

BEEE203L

CIRCUIT THEORY



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MODULE – III

Three Phase Systems

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Network Theorems for AC Circuits

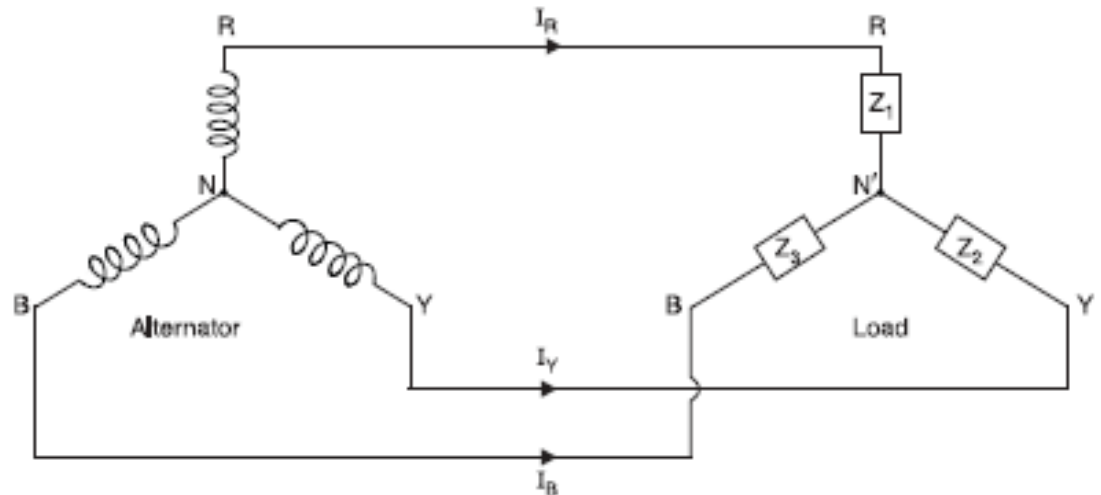
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Unbalanced three wire star-connected load

- In this the supply star point N (normally at zero potential) is isolated from the load star point N'. Since the Y-load is unbalanced and N is isolated from N', the potential of N' will be different from N.
- This results in Load phase voltages will not be equal to the supply phase voltages (e.g $V_{RN} \neq V'_{RN}$)
- The three load phase voltages will be unequal and will differ in phase by unequal angles
- The phasor sum of the three unbalanced line currents is zero

$$I_R + I_Y + I_B = 0$$

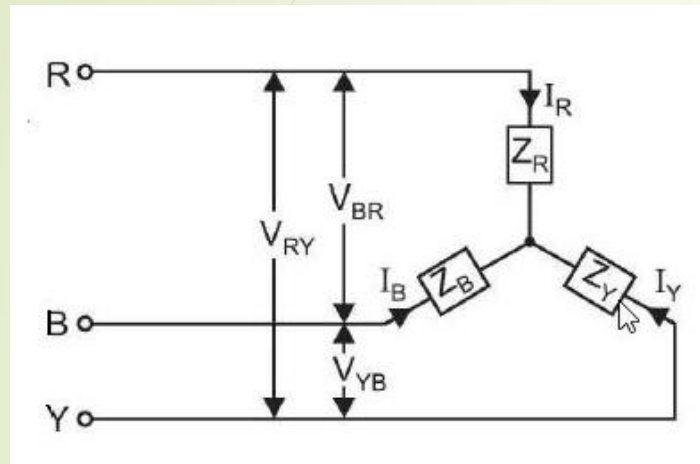


Methods of Solving Unbalanced three wire Y-connected load

- ▶ The unbalanced 3-wire Y load is the most difficult unbalanced 3-phase load to deal with. Load phase voltages cannot be determined directly from the given supply line voltages
- ▶ Several methods have been developed to handle such unbalanced loads, four frequently used methods are
 - ▶ Kirchhoff's laws
 - ▶ Maxwell's mesh current method
 - ▶ Y/ Δ conversion
 - ▶ Millman's theorem

By Kirchoff's Laws

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$$V_{RY} = I_R Z_R - I_Y Z_Y$$

$$V_{YB} = I_Y Z_Y - I_B Z_B$$

$$V_{BR} = I_B Z_B - I_R Z_R$$

$$V_{RY} + V_{BR} = (I_R Z_R - I_Y Z_Y) + (I_B Z_B - I_R Z_R)$$

$$= I_B Z_B - I_Y Z_Y$$

$$I_B = \frac{V_{RY} + V_{BR} + I_Y Z_Y}{Z_B}$$

$$I_R + I_Y + I_B = 0$$

$$V_{BR} = I_B Z_B - I_R Z_R$$

$$-I_R = I_Y + I_B \rightarrow V_{BR} = I_B (Z_B + Z_R) + I_Y Z_R$$

$$I_B = \frac{V_{RY} + V_{BR} + I_Y Z_Y}{Z_B}$$

$$\rightarrow V_{BR} = \frac{V_{RY} + V_{BR} + I_Y Z_Y}{Z_B} (Z_B + Z_R) + I_Y Z_R$$

$$V_{BR}Z_B = V_{RY}Z_B + V_{BR}Z_B + I_Y Z_Y Z_B + V_{RY}Z_R + V_{BR}Z_R \\ + I_Y Z_Y Z_R + I_Y Z_R Z_B$$

$$V_{BR}Z_B = I_Y(Z_R Z_Y + Z_Y Z_B + Z_B Z_R) + Z_R(V_{RY} + V_{BR}) \\ + V_{RY}Z_B + V_{BR}Z_B$$

$$V_{RY} + V_{YB} + V_{BR} = 0 \text{ so that } V_{RY} + V_{BR} = -V_{YB}$$

$$0 = I_Y(Z_R Z_Y + Z_Y Z_B + Z_B Z_R) - V_{YB}Z_R + V_{RY}Z_B$$

$$\text{or } I_Y(Z_R Z_Y + Z_Y Z_B + Z_B Z_R) = V_{YB}Z_R - V_{RY}Z_B$$

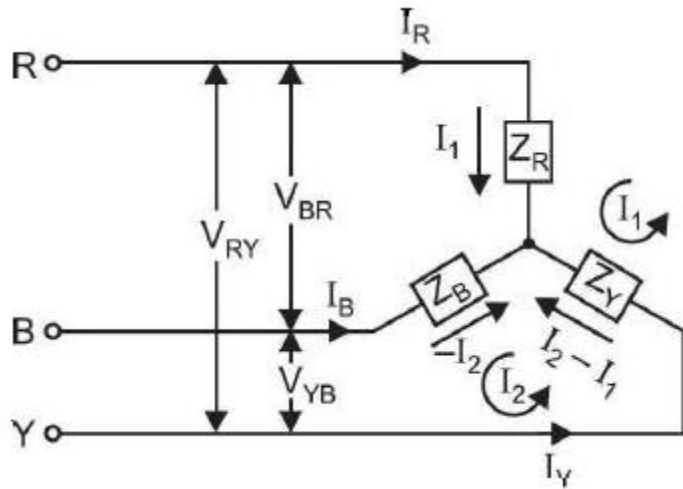
$$I_Y = \frac{V_{YB}Z_R - V_{RY}Z_B}{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}$$

$$I_B = \frac{V_{BR}Z_Y - V_{YB}Z_R}{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}$$

$$I_R = \frac{V_{RY}Z_B - V_{BR}Z_Y}{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}$$

By Maxwell's Mesh current method

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$$I_R = I_1 ; I_Y = I_2 - I_1 ; I_B = -I_2$$

$$V_{RY} = I_R Z_R - I_Y Z_Y = I_1 Z_R - (I_2 - I_1) Z_Y$$

$$V_{RY} = I_1 (Z_R + Z_Y) - I_2 Z_Y$$

$$V_{YB} = I_Y Z_Y - I_B Z_B = (I_2 - I_1) Z_Y - (-I_2 Z_B)$$

$$V_{YB} = I_2 (Z_Y + Z_B) - I_1 Z_Y$$

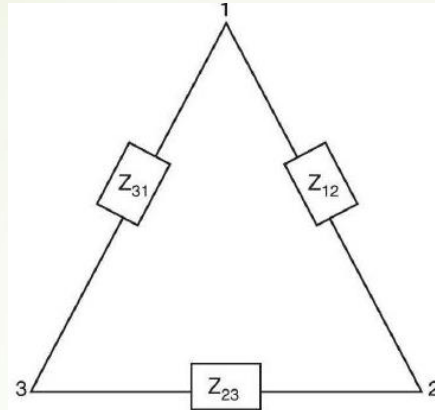
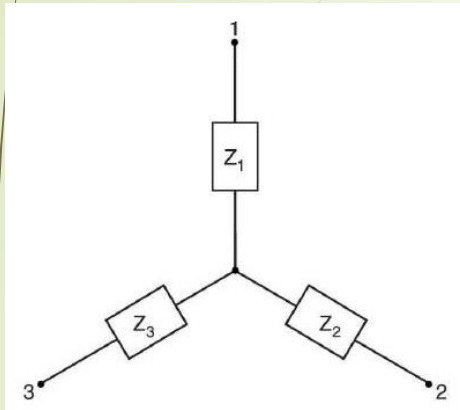
Solving for I_1 and I_2 we get

$$I_1 = \frac{V_{RY} (Z_Y + Z_B) + V_{YB} Z_Y}{(Z_R + Z_Y) (Z_Y + Z_B) - Z_Y^2}$$

$$I_2 = \frac{V_{YB} (Z_R + Z_Y) + V_{RY} Z_Y}{(Z_R + Z_Y) (Z_Y + Z_B) - Z_Y^2}$$

By star-delta Conversion method

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$$Z_{12} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

$$Z_{23} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$Z_{31} = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

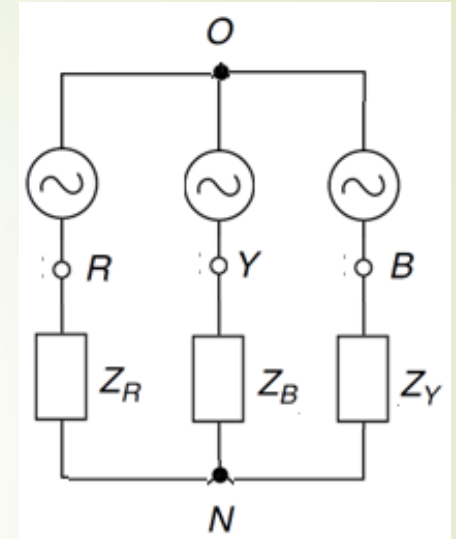
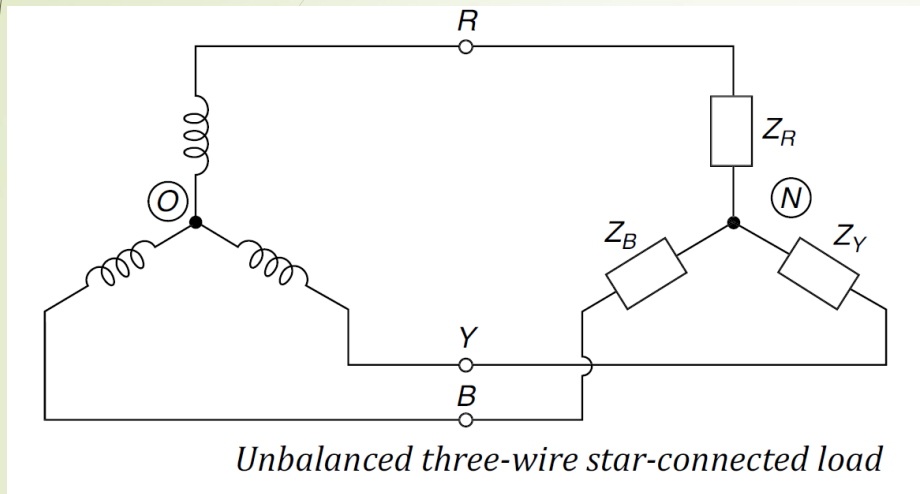
The Y connection of impedances can be replaced by an equivalent Δ and vice-versa.

The equivalent Δ load can be solved to obtain the line currents.

The line currents so calculated will be the line currents taken by the original unbalanced Y-load.

By Millman's Theorem

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Let V_{RO} , V_{YO} and V_{BO} be the phase voltages of the supply which are equal in magnitude but differ in phase from one another by 120° .

Let V_{RN} , V_{YN} and V_{BN} be the load phase voltages which are unequal in magnitude as well as differ in phase by unequal angles.

The voltage between the neutral points, N and O, i.e, V_{NO} , is calculated using Millman's theorem.

$$\mathbf{V}_{NO} = \frac{\mathbf{V}_{RO} \mathbf{Y}_R + \mathbf{V}_{YO} \mathbf{Y}_Y + \mathbf{V}_{BO} \mathbf{Y}_B}{\mathbf{Y}_R + \mathbf{Y}_Y + \mathbf{Y}_B}$$

where $\mathbf{Y}_R$, $\mathbf{Y}_Y$ and $\mathbf{Y}_B$ are the admittances of the branches of the unbalanced star-connected load.

The voltage across phase impedance, i.e., load phase voltage can be calculated as,

$$\mathbf{V}_{RN} = \mathbf{V}_{RO} - \mathbf{V}_{NO}$$

$$\mathbf{V}_{YN} = \mathbf{V}_{YO} - \mathbf{V}_{NO}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{BO} - \mathbf{V}_{NO}$$

The phase currents in the load are given by

$$\mathbf{I}_R = \frac{\mathbf{V}_{RN}}{\mathbf{Z}_R}$$

$$\mathbf{I}_Y = \frac{\mathbf{V}_{YN}}{\mathbf{Z}_Y}$$

$$\mathbf{I}_B = \frac{\mathbf{V}_{BR}}{\mathbf{Z}_B}$$

And in star connection, line current is equal to phase current.