Reg. No:

Final Assessment Test – November 2024

Course: BMAT201L - Complex Variables and Linear Algebra

Class NBR(s): 2483 / 2484 / 2485 / 2486 / 2487 / 2488 Slot: B2+TB2+TBB2

Time: Three Hours Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1.a) Show that $\varphi = x^2 - y^2 + \frac{x}{x^2+y^2}$ can represent the velocity potential in an incompressible fluid flow. Find the corresponding stream function ψ and hence the complex potential $f(z) = \varphi + i\psi$. [10]

OR

1.b) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$. [10]

2. Find the image in the w-plane of the region of the z-plane bounded by the lines $x = 0, y = 0, x + y = 1$ under the transformation [10]
(i) $w = 2z$. (ii) $w = ze^{i\pi/4}$.

3. Find the bilinear transformation that maps the points $z_1 = 0, z_2 = 1$ and $z_3 = \infty$ into the points $w_1 = -5,\ w_2 = -1, and\ w_3 = 3$ and also find its *invariant points.* [10]

4. *Find the Laurent's series expansion of the function* $f(z) = \frac{6z+5}{z(z+1)(z-2)}$ which are valid in the region $(i)\ 1 < |z+1| < 3 \quad (ii)\ |z+1| > 3$. [10]

5. Evaluate $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx$, by contour integration. [10]

6. Find the basis and dimension of column space and null space of [10]

$$A = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}.$$

7.a) Let $T : R^3 \to R^2$ be the linear transformation defined by $T(x;\ y;\ z) = (3x + 2y - 4z;\ x - 5y + 3z)$. Find $[T]_\alpha^\beta$, for $\alpha = \{(1,1,1), (1,1,0), (1,0,0)\}$ and $\beta = \{(1,3), (2,5)\}$. [10]

OR

7.b) Let $S, T : R^2 \to R^2$ be the linear transformation defined by $S(x,y) = (x + 3y, 2x)$ and $T(x,y) = (y, x + 2y)$. Find $[S + T]_\alpha$, $[2T - 3S]_\alpha$, for $\alpha = \{(1,1), (1,2)\}$. [10]

8. Apply the Gram–Schmidt orthogonalization process to find an orthogonal basis and then an orthonormal basis for the subspace U of R^3 spanned by $v_1 = (1, -1, -1),\ v_2 = (0, 2, -2,),\ v_3 = (2, 0, -2)$. [10]

9. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{pmatrix}$, hence find the eigen values of A^{-1}, A^T and A^4. **[10]**

10. Using Gauss-Jordan method, solve the system of equations **[10]**
$x + y + z - w = -2$, $2x - y + z + w = 0$, $3x + 2y - z - w = 1$,
$x + y + 3z - 3w = -8$.

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