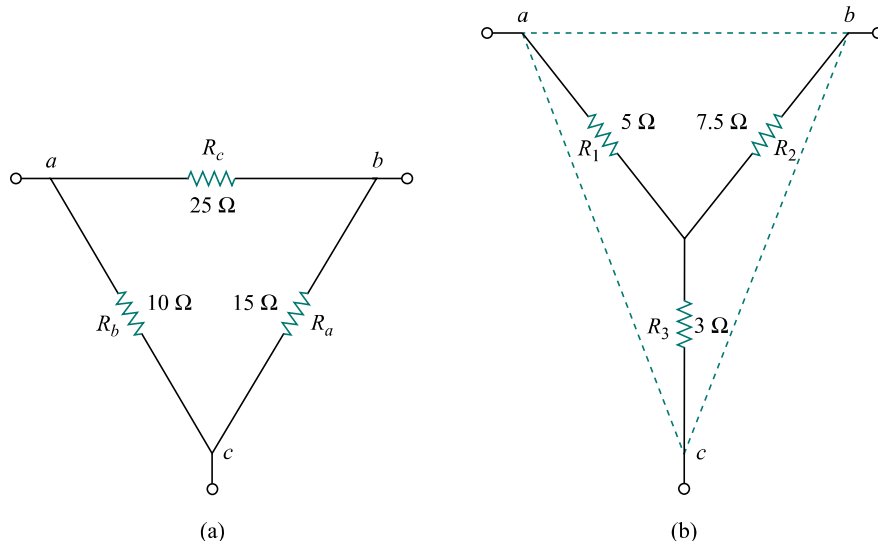


Figure 2.50 For Example 2.14: (a) original  $\Delta$  network, (b) Y equivalent network.

#### Solution:

Using Eqs. (2.49) to (2.51), we obtain

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{25 \times 10}{25 + 10 + 15} = \frac{250}{50} = 5 \Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5 \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3 \Omega$$

The equivalent Y network is shown in Fig. 2.50(b).

### PRACTICE PROBLEM 2.14

Transform the wye network in Fig. 2.51 to a delta network.

**Answer:**  $R_a = 140 \Omega$ ,  $R_b = 70 \Omega$ ,  $R_c = 35 \Omega$ .

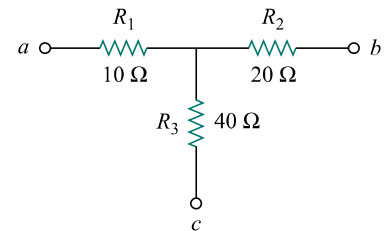


Figure 2.51 For Practice Prob. 2.14.

### EXAMPLE 2.15

Obtain the equivalent resistance  $R_{ab}$  for the circuit in Fig. 2.52 and use it to find current  $i$ .

**Solution:**

In this circuit, there are two Y networks and one  $\Delta$  network. Transforming just one of these will simplify the circuit. If we convert the Y network comprising the 5- $\Omega$ , 10- $\Omega$ , and 20- $\Omega$  resistors, we may select

$$R_1 = 10 \Omega, \quad R_2 = 20 \Omega, \quad R_3 = 5 \Omega$$

Thus from Eqs. (2.53) to (2.55) we have

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = \frac{10 \times 20 + 20 \times 5 + 5 \times 10}{10}$$

$$= \frac{350}{10} = 35 \Omega$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = \frac{350}{20} = 17.5 \Omega$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = \frac{350}{5} = 70 \Omega$$

With the Y converted to  $\Delta$ , the equivalent circuit (with the voltage source removed for now) is shown in Fig. 2.53(a). Combining the three pairs of resistors in parallel, we obtain

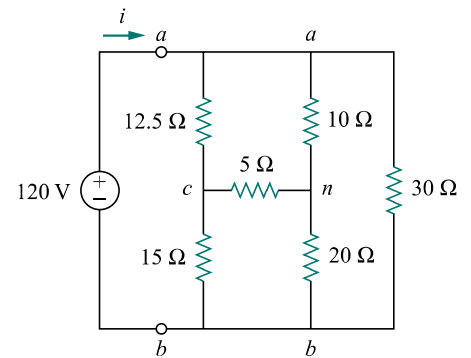


Figure 2.52 For Example 2.15.

$$70 \parallel 30 = \frac{70 \times 30}{70 + 30} = 21 \Omega$$

$$12.5 \parallel 17.5 = \frac{12.5 \times 17.5}{12.5 + 17.5} = 7.2917 \Omega$$

$$15 \parallel 35 = \frac{15 \times 35}{15 + 35} = 10.5 \Omega$$

so that the equivalent circuit is shown in Fig. 2.53(b). Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = \frac{17.792 \times 21}{17.792 + 21} = 9.632 \Omega$$

Then

$$i = \frac{v_s}{R_{ab}} = \frac{120}{9.632} = 12.458 \text{ A}$$

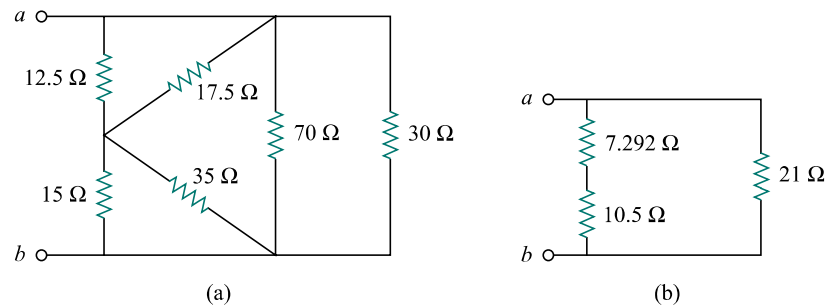
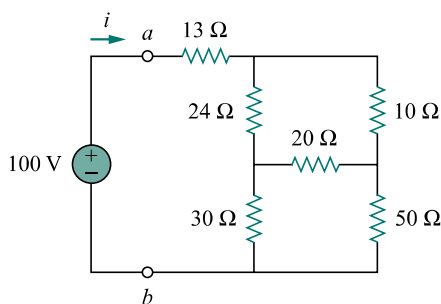


Figure 2.53 Equivalent circuits to Fig. 2.52, with the voltage removed.

## PRACTICE PROBLEM 2.15



For the bridge network in Fig. 2.54, find  $R_{ab}$  and  $i$ .

**Answer:** 40 Ω, 2.5 A.

Figure 2.54 For Practice Prob. 2.15.

## †2.8 APPLICATIONS

Resistors are often used to model devices that convert electrical energy into heat or other forms of energy. Such devices include conducting wire, lightbulbs, electric heaters, stoves, ovens, and loudspeakers. In this