Slot: A2+TA2+TAA2

Vellore Institute of Technology

(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ELECTRONICS ENGINEERING (SENSE)

CONTINUOUS ASSESSMENT TEST- II

Winter Semester: 2022-23

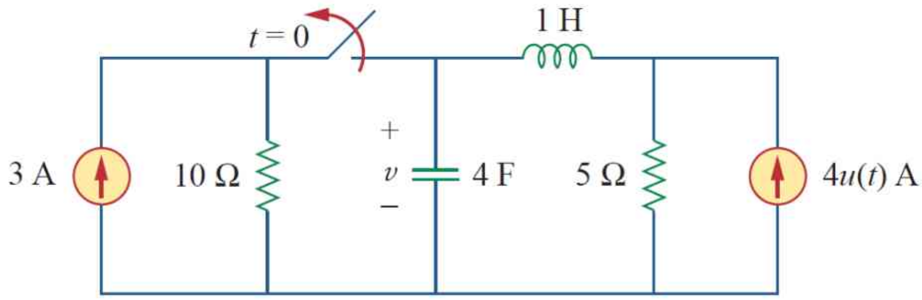
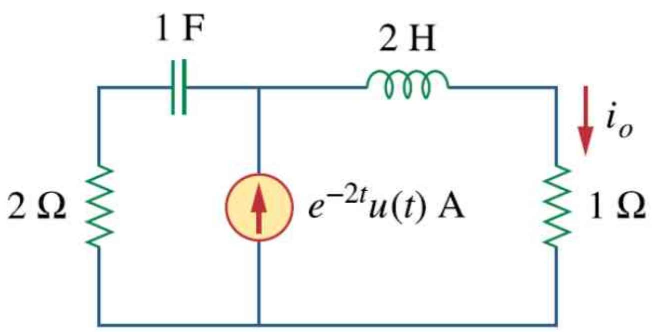
Programme Name & Branch: B.Tech & ECE **Class Number: VL2022230506379**

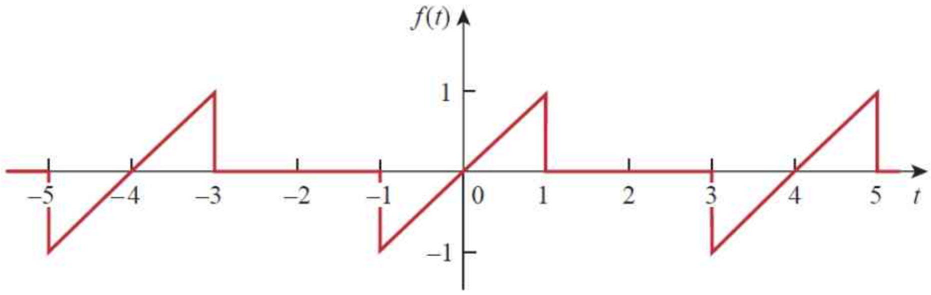
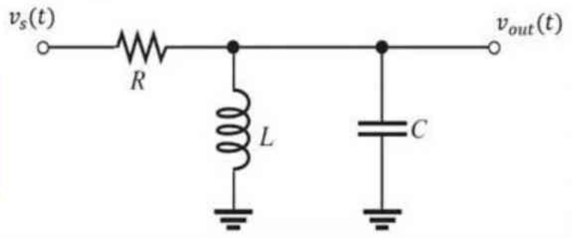
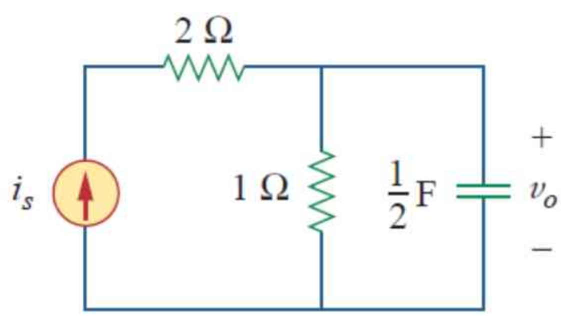
Course Code: BECE203L **Course Title : Circuit Theory**

Faculty Name: Dr. Shanidul Hoque **Date: 07/05/2023**

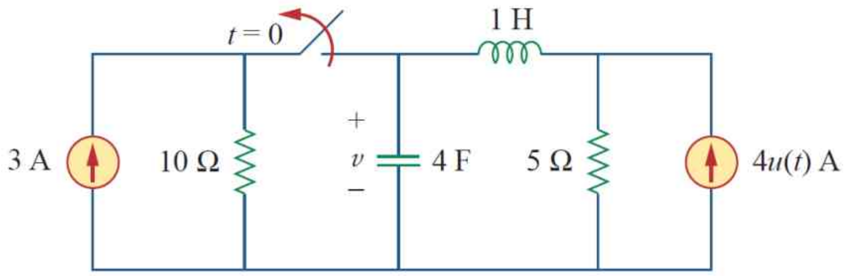
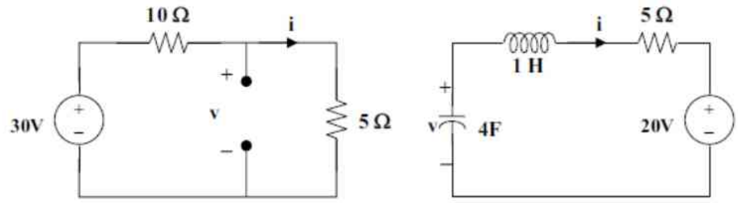
Exam Duration: 90 Minutes **Maximum Marks: 50**

Answer all the following questions

S. No	**Questions**	**Marks**
1	(a) Find the voltage $v(t)$ for $t > 0$ in the circuit of Figure 1. What type of response will it produce?  Figure 1	**10**
2	(a) Find the Laplace transform of the function $f(t) = (e^{-at} - e^{-bt})u(t)$.	**3**
	(b) Find $i_0(t)$ in the circuit of Figure 2 using Laplace transform method.  Figure 2	**7**

3	For the waveform shown in Figure 3, (i) Specify the type of symmetry it has, (ii) Find Fourier series expansion of $f(t)$ and calculate a_3 & b_3, (iii) Find the RMS value using the first five non-zero harmonics.  Figure 3	**10**
4	Determine the output voltage $v_{out}(t)$ in the given circuit of Figure 4 if R= 10 Ω, L=2 H, C=0.25 F and the input voltage $v_s(t)$ given by the Fourier series $v_s(t) = 3 + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi t)}{n}$ V  Figure 4 Also plot the amplitude spectrum and phase spectrum for n=1, 2, 3.	**10**
5	Find the voltage $v_0(t)$ and current $i_0(t)$ using Fourier transform method in the circuit of Figure 5 considering $i_s(t) = 8e^{-t}u(t)$ A.  Figure 5	**10**

Answer:

S. No	**Questions**	**Marks**
1	Find the voltage *v(t)* for *t >0* in the circuit of Figure 1. What type of response will it produce?  Figure 1 We may transform the current sources to voltage sources. For $t = 0^-$, the equivalent circuit is shown in Figure (a).  (a) (b) $i(0) = 30/15 = 2\text{ A},\ v(0) = 5x30/15 = 10\text{ V}$ For t > 0, we have a series RLC circuit, shown in (b). $\alpha = R/(2L) = 5/2 = 2.5$ $\omega_o = 1/\sqrt{LC} = 1/\sqrt{4} = 0.5$, clearly $\alpha > \omega_o$ (overdamped response) $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 0.25} = -4.95, -0.0505$ $v(t) = V_s + [A_1e^{-4.95t} + A_2e^{-0.0505t}],\ V_s = 20.$ $v(0) = 10 = 20 + A_1 + A_2$ or $A_2 = -10 - A_1$ (1) $i(0) = Cdv(0)/dt$ or $dv(0)/dt = 2/4 = 1/2$ Hence, $0.5 = -4.95A_1 - 0.0505A_2$ (2) From (1) and (2), $0.5 = -4.95A_1 + 0.505(10 + A_1)$ or $-4.445A_1 = -0.005$ $A_1 = 0.001125,\ A_2 = -10.001$ $v(t) = \underline{\mathbf{[20 + 0.001125e^{-4.95t} - 10.001e^{-0.05t}]\ V}}$	**10**
2	(a) Find the Laplace transform of the function $f(t)= (e^{-at} - e^{-bt})u(t)$. F(s)=(b-a)/(s+a)(s+b)	**3**

	(b) Find $i_0(t)$ in the circuit of Figure 2 using Laplace transform method.	**7**

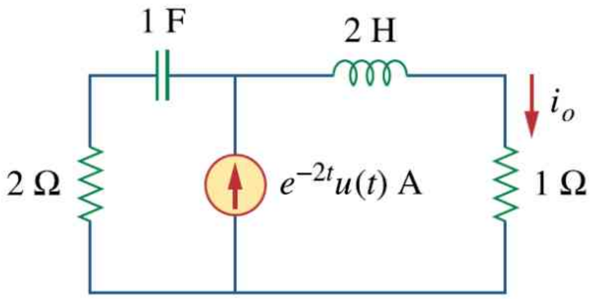
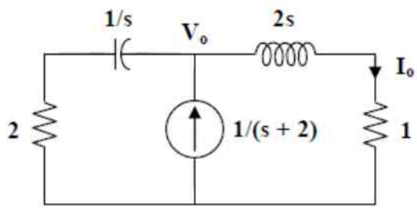


Figure 2

Consider the following circuit.



Applying KCL at node o,

$$\frac{1}{s+2} = \frac{V_o}{2s+1} + \frac{V_o}{2+1/s} = \frac{s+1}{2s+1}V_o$$

$$V_o = \frac{2s+1}{(s+1)(s+2)}$$

$$I_o = \frac{V_o}{2s+1} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$A = 1$, $\quad B = -1$

$$I_o = \frac{1}{s+1} - \frac{1}{s+2}$$

$$i_o(t) = \underline{\mathbf{\left(e^{-t} - e^{-2t}\right)u(t)\ A}}$$

3	For the waveform shown in Figure 3,	
	(i) Specify the type of symmetry it has,	**10**
	(ii) Find Fourier series expansion of $f(t)$ and calculate a_3 & b_3,	
	(iii) Find the RMS value using the first five non-zero harmonics.	

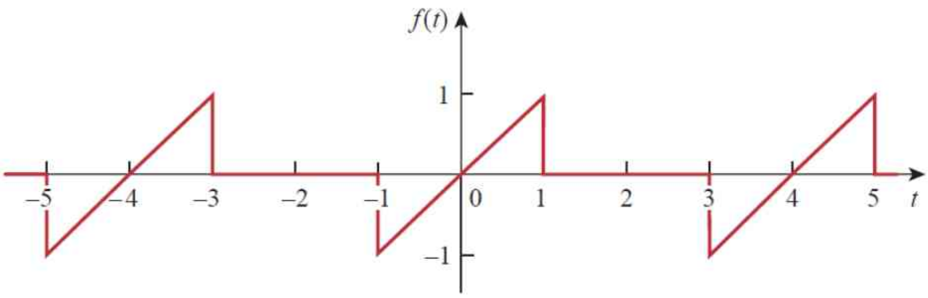
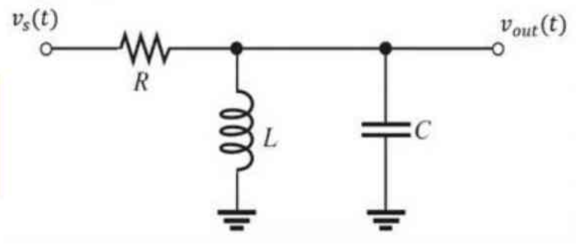


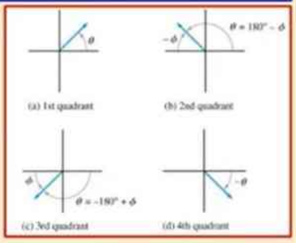
Figure 3

	(a) Odd symmetry (b) $a_o = 0 = a_n,\ T = 4,\ \omega_o = 2\pi/T = \pi/2$ $f(t) = t, \quad 0 < t < 1$ $= 0, \quad 1 < t < 2$ $b_n = \frac{4}{4}\int_0^1 t \sin\frac{n\pi t}{2}\,dt = \left[\frac{4}{n^2\pi^2}\sin\frac{n\pi t}{2} - \frac{2t}{n\pi}\cos\frac{n\pi t}{2}\right]_0^1$ $= \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{2}{n\pi}\cos\frac{n\pi}{2} - 0$ $= 4(-1)^{(n-1)/2}/(n^2\pi^2), \quad n = \text{odd}$ $-2(-1)^{n/2}/(n\pi), \quad n = \text{even}$ $a_3 = \underline{\mathbf{0}},\ b_3 = 4(-1)/(9\pi^2) = \underline{\mathbf{-0.04503}}$ (c) $b_1 = 4/\pi^2,\ b_2 = 1/\pi,\ b_3 = -4/(9\pi^2),\ b_4 = -1/(2\pi),\ b_5 = 4/(25\pi^2)$ $F_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum\left(a_n^2 + b_n^2\right)}$ $F_{rms}^2 = 0.5\Sigma b_n^2 = [1/(2\pi^2)][(16/\pi^2) + 1 + (16/(81\pi^2)) + (1/4) + (16/(625\pi^2))]$ $= (1/19.729)(2.6211 + 0.27 + 0.00259)$ $F_{rms} = \sqrt{0.14667} = \underline{\mathbf{0.383}}$ Compare this with the exact value of $F_{rms} = \sqrt{\frac{2}{T}\int_0^1 t^2 dt} = \sqrt{1/6} = 0.4082$ or	
4	Determine the output voltage $v_{out}(t)$ in the given circuit of Figure 4 if R= 10 Ω, L=2 H, C=0.25 F and the input voltage $v_s(t)$ given by the Fourier series $v_s(t) = 3 + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin(n\pi t)}{n}$ V  Figure 4 Also plot the amplitude spectrum and phase spectrum for n=1, 2, 3.	**10**

Solutions:

The radian frequency is given by $\omega_n = n\omega_o = n\pi$ rad/sec. Using phasors, we get for the output voltage:

$$V_{out}(j\omega) = \frac{X_C || X_L}{R + X_C || X_L} V_s(j\omega) = \frac{\left(\dfrac{\frac{1}{j\omega_n C}\cdot j\omega_n L}{j\omega_n L + \frac{1}{j\omega_n C}}\right)}{R + \left(\dfrac{\frac{1}{j\omega_n C}\cdot j\omega_n L}{j\omega_n L + \frac{1}{j\omega_n C}}\right)} V_s(j\omega) = \frac{\left(\dfrac{j\omega_n L}{-\omega_n^2 LC + 1}\right)}{R + \left(\dfrac{j\omega L}{-\omega_n^2 LC + 1}\right)} V_s(j\omega)$$



$$V_{out}(j\omega) = \frac{j\omega_n L}{R(-\omega_n^2 LC + 1) + j\omega_n L} V_s(j\omega)$$

The real term $5 - 2.5n^2\pi^2 < 0$ for $n \geq 1$, thus $5 - 2.5n^2\pi^2 + jn\pi$ is in the 2nd quadrant.

$$V_{out}(j\omega) = \frac{jn\pi \cdot 2}{10\cdot(-(n\pi)^2\cdot 2\cdot 0.25 + 1) + jn\pi\cdot 2} V_s(j\omega)$$

The phase of the denominator $5 - 2.5n^2\pi^2 + jn\pi$ is given by $180^\circ - \arctan\left(\frac{n\pi}{5-2.5n^2\pi^2}\right)$.

$$V_{out}(j\omega) = \frac{jn\pi}{5 - 2.5n^2\pi^2 + jn\pi} V_s(j\omega)$$

The ***phase*** of $V_{out}(j\omega)$ is given by

$$\theta_n = 0^\circ - \left[180^\circ - \arctan\left(\frac{n\pi}{5 - 2.5n^2\pi^2}\right)\right] \Rightarrow \theta_n = -180^\circ + \arctan\left(\frac{n\pi}{5 - 2.5n^2\pi^2}\right)$$

$$V_{out}(j\omega) = \frac{jn\pi}{5 - 2.5n^2\pi^2 + jn\pi}\cdot\frac{-j4}{n\pi} = \frac{4}{5 - 2.5n^2\pi^2 + jn\pi}\ \text{V}$$

The ***magnitude*** of $V_{out}(j\omega)$ is given by

$$A_n = \frac{4}{\sqrt{(5 - 2.5n^2\pi^2)^2 + (n\pi)^2}}$$

- **For DC** ($n = 0$ or $\omega_n = 0$), we have:

$$V_s = 3\ \text{V} \Longrightarrow V_{out} = 0\ \text{V}$$ *(Inductor is short for DC)*

- **For AC** (n^{th} harmonic), we have:

$$\left.\begin{aligned} \theta_n &= -180^\circ + \arctan\left(\frac{n\pi}{5 - 2.5n^2\pi^2}\right) \\ A_n &= \frac{4}{\sqrt{(5 - 2.5n^2\pi^2)^2 + (n\pi)^2}} \end{aligned}\right\} \begin{aligned} V_{out} &= A_n \angle \theta_n\ \text{V} \\ V_{out} &= \frac{4}{\sqrt{(5 - 2.5n^2\pi^2)^2 + (n\pi)^2}} \angle\left(-180^\circ + \arctan\left(\frac{n\pi}{5 - 2.5n^2\pi^2}\right)\right)\ \text{V} \end{aligned}$$

- Time domain:

$$v_{out}(t) = \sum_{n=1}^{\infty} A_n \cos(n\pi t + \theta_n) = \sum_{n=1}^{\infty} \frac{4}{\sqrt{(5 - 2.5n^2\pi^2)^2 + (n\pi)^2}} \cos\left(n\pi t - 180^\circ + \arctan\left(\frac{n\pi}{5 - 2.5n^2\pi^2}\right)\right)$$

Using $\cos(x - 180^\circ) = -\cos(x)$, we get

$$v_{out}(t) = -\sum_{n=1}^{\infty} \frac{4}{\sqrt{(5 - 2.5n^2\pi^2)^2 + (n\pi)^2}} \cos\left(n\pi t + \arctan\left(\frac{n\pi}{5 - 2.5n^2\pi^2}\right)\right)$$

- The first four terms ($n = 1, 2, 3, 4$) of the harmonics are:

$$n = 1:\ v_{out1}(t) = \frac{-4}{\sqrt{(5-2.5\cdot 1^2\cdot\pi^2)^2 + (1\pi)^2}} \cos\left(1\pi t - \arctan\left(\frac{1\pi}{5-2.5\cdot 1^2\cdot\pi^2}\right)\right) = -0.201\cos(1\pi t - 9.07^\circ)\ \text{V}$$

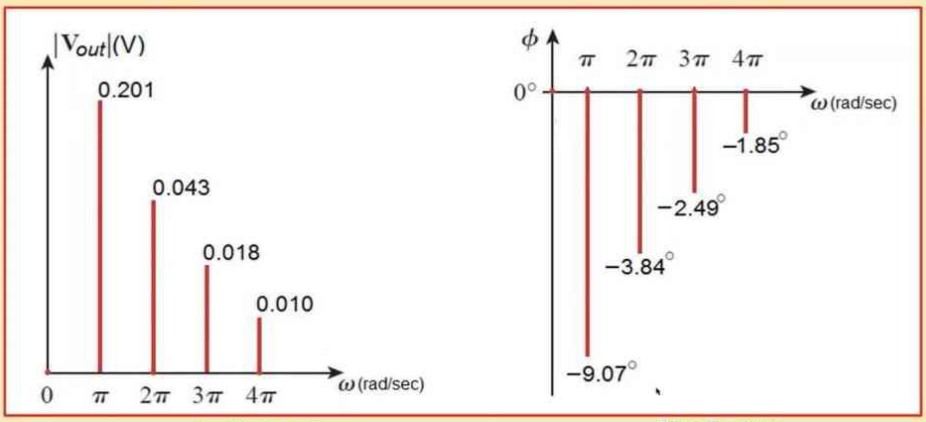
$$n = 2:\ v_{out2}(t) = \frac{-4}{\sqrt{(5-2.5\cdot 2^2\cdot\pi^2)^2 + (2\pi)^2}} \cos\left(2\pi t - \arctan\left(\frac{2\pi}{5-2.5\cdot 2^2\cdot\pi^2}\right)\right) = -0.043\cos(2\pi t - 3.84^\circ)\ \text{V}$$

$$n = 3:\ v_{out3}(t) = \frac{-4}{\sqrt{(5-2.5\cdot 3^2\cdot\pi^2)^2 + (3\pi)^2}} \cos\left(3\pi t - \arctan\left(\frac{3\pi}{5-2.5\cdot 3^2\cdot\pi^2}\right)\right) = -0.018\cos(3\pi t - 2.49^\circ)\ \text{V}$$

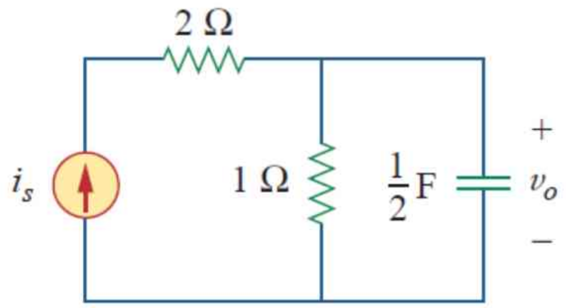
$$n = 4:\ v_{out4}(t) = \frac{-4}{\sqrt{(5-2.5\cdot 4^2\cdot\pi^2)^2 + (4\pi)^2}} \cos\left(4\pi t - \arctan\left(\frac{4\pi}{5-2.5\cdot 4^2\cdot\pi^2}\right)\right) = -0.010\cos(4\pi t - 1.85^\circ)\ \text{V}$$

- **Output Signal Spectrum**

$$\boxed{v_{out}(t) = -0.201\cos(\pi t - 9.07^\circ) - 0.043\cos(2\pi t - 3.84^\circ) - 0.018\cos(3\pi t - 2.49^\circ) - 0.010\cos(4\pi t - 1.85^\circ) + \cdots\ \text{V}}$$



Amplitude Spectrum **Phase Spectrum**

5	Find the voltage $v_0(t)$ and current $i_0(t)$ using Fourier transform method in the circuit of Figure 5 considering $i_s(t) = 8e^{-t}u(t)$ A.  Figure 5 $\frac{1}{2}F \longrightarrow \frac{1}{j\omega C} = \frac{2}{j\omega}$ $\mathrm{I_o} = \frac{1}{1+\frac{2}{j\omega}}\mathrm{I_s}$ $\mathrm{V_o} = \frac{2}{j\omega}\mathrm{I_o} = \frac{\frac{2}{j\omega}}{1+\frac{2}{j\omega}}\mathrm{I_s} = \frac{2}{2+j\omega}\frac{8}{1+j\omega}$ $= \frac{16}{(s+1)(s+2)}, \; s = j\omega$ $= \frac{A}{s+1} + \frac{B}{s+2}$ where A = 16/1 = 16, B = 16/(–1) = –16 Thus, $v_o(t) =$ **$16(e^{-t} - e^{-2t})u(t)$ V**. io(t)=Cdv(t)	**10**