CIRCUIT LAWS

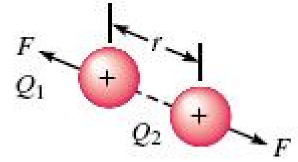
Ohm's Law

Kirchoff's Laws

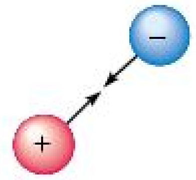
Mathematically, Coulomb's law states

$$F = k\frac{Q_1 Q_2}{r^2} \quad \text{[newtons, N]}$$

$$k = 9 \times 10^9.$$



(a) Like charges repel



(b) Unlike charges attract

Conductors

Materials through which charges move easily are termed **conductors.**
The most familiar examples are metals.
Good metal conductors have large numbers of free electrons that are able to move about easily.
In particular, silver, copper, gold, and aluminum are excellent conductors.
Of these, copper is the most widely used.
Not only is it an excellent conductor, it is inexpensive and easily formed into wire, making it suitable for a broad spectrum of applications ranging from common house wiring to sophisticated electronic equipment.

Aluminum, although it is only about 60% as good a conductor as copper, is also used, mainly in applications where light weight is important, such as in overhead power transmission lines.

Silver and gold are too expensive for general use. However, gold, because it oxidizes less than other materials, is used in specialized applications; for example, some critical electrical connectors use it because it makes a more reliable connection than other materials.

Insulators

Materials that do not conduct (e.g., glass, porcelain, plastic, rubber, and so on) are termed **insulators.**

The covering on electric lamp cords, for example, is an insulator.

It is used to prevent the wires from touching and to protect us from electric shock.

Insulators do not conduct because they have full or nearly full valence shells and thus their electrons are tightly bound.

However, when high enough voltage is applied, the force is so great that electrons are literally torn from their parent atoms, causing the insulation to break down and conduction to occur.

In air, you see this as an arc or flashover. In solids, charred insulation usually results.

Semiconductors

Silicon and germanium (plus a few other materials) have half-filled valence shells and are thus neither good conductors nor good insulators.

Known as **semiconductors,** they have unique electrical properties that make them important to the electronics industry.

The most important material is silicon.

It is used to make transistors, diodes, integrated circuits, and other electronic devices.

Semiconductors have made possible personal computers, VCRs,
portable CD players, calculators, and a host of other electronic products.

The Unit of Electrical Charge: The Coulomb

As noted in the previous section, the unit of electrical charge is the coulomb (C). The **coulomb** is defined as the charge carried by 6.24×10^{18} electrons. Thus, if an electrically neutral (i.e., uncharged) body has 6.24×10^{18} electrons removed, it will be left with a net positive charge of 1 coulomb, i.e., $Q = 1$ C. Conversely, if an uncharged body has 6.24×10^{18} electrons added, it will have a net negative charge of 1 coulomb, i.e., $Q = -1$ C. Usually, however, we are more interested in the charge moving through a wire. In this regard, if 6.24×10^{18} electrons pass through a wire, we say that the charge that passed through the wire is 1 C.

We can now determine the charge on one electron. It is $Q_e = 1/(6.24 \times 10^{18}) = 1.60 \times 10^{-19}$ C.

Definition of Voltage: The Volt

In electrical terms, a difference in potential energy is defined as **voltage.** In general, the amount of energy required to separate charges depends on the voltage developed and the amount of charge moved. By definition, *the voltage between two points is one volt if it requires one joule of energy to move one coulomb of charge from one point to the other.* In equation form,

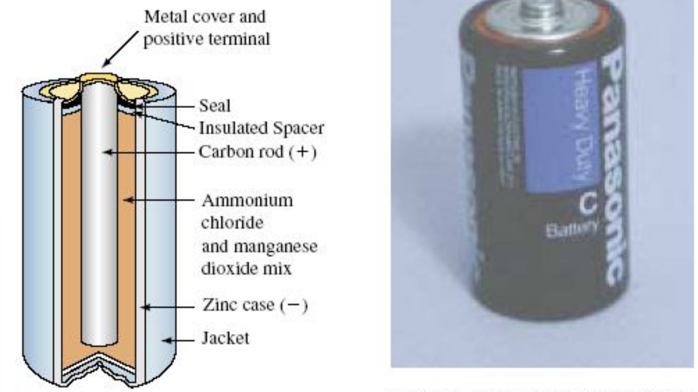
$$V = \frac{W}{Q} \quad \text{[volts, V]}$$

where W is energy in joules, Q is charge in coulombs, and V is the resulting voltage in volts.

$$W = QV \quad \text{[joules, J]}$$

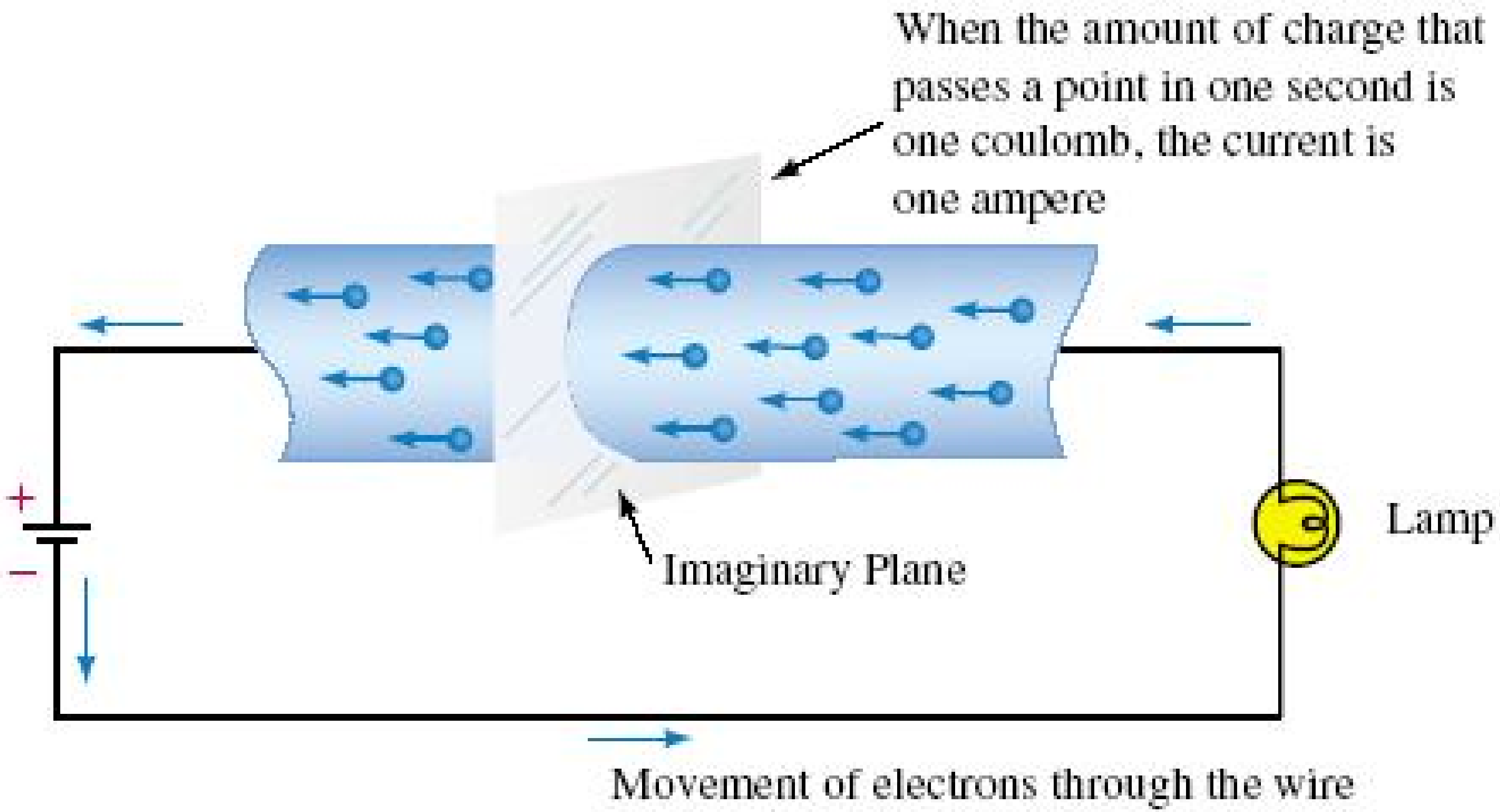
$$Q = \frac{W}{V} \quad \text{[coulombs, C]}$$

voltage is also called **potential difference.**
We often use the terms interchangeably



(a) Basic construction.

(b) C cell, commonly called a flashlight battery.


When the amount of charge that passes a point in one second is one coulomb, the current is one ampere
+
−
Imaginary Plane
Lamp
Movement of electrons through the wire

The Ampere

Since charge is measured in coulombs, its rate of flow is coulombs per second. In the SI system, one coulomb per second is defined as one **ampere** (commonly abbreviated A). From this, we get that *one ampere is the current in a circuit when one coulomb of charge passes a given point in one second* The symbol for current is I. Expressed mathematically,

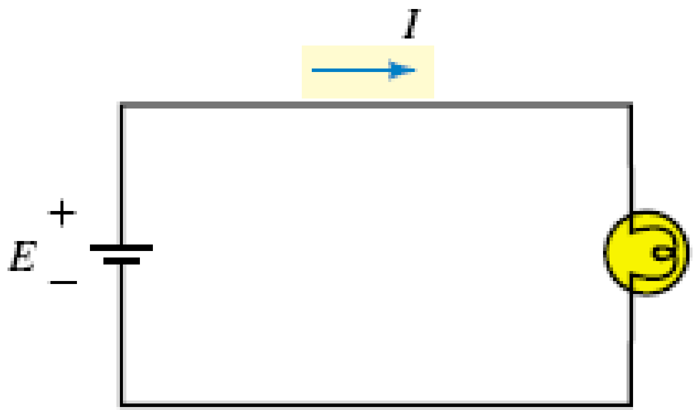
$$I = \frac{Q}{t} \quad \text{[amperes, A]}$$

where Q is the charge (in coulombs) and t is the time interval (in seconds) over which it is measured. *it is important to note that t does not represent a discrete point in time but is the interval of time during which the transfer of charge occurs*. Alternate forms of Equation are

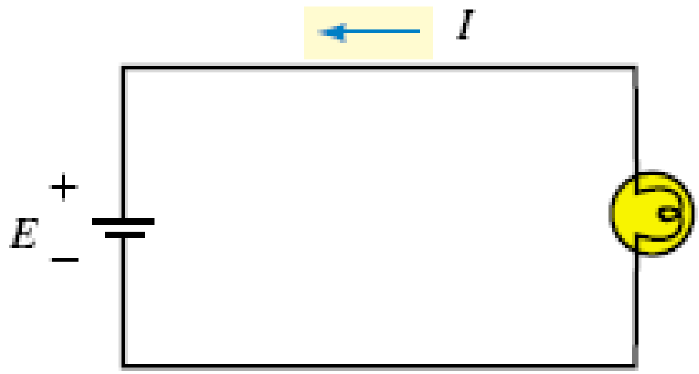
$$Q = It \quad \text{[coulombs, C]}$$

and

$$t = \frac{Q}{I} \quad \text{[seconds, s]}$$



(a) Conventional current direction



(b) Electron flow direction

Resistance of Conductors

the resistance of a material is dependent upon several factors:

- Type of material
- Length of the conductor
- Cross-sectional area
- Temperature

$$R = \frac{\rho \ell}{A} \quad [\text{ohms}, \Omega]$$

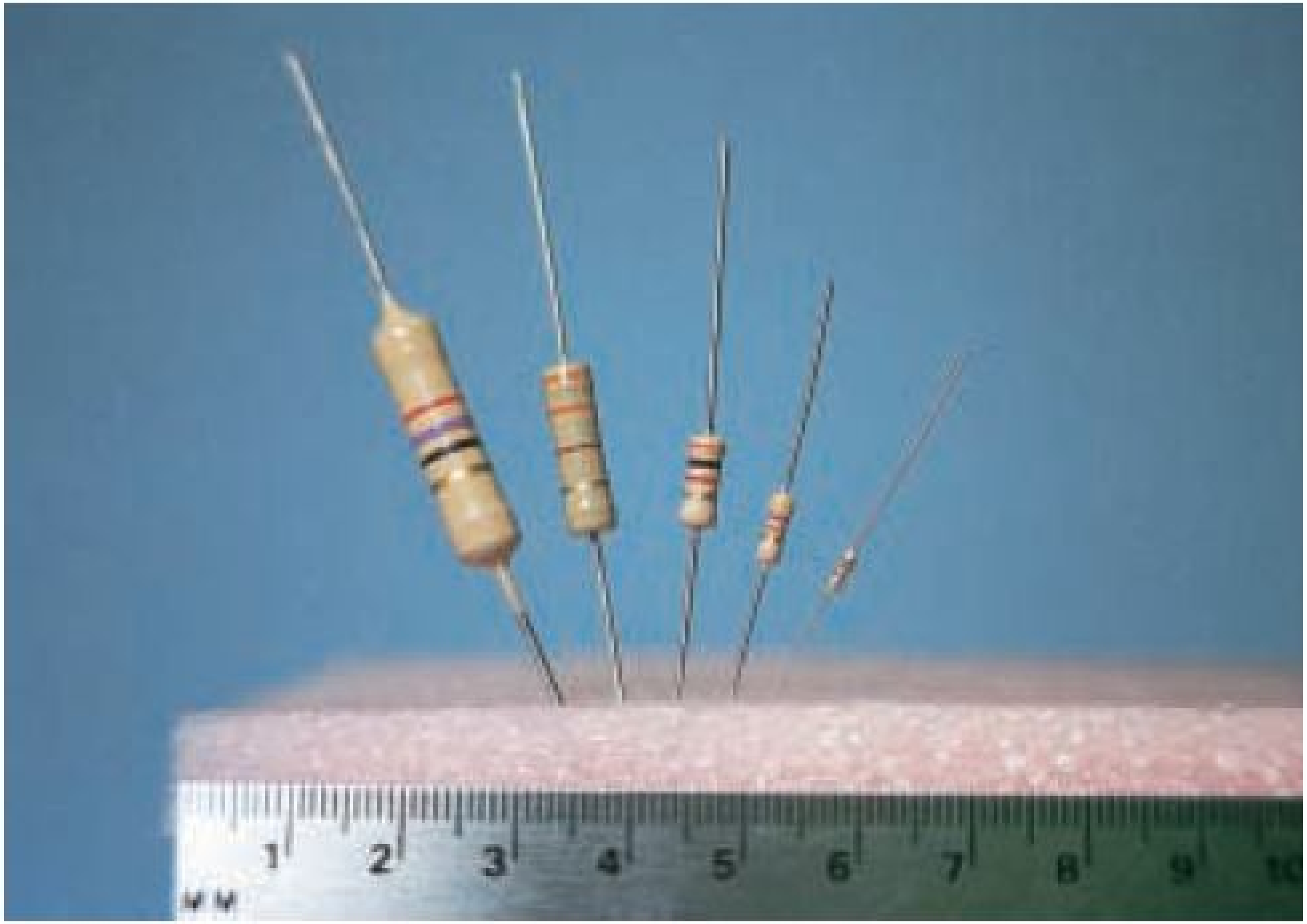
where

ρ = resistivity, in ohm-meters (Ω-m)
ℓ = length, in meters (m)
A = cross-sectional area, in square meters (m^2).

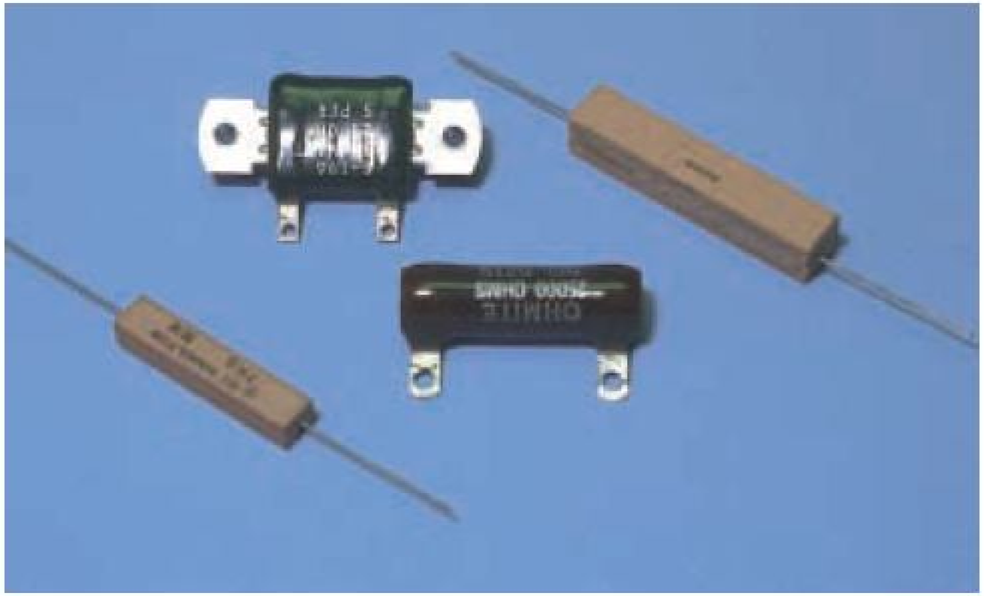
The resistance of a conductor is dependent upon the type of material.

The resistance of a metallic conductor is directly proportional to the length of the conductor.

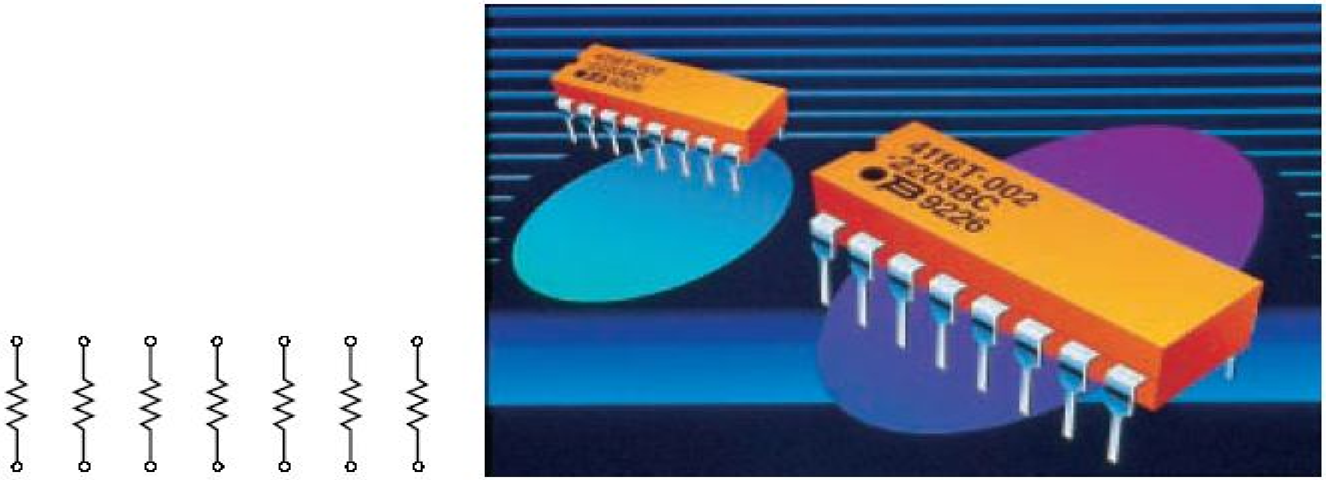
The resistance of a metallic conductor is inversely proportional to the cross-sectional area of the conductor.



Actual size of carbon resistors (2 W, 1 W, ½ W, ¼ W, ⅛ W).



Power resistors.

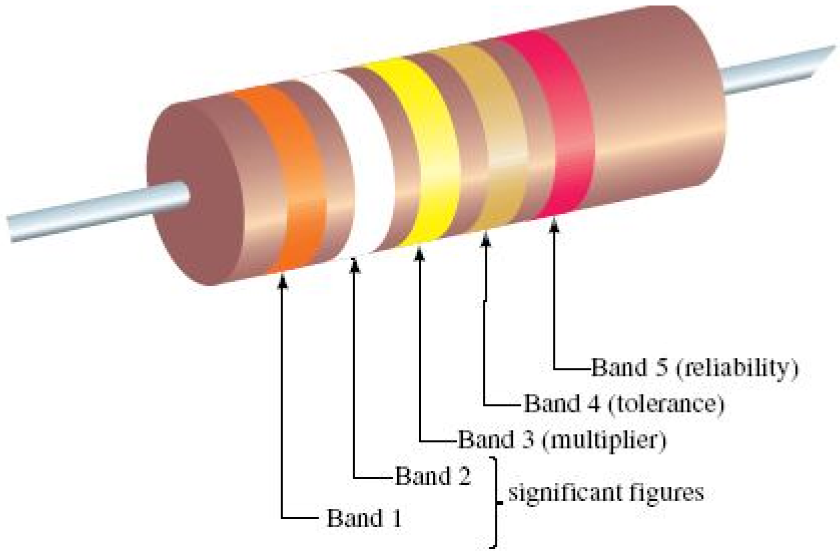


(a) Internal resistor arrangement

(b) Integrated resistor network. *(Courtesy of Bourns, Inc.)*

(a) External view of variable resistors.

(b) Internal view of variable resistor.

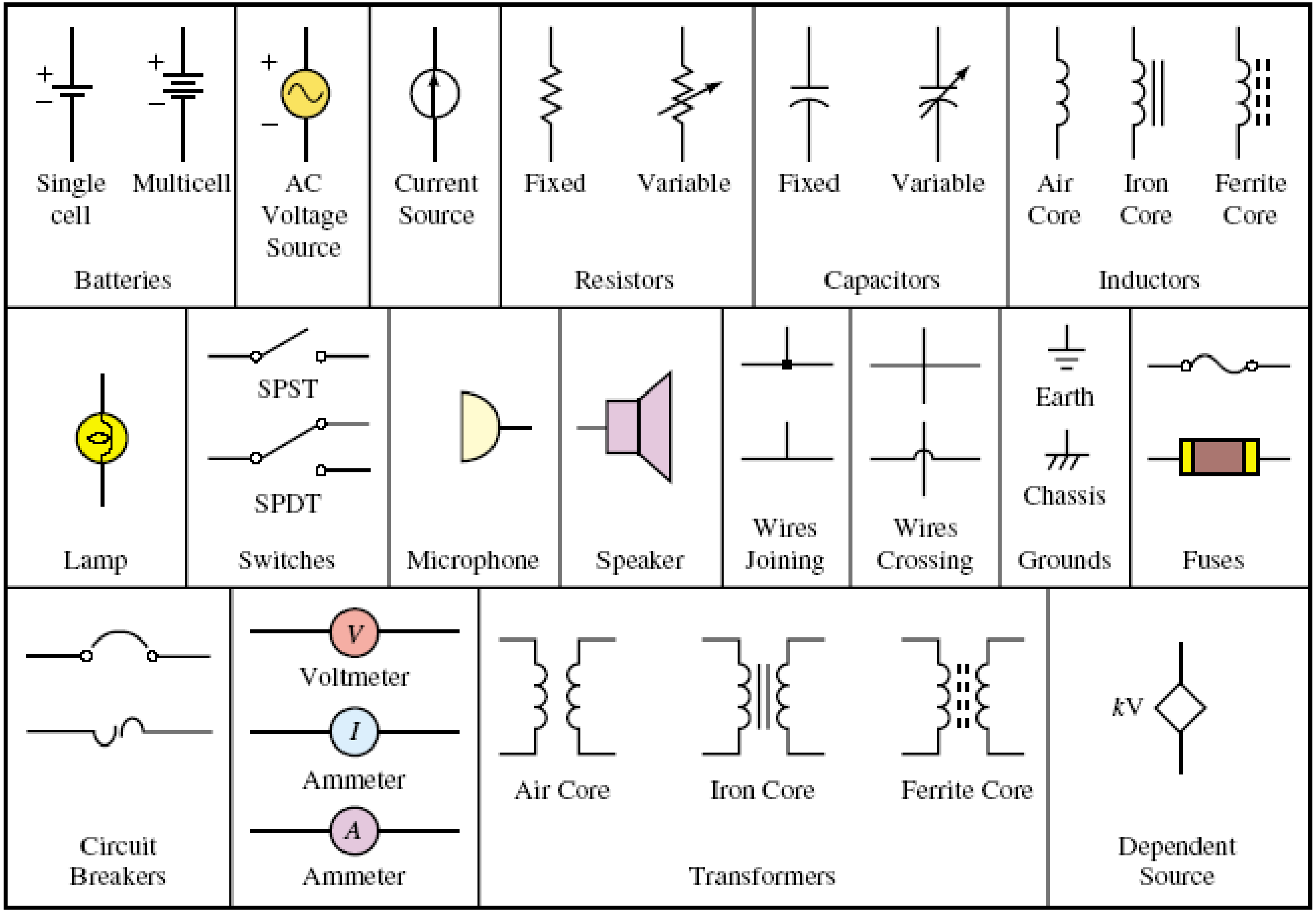


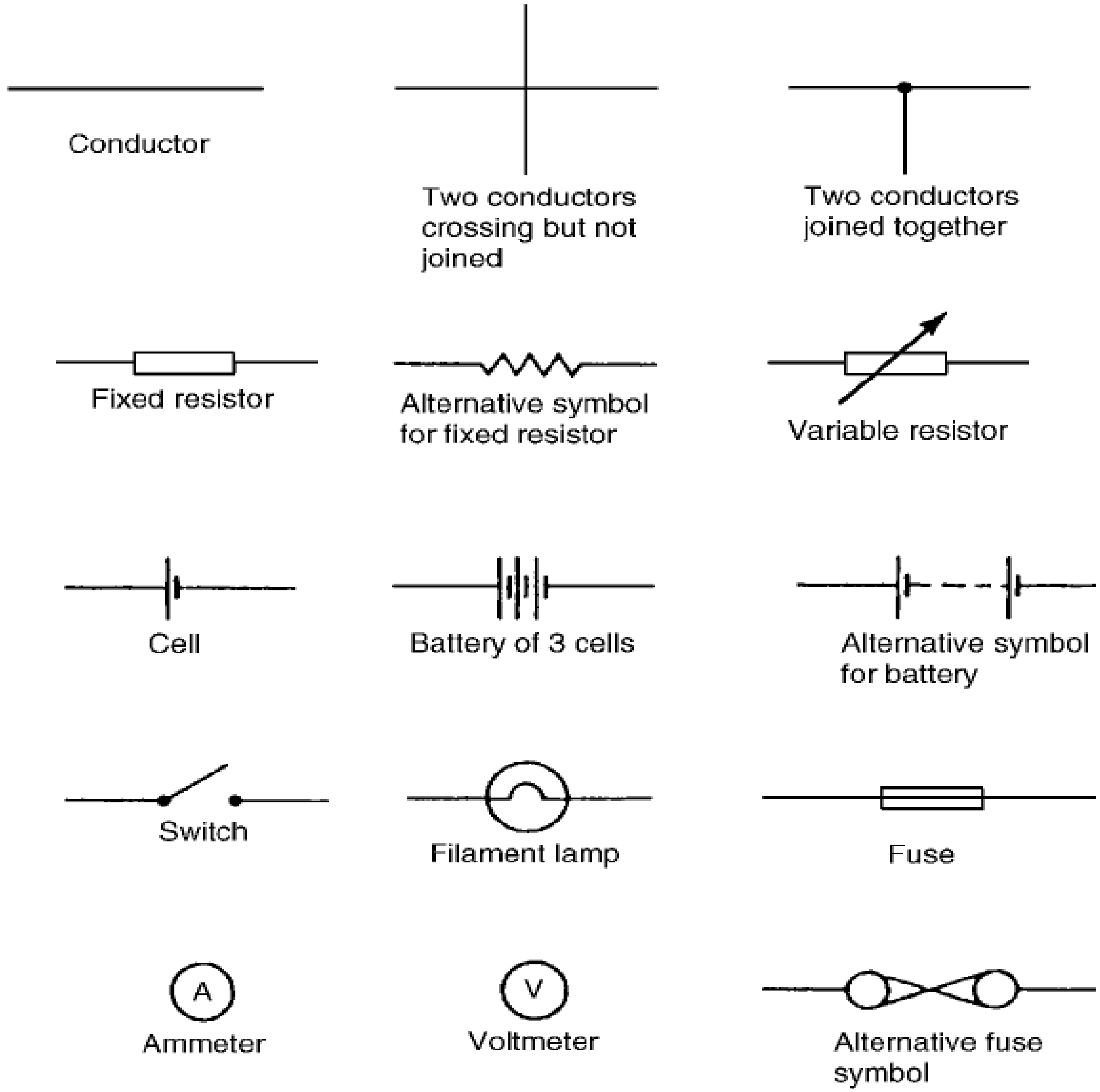
Resistor color codes.

Resistor Color Codes

Color	Band 1 Sig. Fig.	Band 2 Sig. Fig.	Band 3 Multiplier	Band 4 Tolerance	Band 5 Reliability
Black		0	$10^0 = 1$		
Brown	1	1	$10^1 = 10$		1%
Red	2	2	$10^2 = 100$		0.1%
Orange	3	3	$10^3 = 1\ 000$		0.01%
Yellow	4	4	$10^4 = 10\ 000$		0.001%
Green	5	5	$10^5 = 100\ 000$		
Blue	6	6	$10^6 = 1\ 000\ 000$		
Violet	7	7	$10^7 = 10\ 000\ 000$		
Gray	8	8			
White	9	9			
Gold			0.1	5%	
Silver			0.01	10%	
No color				20%	

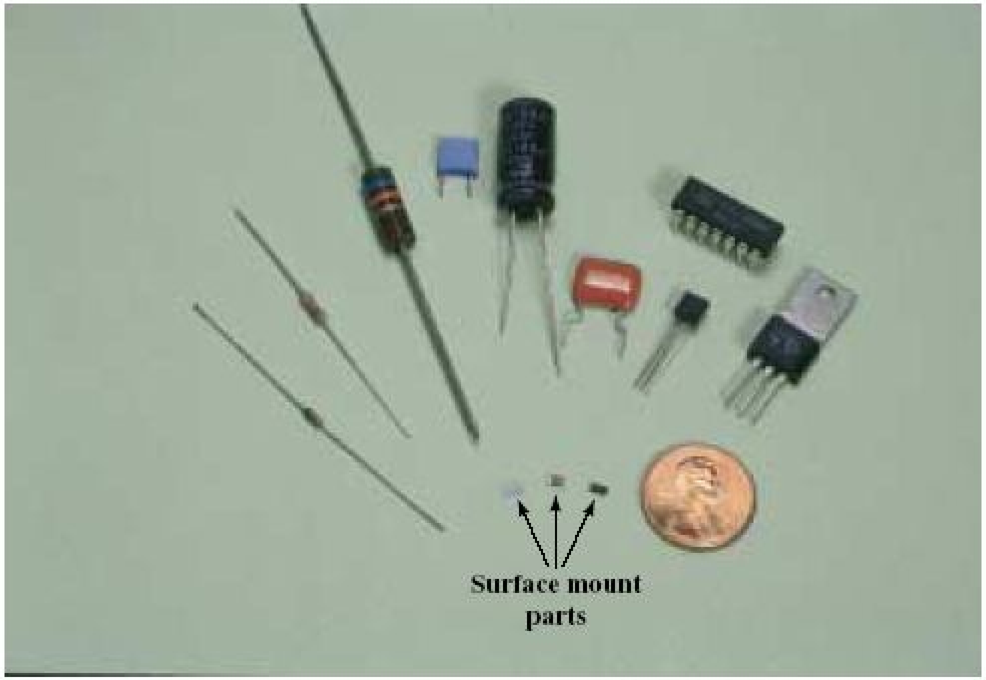
Schematic Circuit Symbols

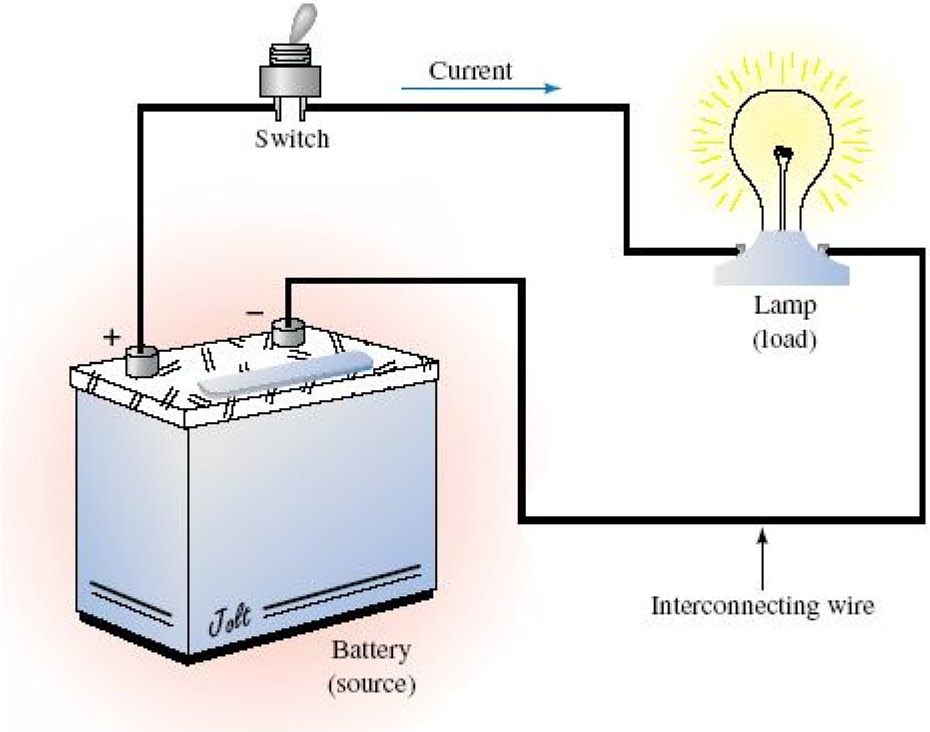


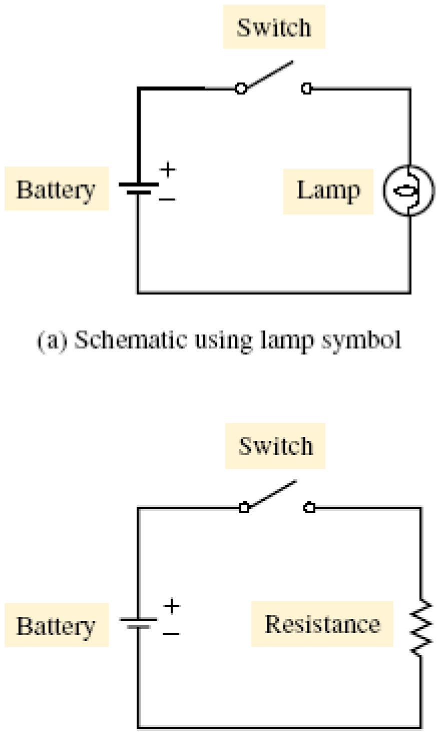


Conductor
Two conductors crossing but not joined
Two conductors joined together
Fixed resistor
Alternative symbol for fixed resistor
Variable resistor
Cell
Battery of 3 cells
Alternative symbol for battery
Switch
Filament lamp
Fuse
A
Ammeter
V
Voltmeter
Alternative fuse symbol

Components R,C, diode, IC







(a) Schematic using lamp symbol

(b) Schematic using resistance symbol

$$1 \text{ coulomb} = 6.24 \times 10^{18} \text{ electrons}$$

$$\text{volts} = \frac{\text{watts}}{\text{amperes}} = \frac{\text{joules/second}}{\text{amperes}} = \frac{\text{joules}}{\text{ampere seconds}} = \frac{\text{joules}}{\text{coulombs}}$$

$$\boxed{Q = It}$$

power, in watts $\boxed{P = VI}$

$$\textbf{Electrical energy} = \text{Power} \times \text{time}$$
$$= VIt \textbf{ Joules}$$

resistance, in ohms $\boxed{R = \frac{V}{I}}$

conductance, in siemens $\boxed{G = \frac{1}{R}}$

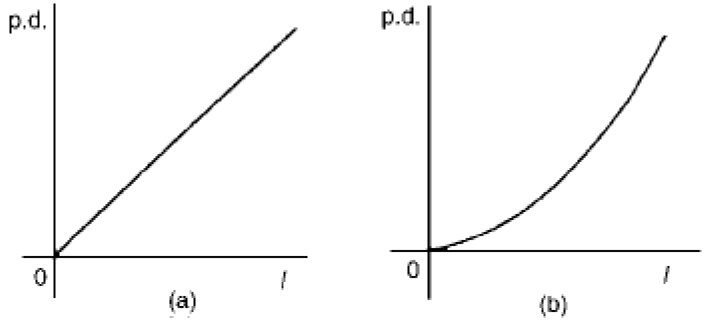
power in watts, $\boxed{P = \frac{W}{t}}$

energy, in joules, $\boxed{W = Pt}$

$$1 \text{ volt} = \frac{1 \text{ joule}}{\text{coulomb}}$$

Quantity	Quantity Symbol	Unit	Unit symbol
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Velocity	v	metres per second	m/s or m s^{-1}
Acceleration	a	metres per second squared	m/s^2 or m s^{-2}
Force	F	newton	N
Electrical charge or quantity	Q	coulomb	C
Electric current	I	ampere	A
Resistance	R	ohm	Ω
Conductance	G	siemen	S
Electromotive force	E	volt	V
Potential difference	V	volt	V
Work	W	joule	J
Energy	E (or W)	joule	J
Power	P	watt	W

Ohm's law



Ohm's law states that the current I flowing in a circuit is directly proportional to the applied voltage V and inversely proportional to the resistance R, provided the temperature remains constant.

A **conductor** is a material having a low resistance which allows electric current to flow in it. All metals are conductors and some examples include copper, aluminium, brass, platinum, silver, gold and carbon.

An **insulator** is a material having a high resistance which does not allow electric current to flow in it. Some examples of insulators include plastic, rubber, glass, porcelain, air, paper, cork, mica, ceramics and certain oils.

$$P = V \times I \text{ watts}$$

$$P = I^2 R \text{ watts}$$

$$P = \frac{V^2}{R} \text{ watts}$$

Electrical energy

$$\textbf{Electrical energy} = \textbf{power} \times \textbf{time}$$

If the power is measured in watts and the time in seconds then the unit of energy is watt-seconds or **joules**. If the power is measured in kilowatts and the time in hours then the unit of energy is **kilowatt-hours**, often called the '**unit of electricity**'. The 'electricity meter' in the home records the number of kilowatt-hours used and is thus an energy meter.

resistance $\boxed{R = \frac{\rho l}{a} \text{ ohms}}$

ρ is measured in ohm metres (Ωm)

Copper 1.7×10^{-8} Ωm (or 0.017 $\mu\Omega$m)

Aluminium 2.6×10^{-8} Ωm (or 0.026 $\mu\Omega$m)

Carbon (graphite) 10×10^{-8} Ωm (or 0.10 $\mu\Omega$m)

Glass 1×10^{10} Ωm (or 10^4 $\mu\Omega$m)

Mica 1×10^{13} Ωm (or 10^7 $\mu\Omega$m)

Note that good conductors of electricity have a low value of resistivity and good insulators have a high value of resistivity.

$$\boldsymbol{R_\theta = R_0(1 + \alpha_0\theta)}$$

where R_0 = resistance at 0°C

R_θ = resistance at temperature θ°C

α_0 = temperature coefficient of resistance at 0°C

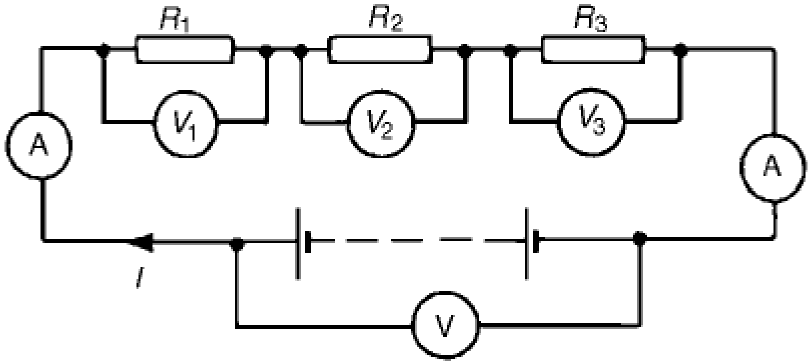
then the resistance R_θ at temperature θ°C is given by:

$$\boldsymbol{R_\theta = R_{20}[1 + \alpha_{20}(\theta - 20)]}$$

$R_1 = R_0(1 + \alpha_0\theta_1)$ and $R_2 = R_0(1 + \alpha_0\theta_2)$

$$\boldsymbol{\frac{R_1}{R_2} = \frac{1 + \alpha_0\theta_1}{1 + \alpha_0\theta_2}}$$

where R_2 = resistance at temperature θ_2



In a series circuit

(a) the current I is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters shown, and

(b) the sum of the voltages V_1, V_2 and V_3 is equal to the total applied voltage, V, i.e.

$$V = V_1 + V_2 + V_3$$

From Ohm's law:

$$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3 \text{ and } V = IR$$

where R is the total circuit resistance.

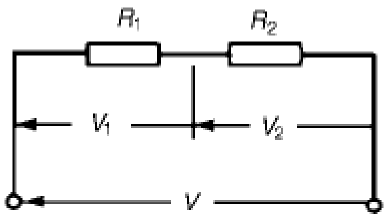
Since $V = V_1 + V_2 + V_3$

then $IR = IR_1 + IR_2 + IR_3$

Dividing throughout by I gives

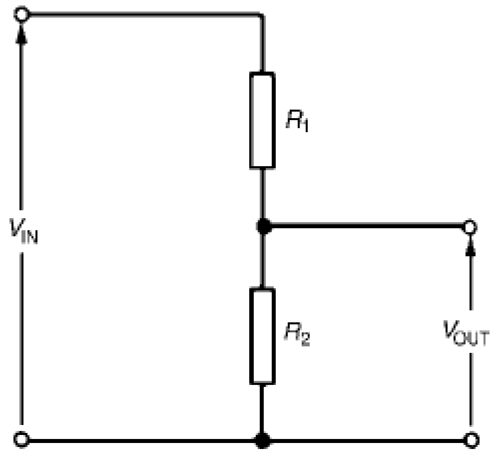
$$\boxed{\boldsymbol{R = R_1 + R_2 + R_3}}$$

Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances.



$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) \mathbf{V}$$

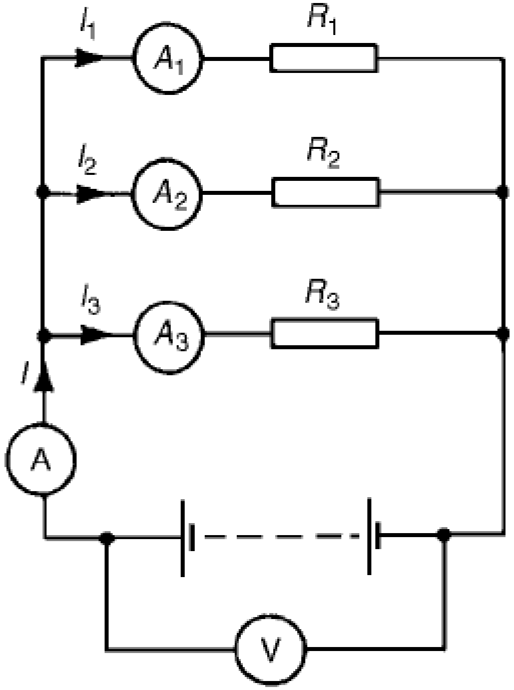
$$V_2 = \left(\frac{R_2}{R_1 + R_2}\right) \mathbf{V}$$



$$V_{OUT} = \left(\frac{R_2}{R_1 + R_2}\right) V_{IN}$$

In a parallel circuit:

(a) the sum of the currents I_1, I_2 and I_3 is equal to the total circuit current, I, i.e. $\boldsymbol{I = I_1 + I_2 + I_3}$, and
(b) the source p.d., V volts, is the same across each of the resistors.



From Ohm's law:

$$I_1 = \frac{V}{R_1},\ I_2 = \frac{V}{R_2},\ I_3 = \frac{V}{R_3} \text{ and } I = \frac{V}{R}$$

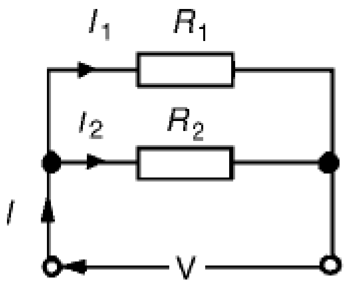
where R is the total circuit resistance.

Since $I = I_1 + I_2 + I_3$

then, $\dfrac{V}{R} = \dfrac{V}{R_1} + \dfrac{V}{R_2} + \dfrac{V}{R_3}$

Dividing throughout by V gives:

$$\boldsymbol{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$



$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

and $V = IR_T = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$

Current $I_1 = \frac{V}{R_1} = \frac{I}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \left(\frac{\boldsymbol{R_2}}{\boldsymbol{R_1 + R_2}}\right)(\boldsymbol{I})$

Similarly,

current $I_2 = \frac{V}{R_2} = \frac{I}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2}\right) = \left(\frac{\boldsymbol{R_1}}{\boldsymbol{R_1 + R_2}}\right)(\boldsymbol{I})$

Summarizing, with reference to Figure 5.18

$$\boldsymbol{I_1 = \left(\frac{R_2}{R_1 + R_2}\right)(I)}$$ **and** $$\boldsymbol{I_2 = \left(\frac{R_1}{R_1 + R_2}\right)(I)}$$

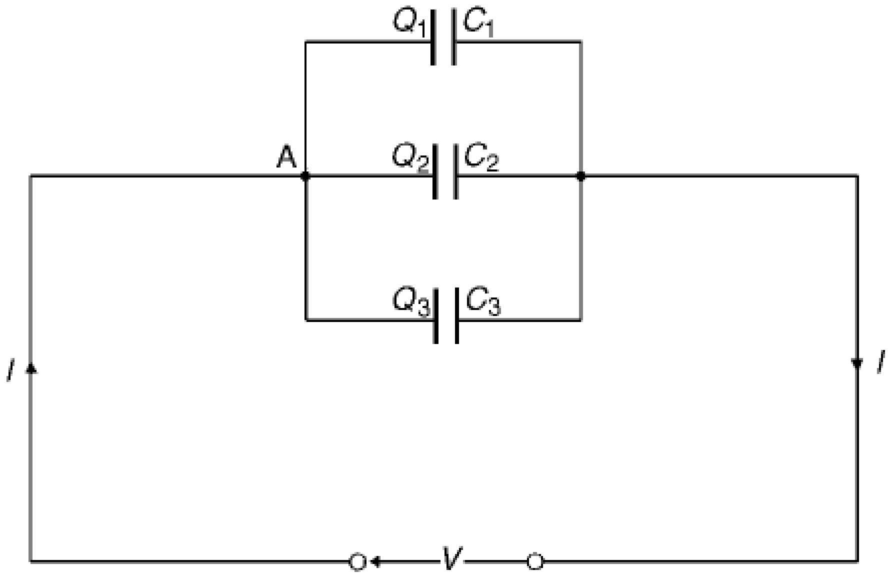
$$\text{capacitance } C = \frac{Q}{V}$$

$$\text{Capacitance, } C = \frac{\varepsilon_0 \varepsilon_r A}{d} \text{ farads}$$

where ε_0 is called the **permittivity of free space** or the free space constant. The value of ε_0 is 8.85×10^{-12} F/m.

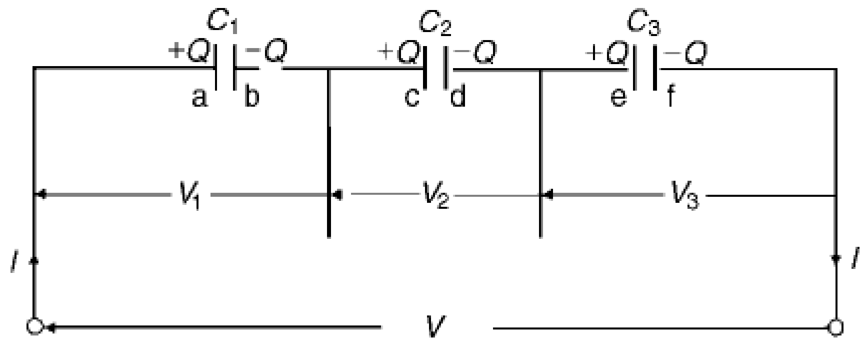
The energy, W, stored by a capacitor is given by

$$W = \frac{1}{2} CV^2 \text{ joules}$$



Total charge, $Q_T = Q_1 + Q_2 + Q_3$

$$C = C_1 + C_2 + C_3$$



Charge on each capacitor = Q

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Magnetic flux is the amount of magnetic field (or the number of lines of force) produced by a magnetic source. The symbol for magnetic flux is Φ (Greek letter 'phi'). The unit of magnetic flux is the **weber**, **Wb**

Magnetic flux density is the amount of flux passing through a defined area that is perpendicular to the direction of the flux:

$$\textbf{Magnetic flux density} = \frac{\textbf{magnetic flux}}{\textbf{area}}$$

$$\boxed{B = \frac{\Phi}{A}\ \textbf{tesla}}, \quad \text{where } A(\text{m}^2) \text{ is the area}$$

Magnetomotive force (mmf) is the cause of the existence of a magnetic flux in a magnetic circuit,

$$\boxed{\textbf{mmf}, F_m = NI \textbf{ amperes}}$$

where N is the number of conductors (or turns) and I is the current in amperes. The unit of mmf is sometimes expressed as 'ampere-turns'.

Magnetic field strength (or **magnetizing force**),

$$\boxed{H = NI/l \textbf{ ampere per metre,}}$$

where l is the mean length of the flux path in metres.

Thus **mmf** $= NI = Hl$ **amperes**.

For all media other than free space, $\boldsymbol{B/H = \mu_0 \mu_r}$

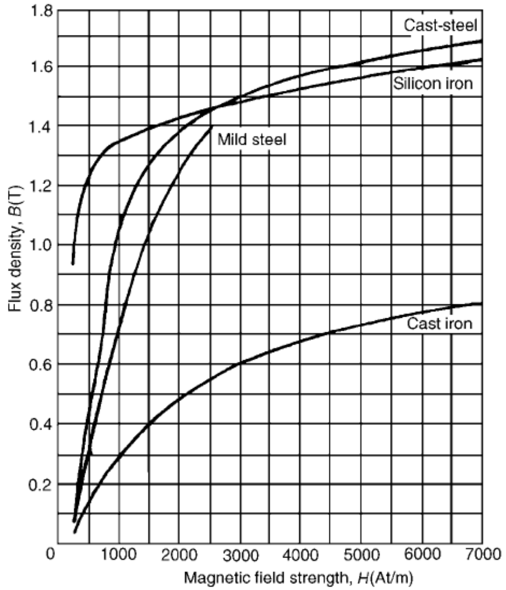
where u_r is the relative permeability, and is defined as

$$\mu_r = \frac{\textbf{flux density in material}}{\textbf{flux density in a vacuum}}$$

$\mu_0 \mu_r = \mu$, called the **absolute permeability**

The **relative permeability** of a ferromagnetic material is proportional to the slope of the B–H curve and thus varies with the magnetic field strength. The approximate range of values of relative permeability μ_r for some common magnetic materials are:

Cast iron	$\mu_r = 100–250$	Mild steel	$\mu_r = 200–800$
Silicon iron	$\mu_r = 1000–5000$	Cast steel	$\mu_r = 300–900$
Mumetal	$\mu_r = 200–5000$	Stalloy	$\mu_r = 500–6000$



Reluctance S (or R_M) is the 'magnetic resistance' of a magnetic circuit to the presence of magnetic flux.

$$\textbf{Reluctance } S = \frac{F_M}{\Phi} = \frac{NI}{\Phi} = \frac{Hl}{BA} = \frac{l}{(B/H)A} = \frac{l}{\mu_0 \mu_r A}$$

The unit of reluctance is 1/H (or H^{-1}) or A/Wb

Ferromagnetic materials have a low reluctance and can be used as **magnetic screens** to prevent magnetic fields affecting materials within the screen.

For a series magnetic circuit having n parts, the **total reluctance** S is given by:

$$S = S_1 + S_2 + \ldots + S_n$$

(This is similar to resistors connected in series in an electrical circuit.)

Electrical circuit	Magnetic circuit
e.m.f. E (V) current I (A) resistance R (Ω) $I = \frac{E}{R}$ $R = \frac{\rho l}{A}$	mmf F_m (A) flux Φ (Wb) reluctance S (H^{-1}) $\Phi = \frac{\text{mmf}}{S}$ $S = \frac{l}{\mu_0 \mu_r A}$

General

Charge $Q = It$ Force $F = ma$ Work $W = Fs$ Power $P = \frac{W}{t}$

Energy $W = Pt$

Ohm's law $V = IR$ or $I = \frac{V}{R}$ or $R = \frac{V}{I}$ Conductance $G = \frac{1}{R}$

Power $P = VI = I^2R = \frac{V^2}{R}$ Resistance $R = \frac{\rho l}{a}$

Resistance at $\theta°C$, $R_\theta = R_0(1 + \alpha_0\theta)$

Terminal p.d. of source, $V = E - Ir$

Series circuit $R = R_1 + R_2 + R_3 + \cdots$

Parallel network $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$

Capacitors and capacitance

$E = \frac{V}{d}$ $C = \frac{Q}{V}$ $Q = It$ $D = \frac{Q}{A}$ $\frac{D}{E} = \varepsilon_0\varepsilon_r$

$C = \frac{\varepsilon_0\varepsilon_r A(n-1)}{d}$

Capacitors in parallel $C = C_1 + C_2 + C_3 + \cdots$

Capacitors in series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$ $W = \frac{1}{2}CV^2$

Magnetic circuits

$$B = \frac{\Phi}{A} \qquad F_m = NI \qquad H = \frac{NI}{l} \qquad \frac{B}{H} = \mu_0\mu_r$$

$$S = \frac{\text{mmf}}{\Phi} = \frac{1}{\mu_0\mu_r A}$$

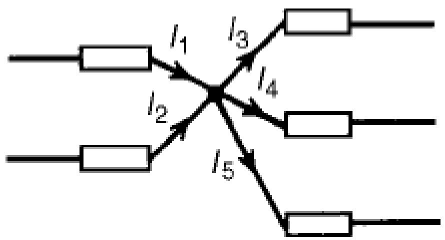
Electromagnetism

$$F = Bil\sin\theta \qquad F = QvB$$

Electromagnetic induction

$$E = Blv\sin\theta \qquad E = -N\frac{d\Phi}{dt} = -L\frac{dI}{dt} \qquad W = \frac{1}{2}LI^2 \qquad L = \frac{N\Phi}{I}$$

$$E_2 = -M\frac{dI_1}{dt}$$



Kirchhoff's laws state:

(a) **Current Law.** *At any junction in an electric circuit the total current flowing towards that junction is equal to the total current flowing away from the junction*, i.e. $\Sigma I = 0$
Thus, referring to Figure 13.1:

$$I_1 + I_2 = I_3 + I_4 + I_5 \quad \text{or} \quad I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

KIRCHOFF’S LAWS

KVL – Kirchoff’s voltage law

In a closed loop, the sum of potential raise is equal to potential drop or the algebric sum of voltages around a closed path is equal to zero

Through the source

Potential raise , +ve, current flows from ‘-ve’ end to ‘+ve’ end of the source and we go around the loop in the direction of current

Potential drop. –ve, current flows from ‘+ve’ end to ‘-ve’ end of the source and we go around the loop in the direction of current

- Through resistor

Always Current entering is positive

potential drop, -ve, when we go around the loop through the resistor from positive to negative

potential raise, +ve, when we go around the loop through the resistor from negative to positive

$$\sum V = 0$$

Things placed in parallel have same voltage drop across them

KCL – Kirchoff's Current law

The sum of incoming currents is equal to sum of outgoing currents or the algebric sum of currents meeting at node is equal to zero

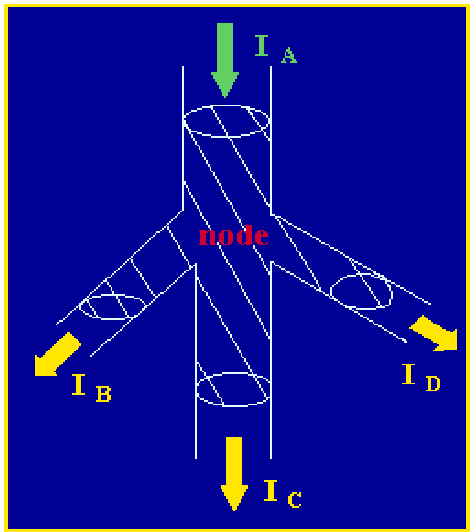
If current entering is taken '+ve', then current leaving the node is taken as '-ve'

If current entering is taken '-ve', then current leaving the node is taken as '+ve'

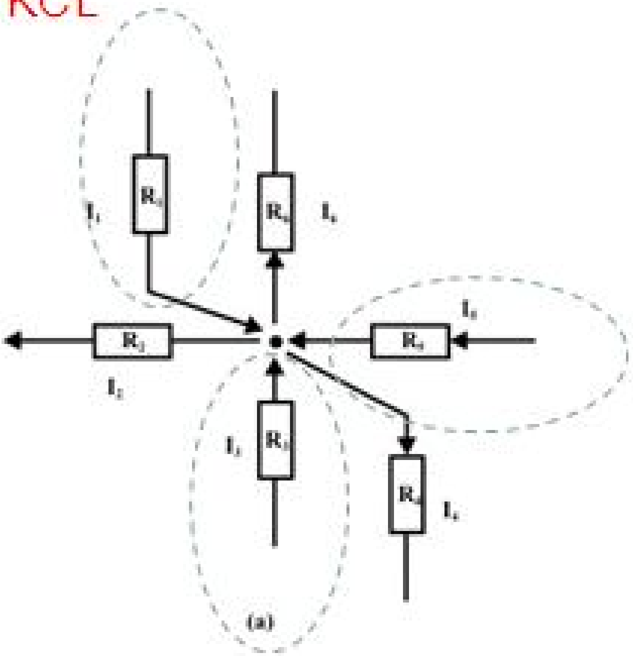
$$\sum I = 0$$

This fundamental law results from the conservation of charge.

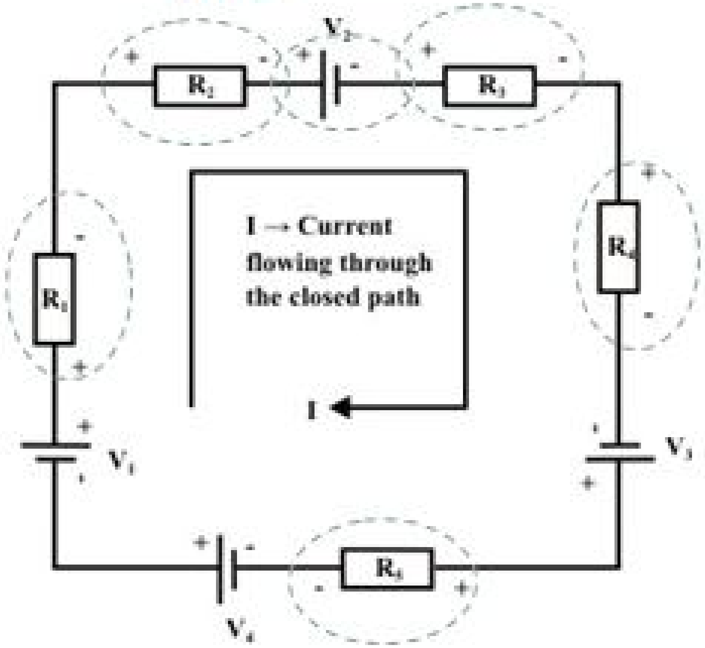
For a series circuit, the current is same everywhere



KCL



KVL



$I_1 - I_2 + I_3 - I_4 + I_5 - I_6 = 0$

or

$I_1 + I_3 + I_5 = I_2 + I_6 + I_4$

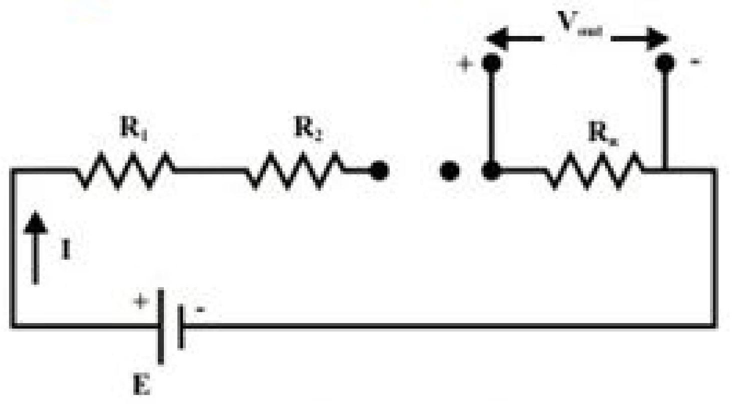
$V_1 - IR_1 - IR_2 - V_2 - IR_3 - IR_4 + V_3 - IR_5 + V_4 = 0$

or

$V_1 + V_3 + V_4 = IR_1 + IR_2 + IR_3 + IR_4 + IR_5 + V_2$

Voltage and Current division

Voltage division technique in Series circuit

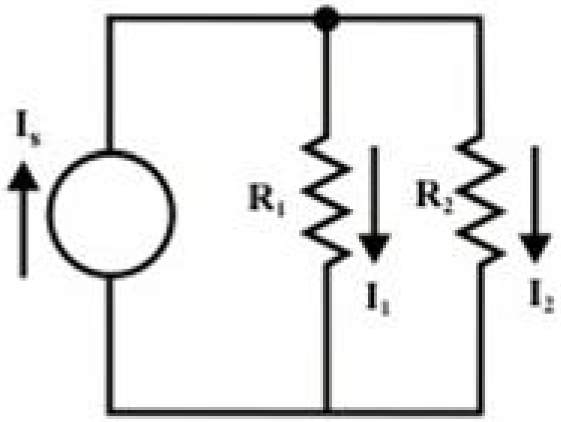


$$V_{out} = IR_n \quad but \quad I = \frac{E}{R_1 + R_2 + \ldots R_n}$$

$$so\ V_{out} = \frac{E}{R_1 + R_2 + \ldots R_n} R_n \ or\ E\frac{R_n}{R_1 + R_2 + \ldots R_n}$$

= Total voltage X $\frac{\text{same resistance}}{\text{Total resistance}}$

Current division technique in parallel circuit



$$\frac{I_1}{I_s} = \frac{\frac{V}{R_1}}{\frac{V}{R_T}} = \frac{R_T}{R_1} = \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1} = \frac{R_2}{R_1 + R_2}$$

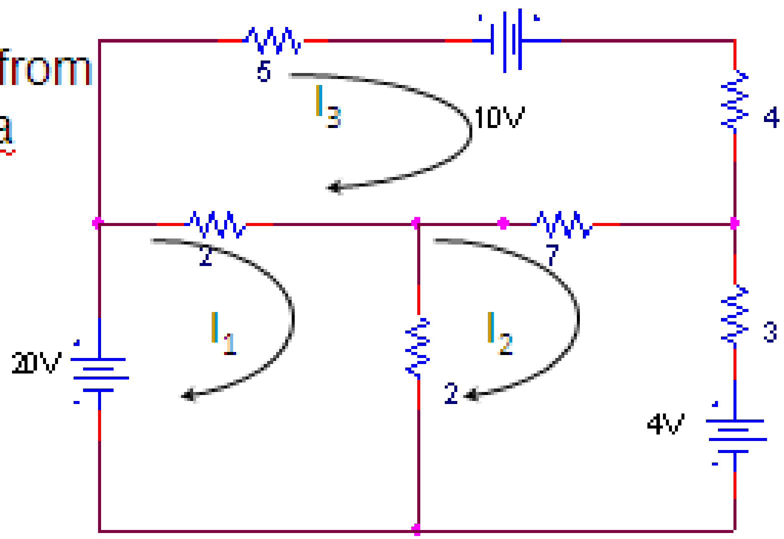
$$so\ I_1 = \frac{I_s}{R_1 + R_2} R_2 \ or\ I_s \frac{R_2}{R_1 + R_2}$$

= Total current X opposite resistance / Total resistance

Mesh Analysis

Writing loop equations

- Assume current in every loop in clock wise direction
- Number of equations will be equal to number of loop currents
- When writing equations for loop 1, for LHS, the co-efficient of I_1 is equal to sum of resistance in that loop, co-efficient of I_2 is the sum of resistance common between loop 1 and loop 2 (-ve as the currents will be in opposite direction) and co-efficient of I_3 is equal to the value of resistance common between loop 1 and loop3 (-ve)
- RHS will be the sum of voltages in that loop (+ve when current flows from –ve to +ve of source and viceversa



Loop 1: $4I_1 - 2I_2 - 2I_3 = 20$
Loop 2: $-2I_1 + 12I_2 - 7I_3 = -4$
Loop 3: $-2I_1 - 7I_2 + 18I_3 = 10$

Solve these by any method to get loop currents

Nodal analysis

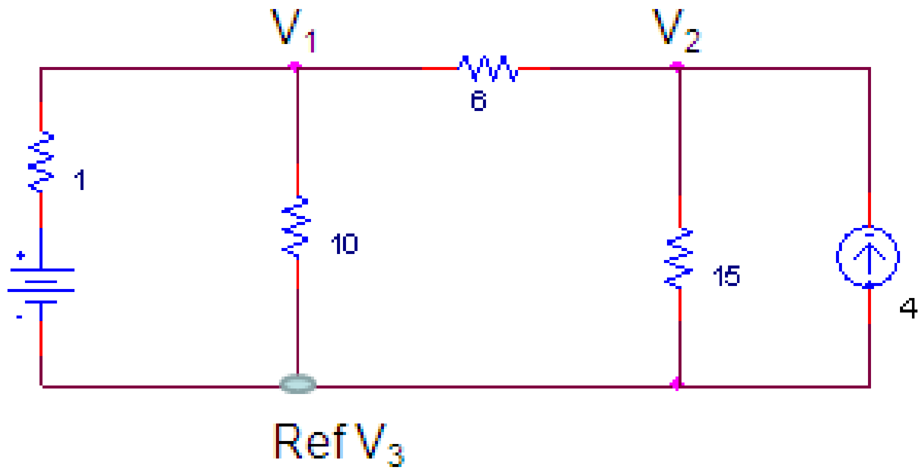
- Assume node voltages at all nodes and take one node (normally –ve of source) as reference
- When applying KCL for a node, assume all currents are leaving and the currents will be the difference of voltage between the nodes of the branch divided by the resistance, equate the sum of current meeting at node to zero.
- Here V1 has three branches 1 ohm, 10 ohm and 6 ohm. The voltage at other end of 1 ohm is 2V, voltage at other end of 10 ohm is 0 volts(ref) and voltage at other end of 6 ohm is V_2. hence the equation of current at node 1 is

$$\frac{V_1 - 2}{1} + \frac{V_1 - 0}{10} + \frac{V_1 - V_2}{6} = 0$$

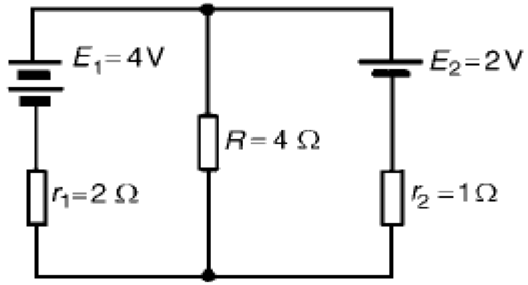
And that at node 2 is

$$\frac{V_2 - 0}{15} - 4 + \frac{V_2 - V_1}{6} = 0$$

Solve these equations for node Voltages and hence find branch Currents



Superposition Theorem

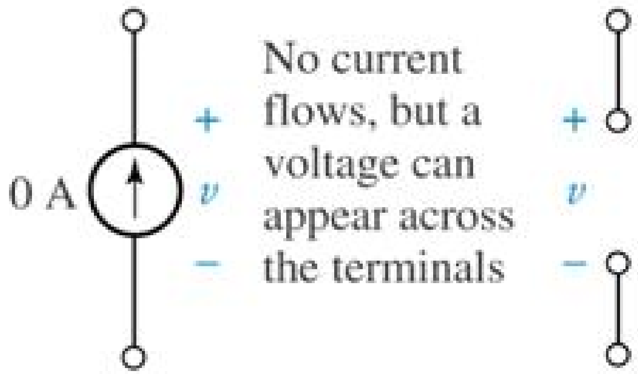


A linear system obeys the principle of superposition

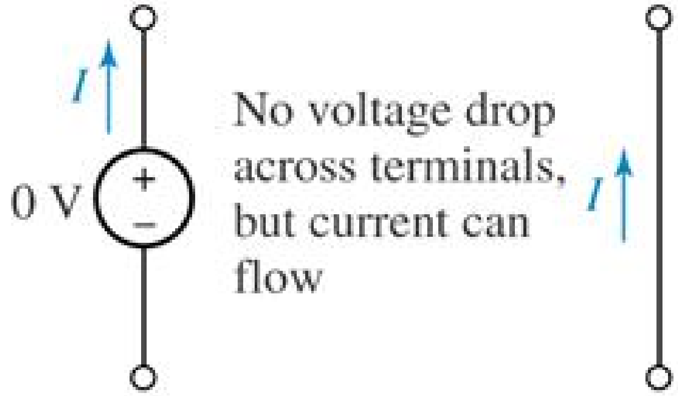
It states that whenever a linear system is excited by more than one independent source of energy, the total response is the sum of the individual responses

To deactivate a voltage source and current source:

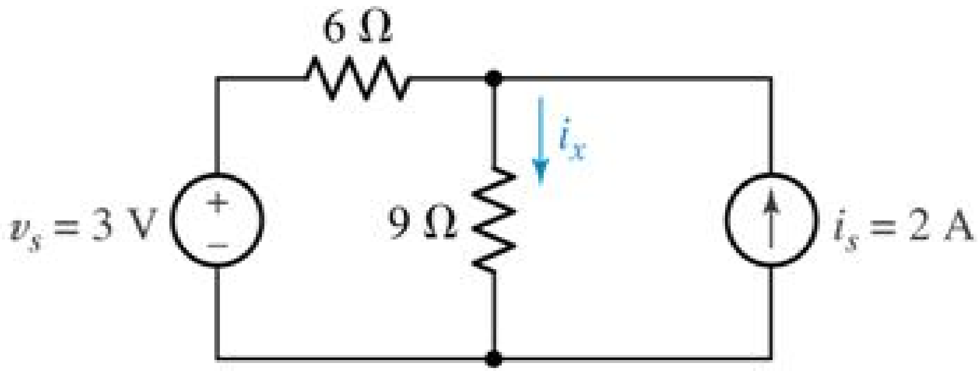
(a) A current source set to zero acts like an open circuit.



(b) A voltage source set to zero acts like a short circuit.



Example : Use superposition to find the current i_x



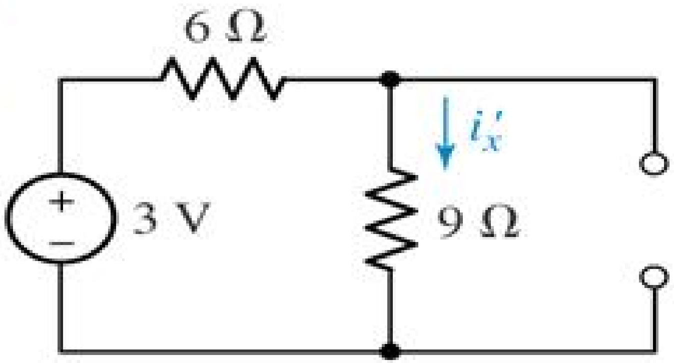
Solution

(a)

1. Find branch currents due to voltage source alone Replace the ideal current source by an open circuit

Current due to 3V source alone is Denoted by i_x'

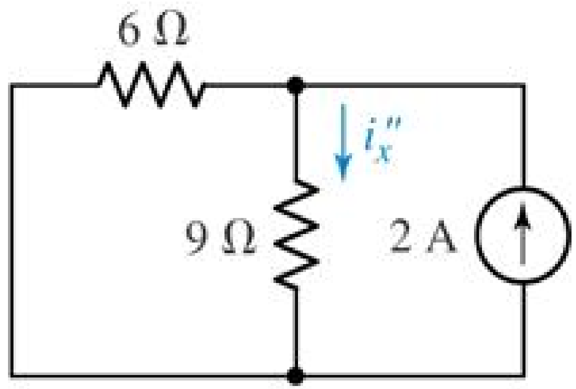
$i_x' = 3V/ (6+9)\Omega = 0.2A$



(b)

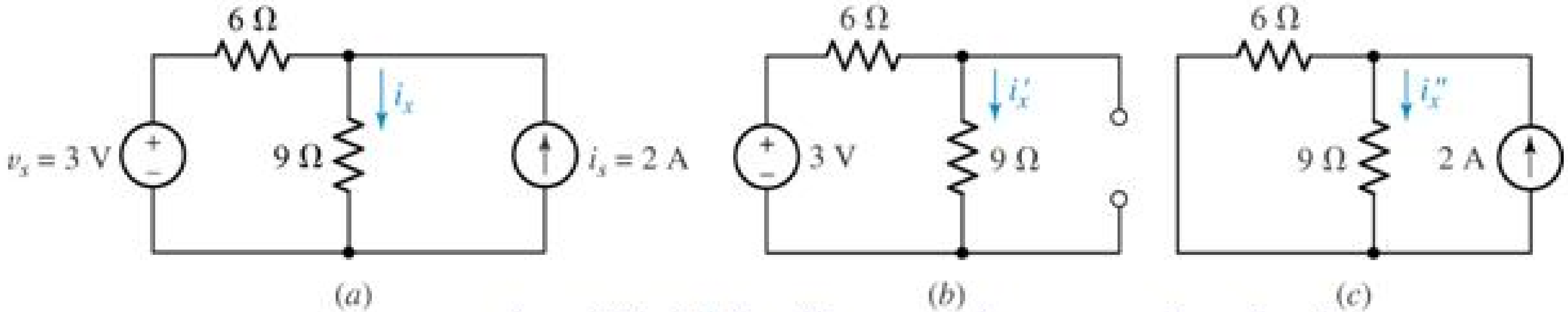
2. To find the branch currents due to current source alone Replace the ideal voltage source by a short circuit

Current due to 2A current source alone is Denoted by i_x''



$$i_x'' = 2 \times \frac{6}{6+9} = 0.8A$$

3. Total response is equal to sum of individual responses.



$i_x = i_x' + i_x''$ (as they are in same direction)

$= 0.2 + 0.8$

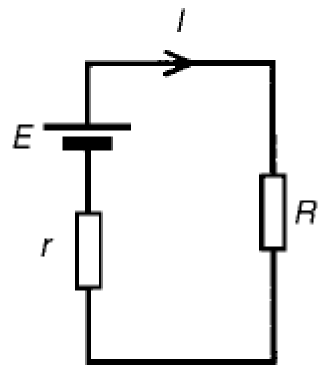
$= 1A$

Thévenin's theorem states:

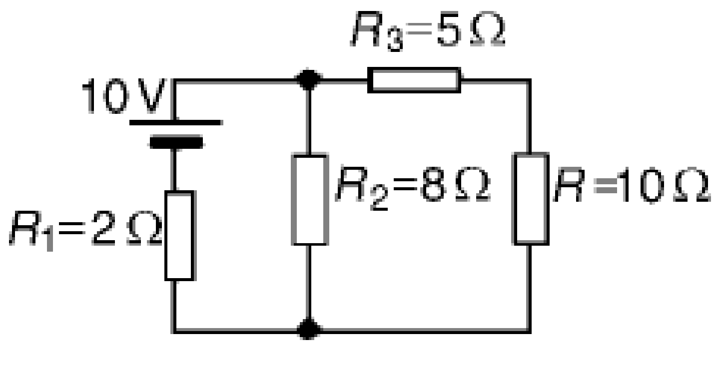
'The current in any branch of a network is that which would result if an e.m.f. equal to the p.d. across a break made in the branch, were introduced into the branch, all other e.m.f.'s being removed and represented by the internal resistances of the sources.'

The procedure adopted when using Thévenin's theorem is summarized below. To determine the current in any branch of an active network (i.e. one containing a source of e.m.f.):

(i) remove the resistance R from that branch,
(ii) determine the open-circuit voltage, E, across the break,
(iii) remove each source of e.m.f. and replace them by their internal resistances and then determine the resistance, r, 'looking-in' at the break,
(iv) determine the value of the current from the equivalent circuit shown in Figure 13.27, i.e. $\boldsymbol{I = \frac{E}{R + r}}$

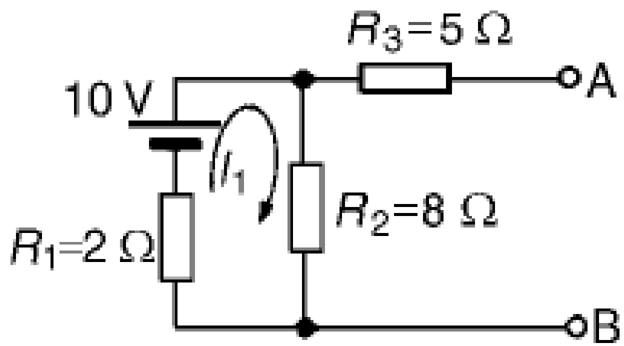


Problem 7. Use Thévenin's theorem to find the current flowing in the 10 Ω resistor for the circuit shown in Figure 13.28(a).



(a)

(i) The 10 Ω resistance is removed from the circuit as shown in Figure 13.28(b)



(b)

(ii) There is no current flowing in the 5 Ω resistor and current I_1 is given by:

$$I_1 = \frac{10}{R_1 + R_2} = \frac{10}{2+8} = 1 \text{ A}$$

P.d. across $R_2 = I_1 R_2 = 1 \times 8 = 8$ V

Hence p.d. across AB, i.e. the open-circuit voltage across the break, $E = 8$ V.

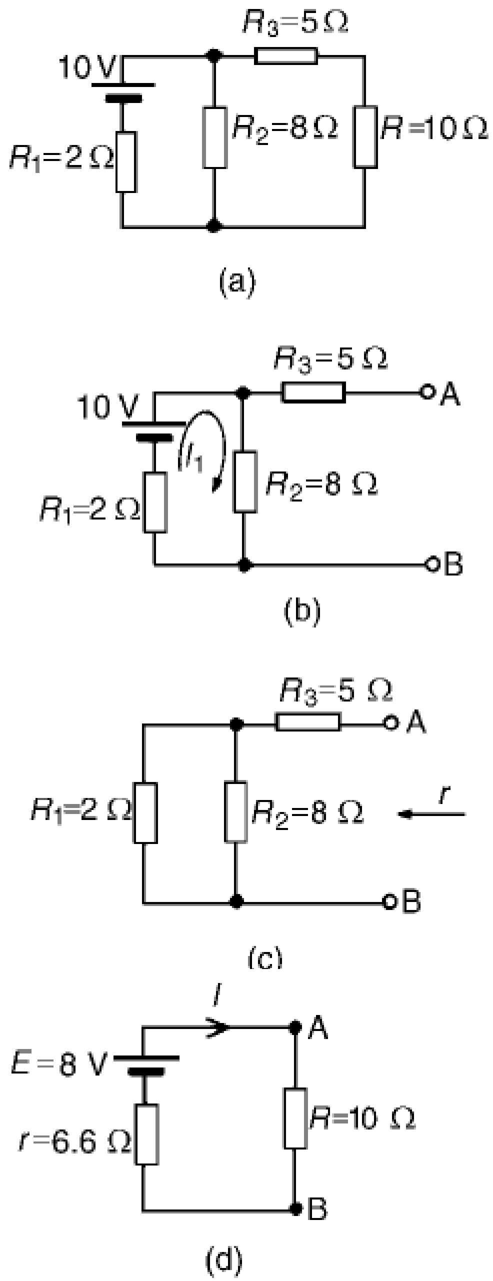
(iii) Removing the source of e.m.f. gives the circuit of Figure 13.28(c).

$$\text{Resistance, } r = R_3 + \frac{R_1 R_2}{R_1 + R_2} = 5 + \frac{2 \times 8}{2+8}$$
$$= 5 + 1.6 = 6.6\ \Omega$$

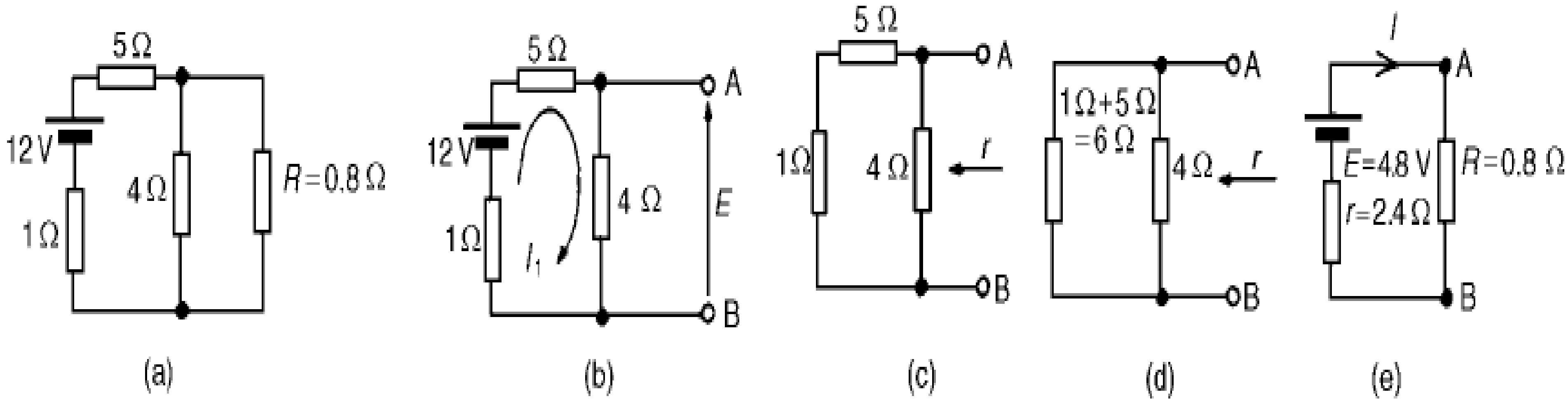
(iv) The equivalent Thévenin's circuit is shown in Figure 13.28(d).

$$\text{Current } I = \frac{E}{R+r} = \frac{8}{10+6.6} = \frac{8}{16.6} = 0.482 \text{ A}$$

Hence the current flowing in the 10 Ω resistor of Figure 28(a) is **0.482 A**



Problem 8. For the network shown in Figure 13.29(a) determine the current in the 0.8 Ω resistor using Thévenin's theorem.



$$I = 1.5\ \text{A} = \textbf{current in the 0.8 Ω resistor}$$

(i) The 0.8 Ω resistor is removed from the circuit as shown in Figure 13.29(b).

(ii) Current $I_1 = \frac{12}{1+5+4} = \frac{12}{10} = 1.2$ A

P.d. across 4 Ω resistor $= 4I_1 = (4)(1.2) = 4.8$ V

Hence p.d. across AB, i.e. the open-circuit voltage across AB,

$E = 4.8$ V

(iii) Removing the source of e.m.f. gives the circuit shown in Figure 13.29(c). The equivalent circuit of Figure 13.29(c) is shown in Figure 13.29(d), from which,

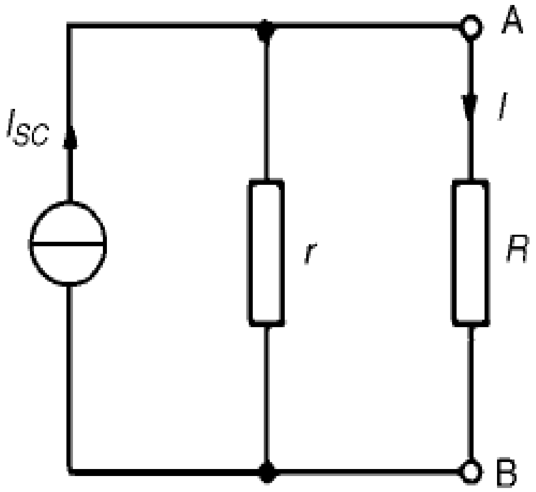
$$\text{resistance } r = \frac{4 \times 6}{4+6} = \frac{24}{10} = 2.4\ \Omega$$

(iv) The equivalent Thévenin's circuit is shown in Figure 13.29(e), from which,

$$\text{current } I = \frac{E}{r+R} = \frac{4.8}{2.4+0.8} + \frac{4.8}{3.2}$$

$$\mathbf{I = 1.5\ A = current\ in\ the\ 0.8\ \Omega\ resistor}$$

13.6 Constant-current source



A source of electrical energy can be represented by a source of e.m.f. in series with a resistance. In Section 13.5, the Thévenin constant-voltage source consisted of a constant e.m.f. E in series with an internal resistance r. However this is not the only form of representation. A source of electrical energy can also be represented by a constant-current source in parallel with a resistance. It may be shown that the two forms are equivalent. An **ideal constant-voltage generator** is one with zero internal resistance so that it supplies the same voltage to all loads. An **ideal constant-current generator** is one with infinite internal resistance so that it supplies the same current to all loads.

Note the symbol for an ideal current source (BS 3939, 1985), shown in Figure 13.33.

Norton's theorem states:

'The current that flows in any branch of a network is the same as that which would flow in the branch if it were connected across a source of electrical energy, the short-circuit current of which is equal to the current that would flow in a short-circuit across the branch, and the internal resistance of which is equal to the resistance which appears across the open-circuited branch terminals.'

The procedure adopted when using Norton's theorem is summarized below.

To determine the current flowing in a resistance R of a branch AB of an active network:

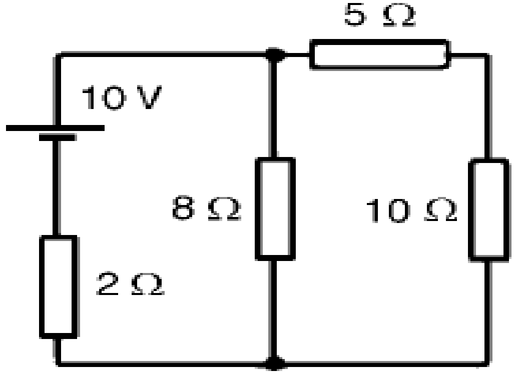
(i) short-circuit branch AB

(ii) determine the short-circuit current I_{SC} flowing in the branch

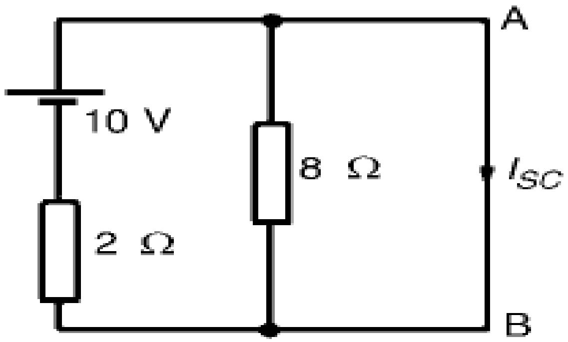
(iii) remove all sources of e.m.f. and replace them by their internal resistance (or, if a current source exists, replace with an open-circuit), then determine the resistance r, 'looking-in' at a break made between A and B

(iv) determine the current I flowing in resistance R from the Norton equivalent network shown in Figure 13.33, i.e.

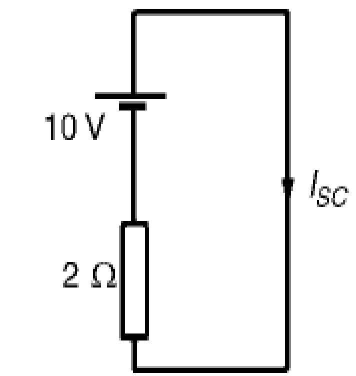
$$I = \left(\frac{r}{r + R}\right) I_{SC}$$



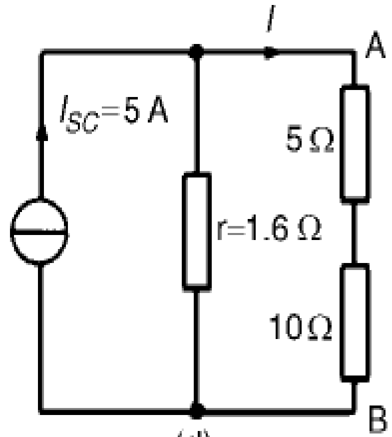
(a)



(b)



(c)



(d)

Following the above procedure:

(i) The branch containing the 10 Ω resistance is short-circuited as shown in Figure 13.34(b).

(ii) Figure 13.34(c) is equivalent to Figure 13.34(b). Hence

$$I_{SC} = \frac{10}{2} = 5 \text{ A}$$

(iii) If the 10 V source of e.m.f. is removed from Figure 13.34(b) the resistance 'looking-in' at a break made between A and B is given by:

$$r = \frac{2 \times 8}{2 + 8} = 1.6\ \Omega$$

(iv) From the Norton equivalent network shown in Figure 13.34(d) the current in the 10 Ω resistance, by current division, is given by:

$$I = \left(\frac{1.6}{1.6 + 5 + 10}\right)(5) = \mathbf{0.482\ A}$$

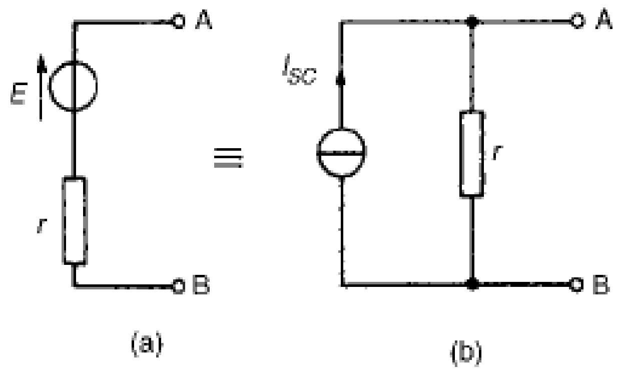
The Thévenin and Norton networks shown in Figure 13.38 are equivalent to each other. The resistance 'looking-in' at terminals AB is the same in each of the networks, i.e. r.

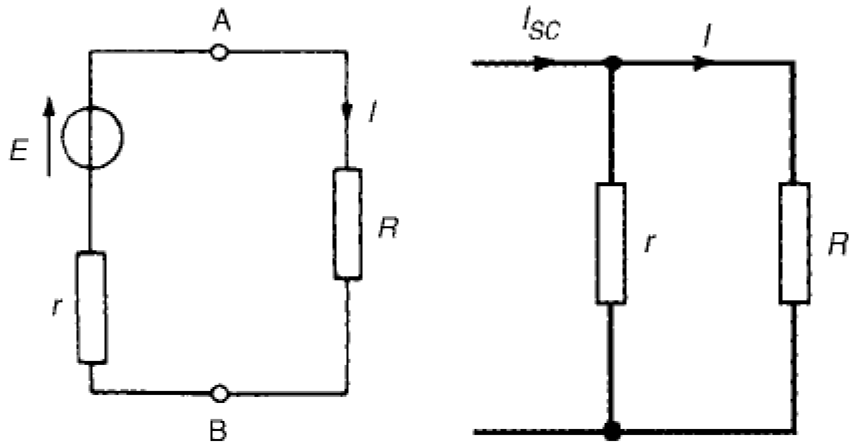
If terminals AB in Figure 13.38(a) are short-circuited, the short-circuit current is given by E/r. If terminals AB in Figure 13.38(b) are short-circuited, the short-circuit current is I_{SC}. For the circuit shown in Figure 13.38(a) to be equivalent to the circuit in Figure 13.38(b) the same short-circuit current must flow. Thus $I_{SC} = E/r$.

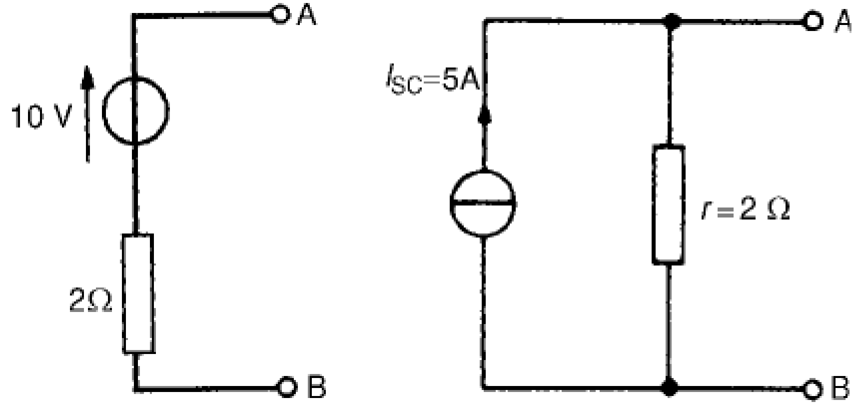
Figure 13.39 shows a source of e.m.f. E in series with a resistance r feeding a load resistance R.

From Figure 13.39, $I = \dfrac{E}{r+R} = \dfrac{E/r}{(r+R)/r} = \left(\dfrac{r}{r+R}\right)\dfrac{E}{r}$

$$\text{i.e. } I = \left(\frac{r}{r+R}\right) I_{SC}$$

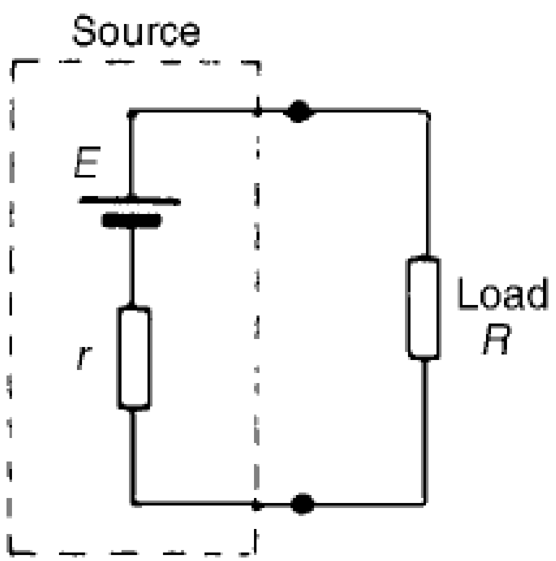






From Figure 13.40 it can be seen that, when viewed from the load, the source appears as a source of current I_{SC} which is divided between r and R connected in parallel.

Thus the two representations shown in Figure 13.38 are equivalent.



The **maximum power transfer theorem** states:

'The power transferred from a supply source to a load is at its maximum when the resistance of the load is equal to the internal resistance of the source.'

Hence, in Figure 13.47, when $R = r$ the power transferred from the source to the load is a maximum.