Final Assessment Test – June 2023

VIT
Vellore Institute of Technology

Course: BMAT102L - Differential Equations and Transforms

Class NBR(s): 0379 / 4501 / 4588 / 4590 / 4592 / 4661 / 4663 / 4665 / 4880 / 4882 / 4884 / 4886 / 4888 / 4890 / 4892 / 4894 / 4896 / 4909 / 4911 Slot: C1+TC1+TCC1

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. Find the solution of $\frac{d^2x}{dt^2}-4\frac{dx}{dt}-12x=te^{4t}$ using the method of undetermined coefficients.

2. An LCR circuit connected in series has $R=10$ ohms, $C=10^{-2}$ farad, $L=\frac{1}{2}$ henry and an applied voltage $E=12$ volts. Assuming no initial current and no initial charge at $t=0$ when the voltage is first applied, find the subsequent current in the system at any time t without finding charge.

3. Solve the Lagrange's equation $x(y^2-z^2)p-y(z^2+x^2)q=z(x^2+y^2)$.

4. Solve $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial t}=3u$ with $u(0,\ t)=e^{-5t}$ using the method of separation of variables.

5. Express $f(t)=\begin{cases} t & for\ 0<t<1 \\ e^{-t} & for\ 1<t<2 \\ 1 & for\ 2>t \end{cases}$ in terms of unit step function and hence find its Laplace transform.

6. Find $L^{-1}\left\{\frac{s}{(s+1)(s^2+4)}\right\}$ using convolution theorem.

7. Find the solution of $x''(t)+7x'(t)+6x(t)=H(t-3)$ with $x(0)=0$ and $x'(0)=0$ using Laplace transform.

8. Obtain the half range cosine series for $f(x)=x(2-x)$ in $0\le x\le 2$ and hence find the sum of the series $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots$

9. Obtain the Fourier series for $f(x)=x^3-\pi^2x$ in the interval $(-\pi,\ \pi)$ and hence find the sum of the series $\sum_{n=1}^{\infty}\frac{1}{n^6}$ using Parseval's identity.

10. Find the Fourier sine integral for $f(x)=e^{-\beta x},\ (\beta>0)$ hence evaluate $\int_0^{\infty}\frac{\lambda\sin\lambda x}{\beta^2+\lambda^2}d\lambda$.

11. a) Find $Z[(n+1)^2]$. [5]

    b) Find $Z^{-1}\left\{\frac{z}{z^2+11z+24}\right\}$ using partial fractions method. [5]

12. Obtain the solution of $u_{n+2}-6u_{n+1}+8u_n=n$ with $u_0=0,\ u_1=0$ using Z-Transform.

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