

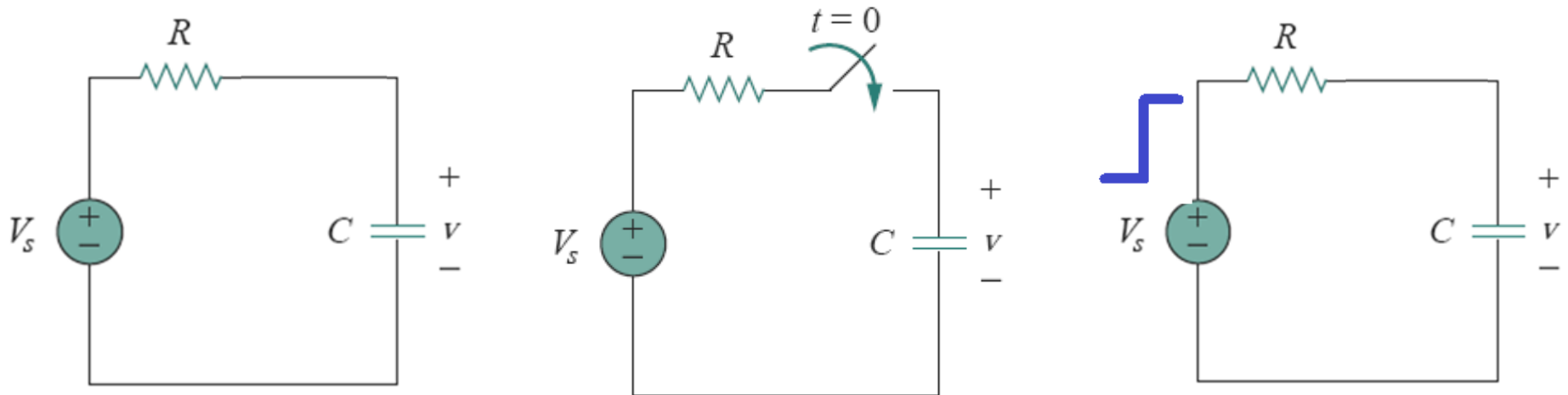
In general, it is necessary to analyse the complete response of the system to determine its performance, namely

- **Speed of response (transient performance)**
- **Error (steady state performance)**

for standard inputs.

However, the equations of the system may be integro-differential.

What is the difference between three circuits?



Time domain analysis

The dynamic behavior of a system is judged under application of standard test signals –

an impulse,

a step,

a constant velocity (Ramp), and

constant acceleration (Parabolic).

The output of the system for time domain signal (**time dependent signal**) is called as **time response**.

The **time response** of any system **has** two components: **transient response** and the **steady-state response**.

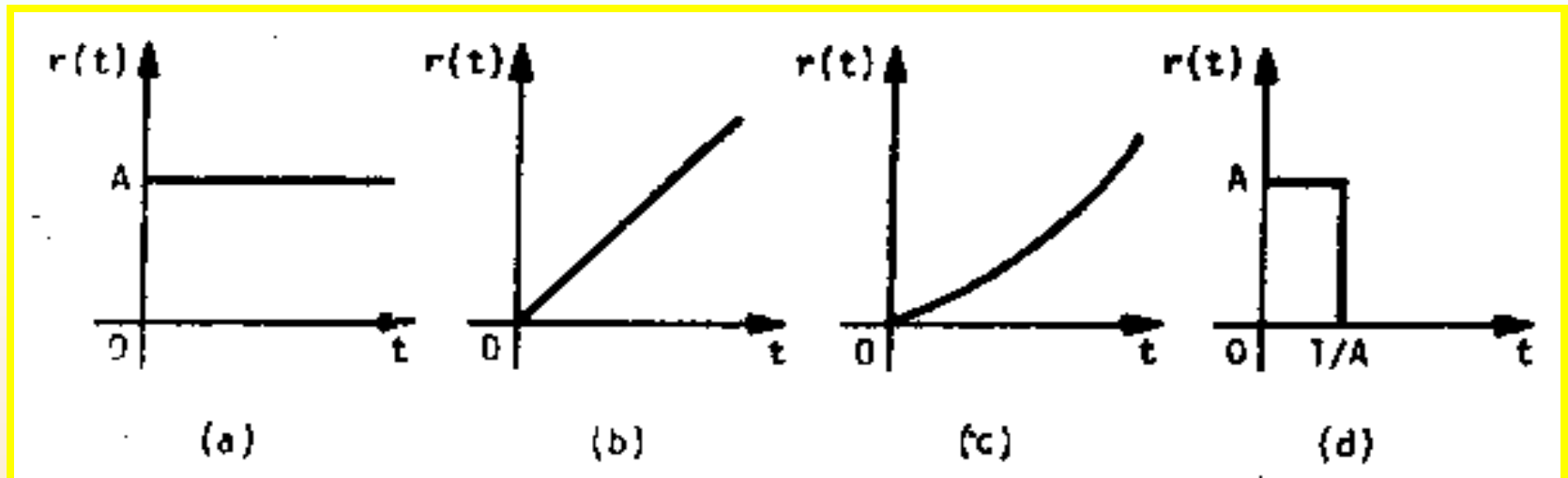
Graph of Standard test signals

a) Step signal: $r(t) = Au(t)$.

b) Ramp signal: $r(t) = At; \quad t > 0$.

c) Parabolic signal: $r(t) = At^2 / 2; \quad t > 0$.

d) Impulse signal: $r(t) = \delta(t)$.



Laplace transform, an analytical tool for analyzing electric circuits.

Advantages of LT:

1. It can be applied to circuits with sinusoidal *or* non-sinusoidal inputs.
2. It provides an easy way to solve circuit problems involving initial conditions, because it allows us to work with algebraic equations instead of differential equations.
3. Through Transfer Function, it is capable of providing the total response of the circuit comprising both the natural and forced responses.

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$

$f(t)$	$F(s)$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
$\cos(\omega t + \theta)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
$\frac{df}{dt}$	$sF(s) - f(0^-)$
$\int_0^t f(t) dt$	$\frac{1}{s} F(s)$

$$\frac{d^2 f}{dt^2}$$

$$s^2 F(s) - sf(0^-) - f'(0^-)$$

$$\frac{d^3 f}{dt^3}$$

$$s^3 F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$$

$$\frac{d^n f}{dt^n}$$

$$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \dots - f^{(n-1)}(0^-)$$

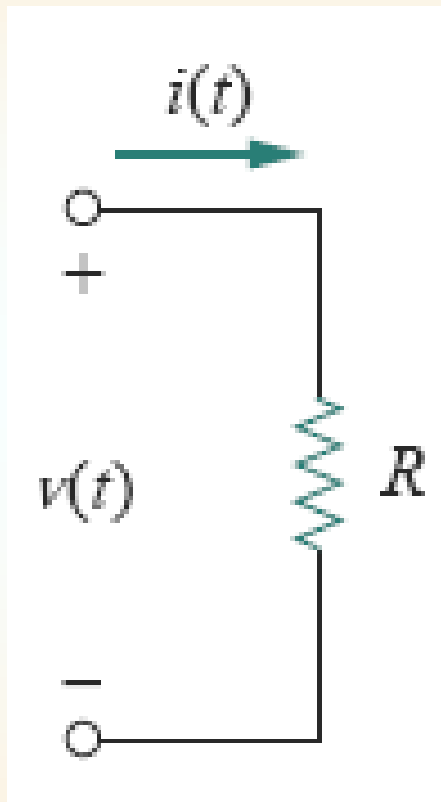
Steps in applying the Laplace transform:

1. Transform the circuit from the time domain to the s domain.
2. Solve the circuit using nodal analysis, mesh analysis, source transformation, superposition, or any circuit analysis technique with which we are familiar.
3. Take the inverse transform of the solution and thus obtain the solution in the time domain.

Transform the circuit from the time domain to s domain

Resistive circuit

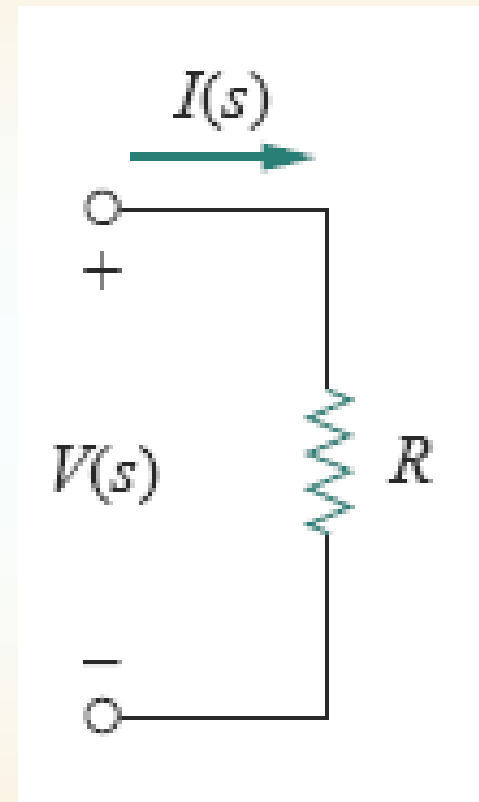
in time domain



impedance

$$\frac{V(s)}{I(s)} = Z(s) = R$$

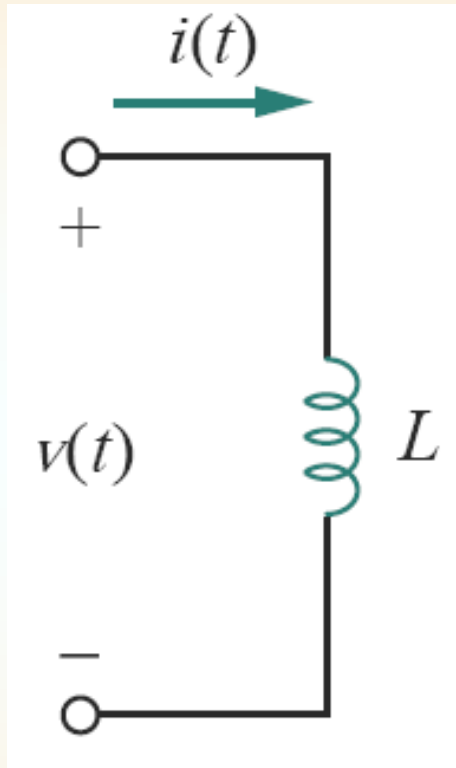
in frequency domain



$$v(t) = Ri(t) \Rightarrow L[v(t)] = L[Ri(t)] \Rightarrow V(s) = RI(s)$$

Inductive circuit

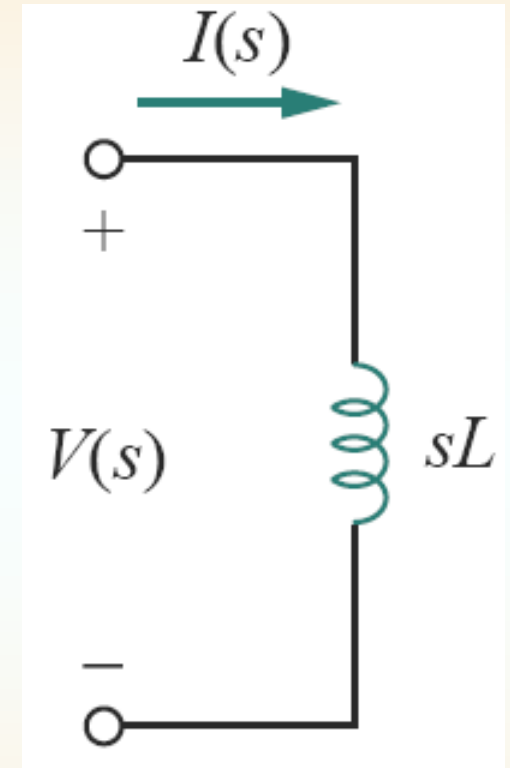
in time domain



impedance

$$\frac{V(s)}{I(s)} = Z(s) = sL$$

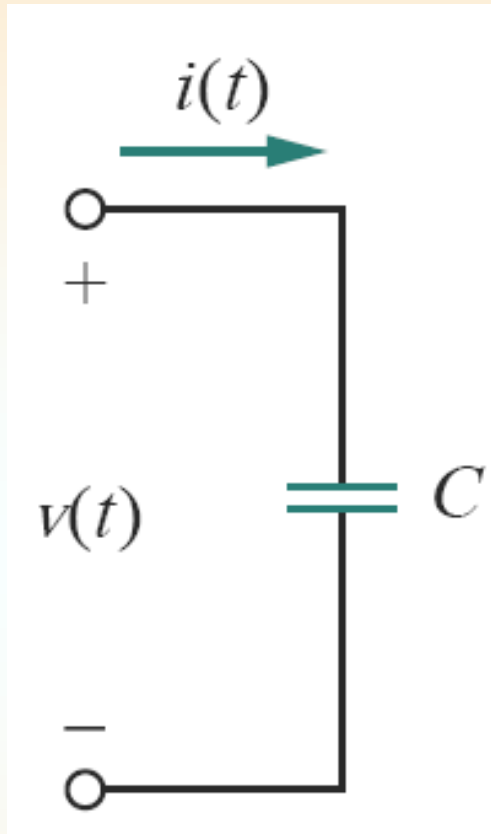
in frequency domain



$$v(t) = L \frac{di(t)}{dt} \quad \Rightarrow \quad \mathcal{L}[v(t)] = \mathcal{L}\left[L \frac{di(t)}{dt}\right] \quad \Rightarrow \quad V(s) = sLI(s)$$

Capacitive circuit

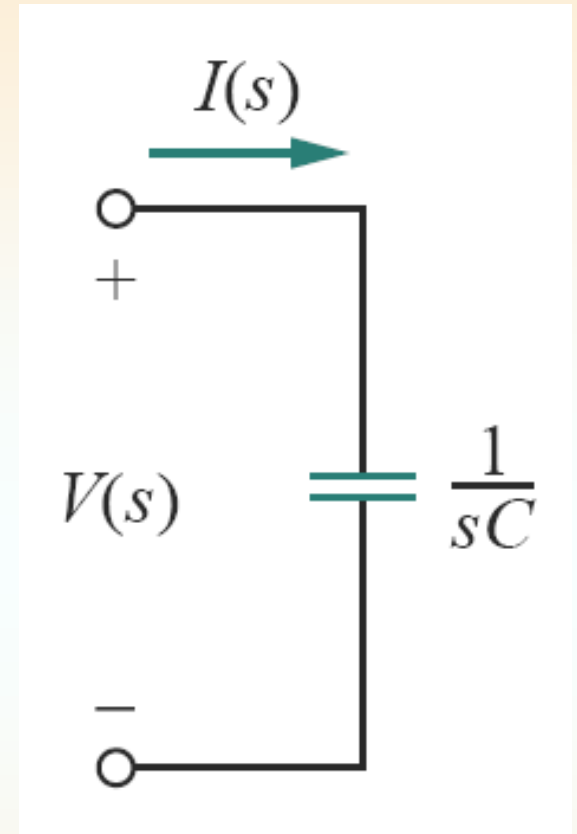
in time domain



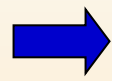
impedance

$$\frac{V(s)}{I(s)} = Z(s) = \frac{1}{sC}$$

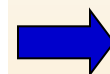
in frequency domain



$$i(t) = C \frac{dv(t)}{dt}$$



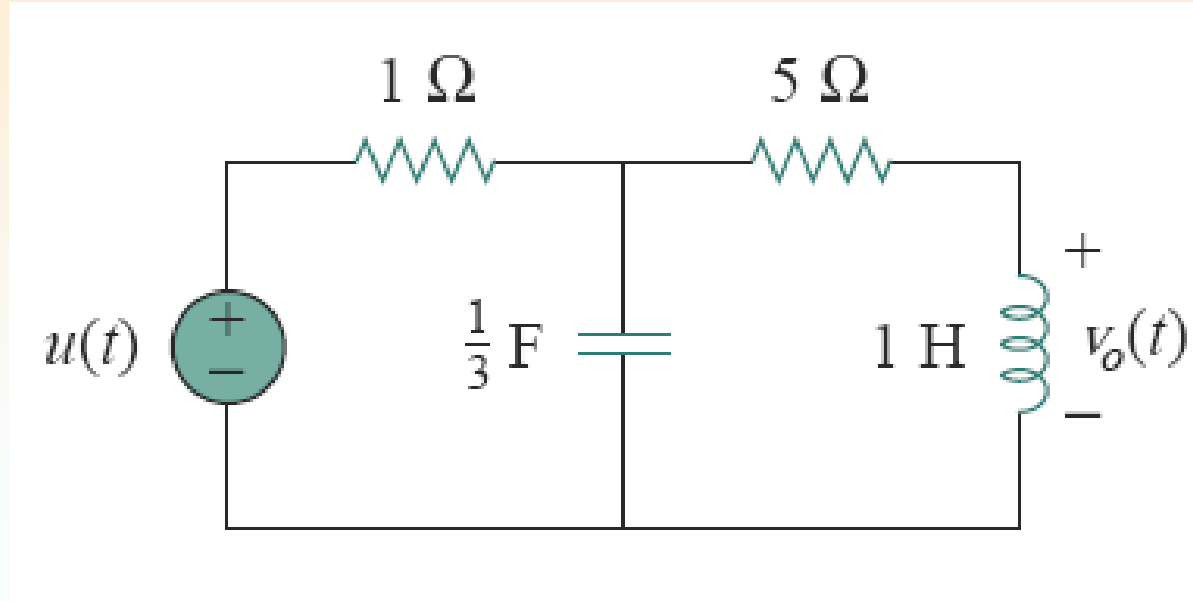
$$\mathcal{L}[i(t)] = \mathcal{L}\left[C \frac{dv(t)}{dt}\right]$$



$$I(s) = sC V(s)$$

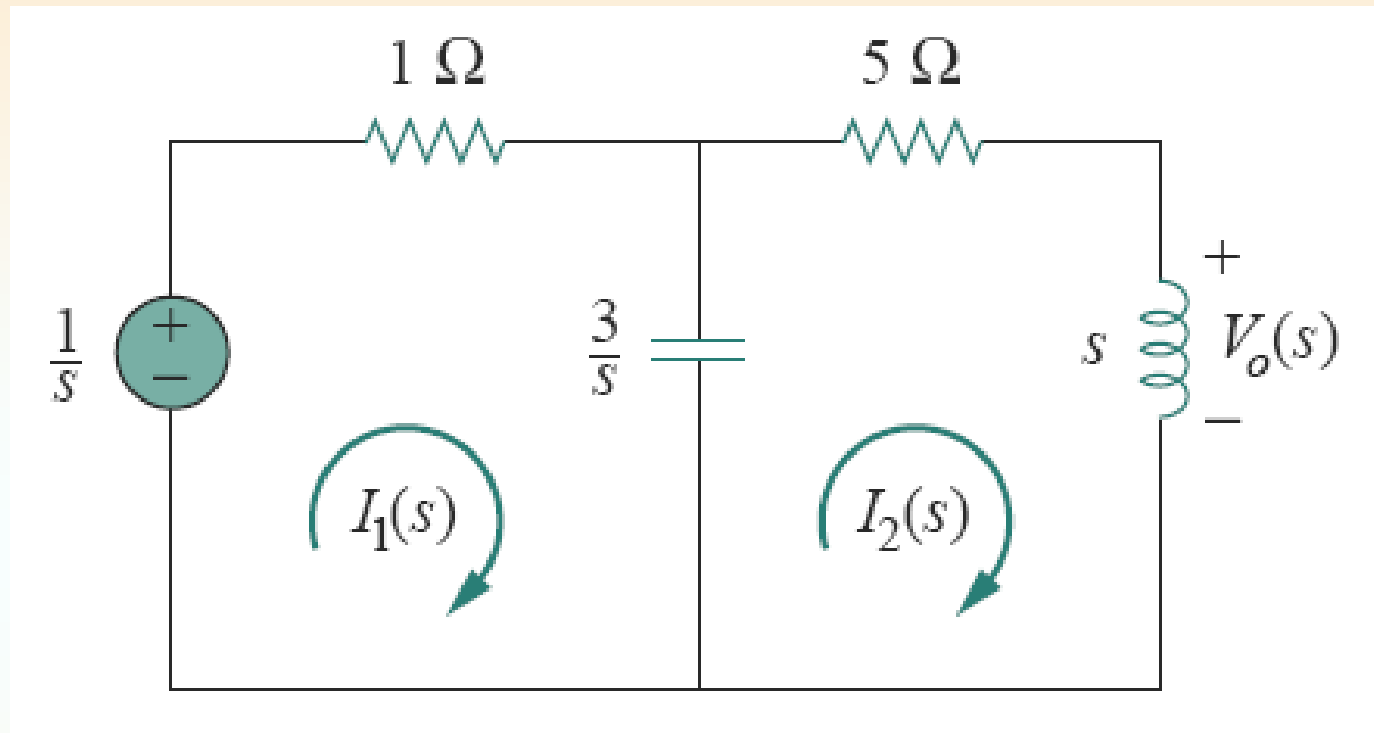
$$V(s) = \frac{1}{sC} I(s)$$

P.1: Find $v_o(t)$ in the circuit in Fig, assuming zero initial conditions.



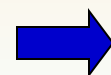
Applying laplace transform to the circuit elements

In frequency domain



$$\frac{1}{s} = \left(1 + \frac{3}{s}\right) I_1 - \frac{3}{s} I_2$$

$$0 = -\frac{3}{s} I_1 + \left(s + 5 + \frac{3}{s}\right) I_2$$



$$I_2 = \frac{3}{s^3 + 8s^2 + 18s}$$

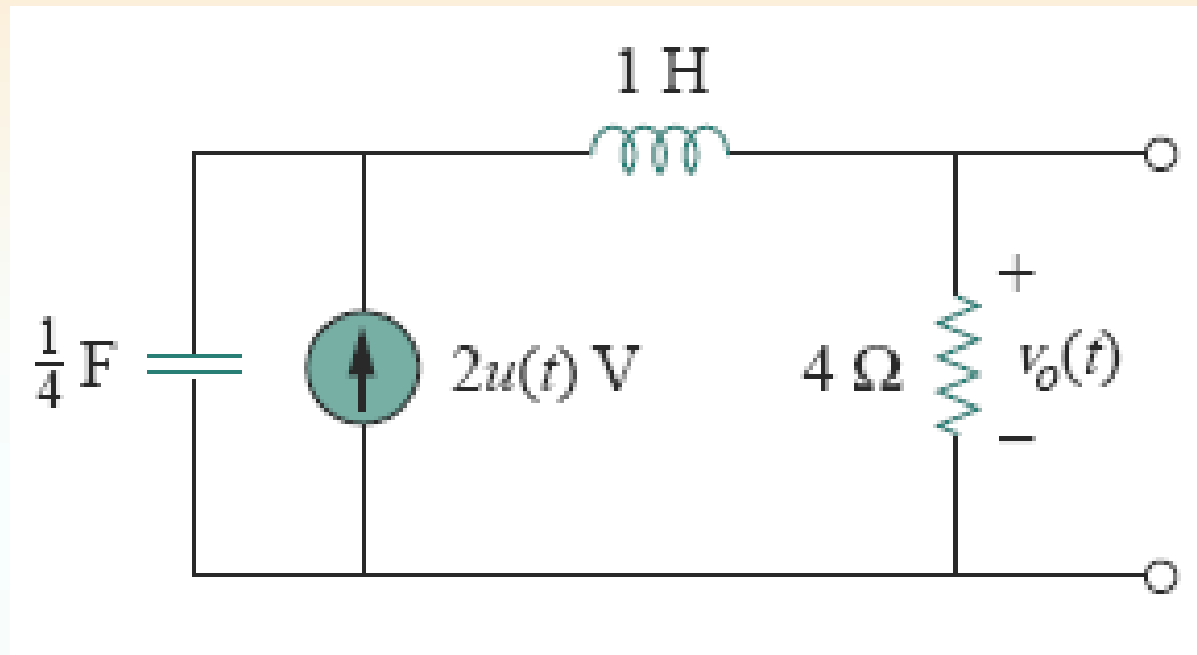
$$I_1 = \frac{1}{3}(s^2 + 5s + 3) I_2$$

$$V_o(s) = s I_2 = \frac{3}{s^2 + 8s + 18} = \frac{3}{\sqrt{2}} \frac{\sqrt{2}}{(s + 4)^2 + (\sqrt{2})^2}$$

Taking the inverse transform yields

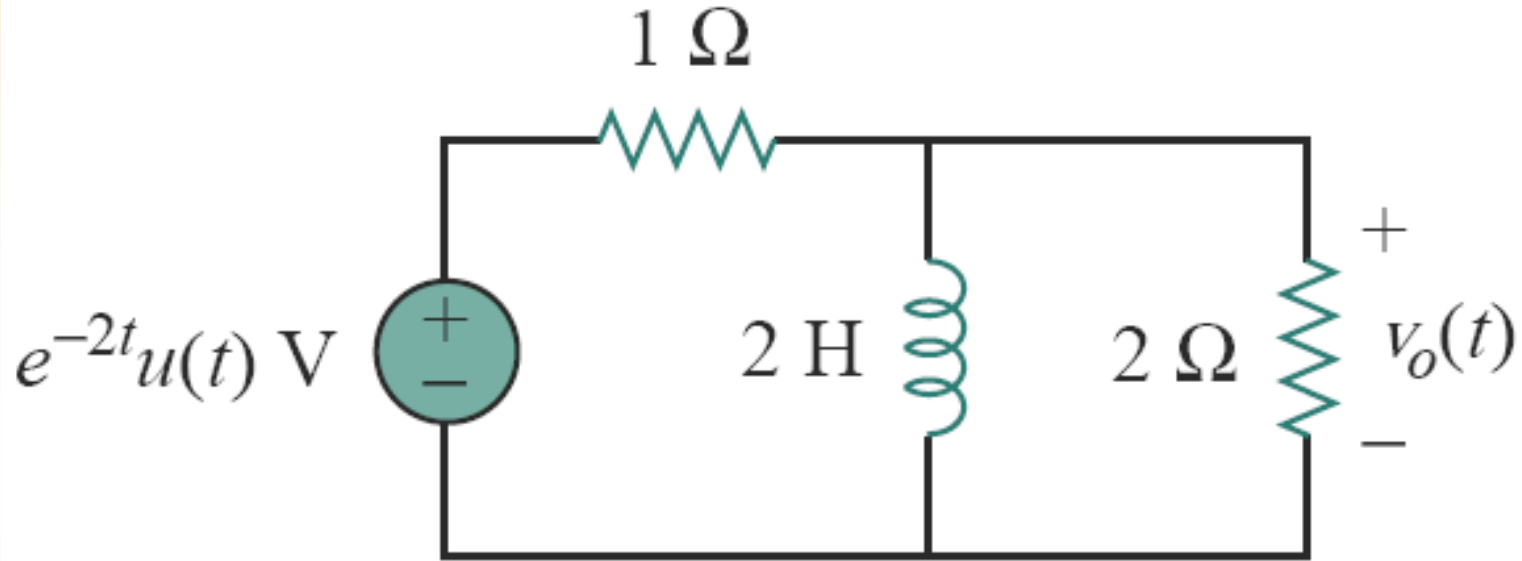
$$v_o(t) = \frac{3}{\sqrt{2}} e^{-4t} \sin \sqrt{2} t \text{ V}, \quad t \geq 0$$

P.2: Find $v_o(t)$ in the circuit in Fig, assuming zero initial conditions.



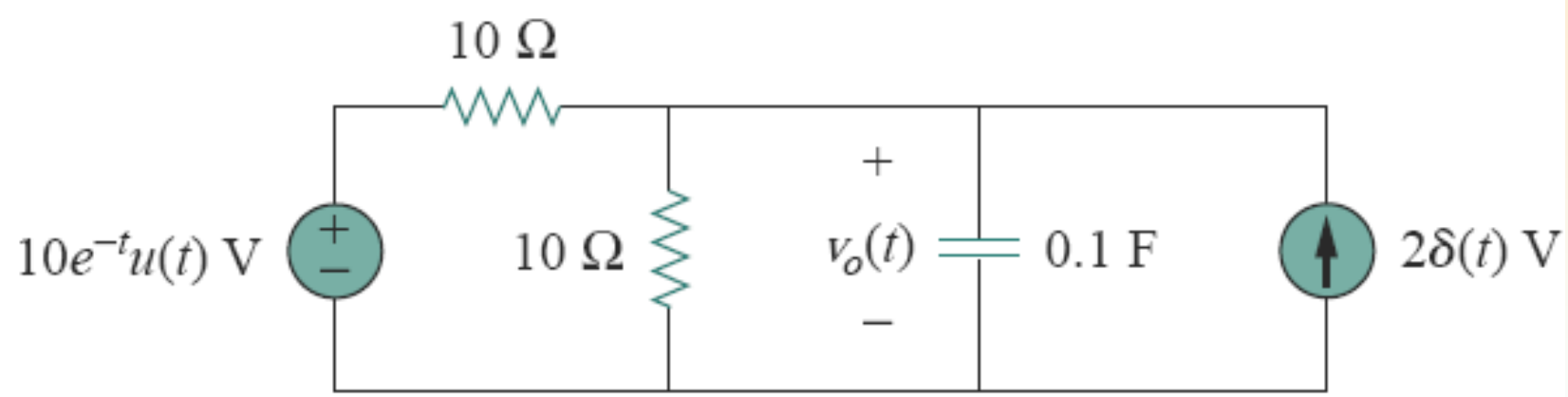
$$8(1 - e^{-2t} - 2te^{-2t})u(t) \text{ V.}$$

P.3: Find $v_o(t)$ in the circuit in Fig, assuming zero initial conditions.

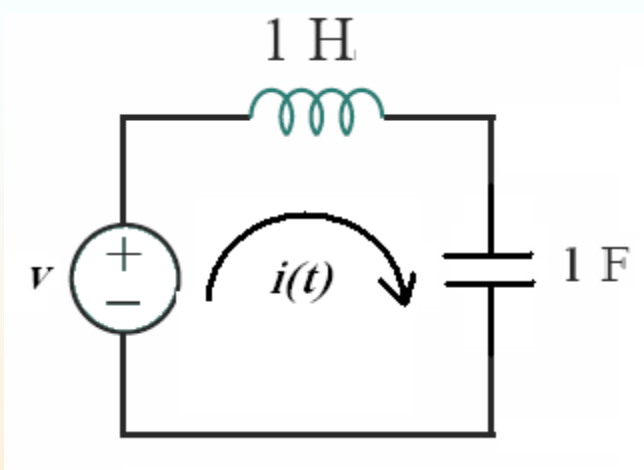


$$\left(\frac{4}{5}e^{-2t} + \frac{8}{15}e^{-t/3}\right)u(t).$$

P.4: Find $v_o(t)$ in the circuit in Fig, assuming zero initial conditions.



P.5: Find $i(t)$ in the circuit with $v = V_m\cos(t)$, assuming zero initial conditions.



$$I(s) = \frac{V_m s^2}{(s^2 + 1)^2} = \frac{V_m s^2}{(s + j)^2 (s - j)^2} = \frac{A}{(s + j)^2} + \frac{B}{(s - j)^2} + \frac{C}{(s + j)} + \frac{D}{(s - j)}$$

$$A = I(s)(s + j)^2 \Big|_{s = -j}$$

$$= \frac{V_m}{4}$$

$$B = A^* = \frac{V_m}{4}$$

$$C = \frac{d}{ds} [I(s)(s + j)^2] \Big|_{s = -j}$$

$$= j \frac{V_m}{4}$$

$$D = C^* = -j \frac{V_m}{4}$$

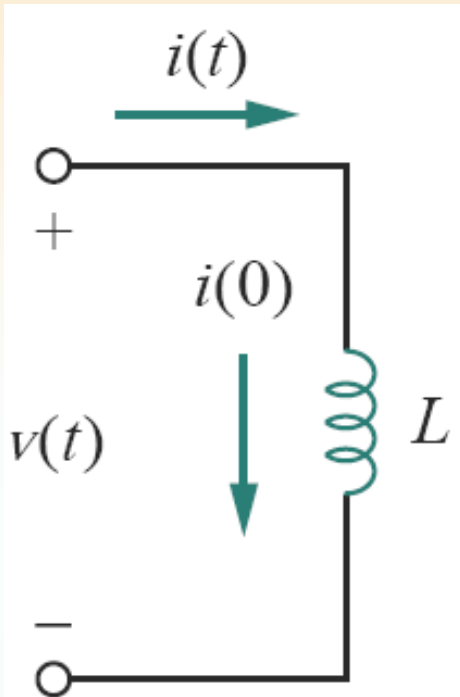
$$i(t) = \frac{V_m}{4} \left[2t \frac{(e^{jt} + e^{-jt})}{2} - j(2j) \frac{(e^{jt} - e^{-jt})}{2j} \right]$$

$$i(t) = \frac{V_m}{2} [t \cos t + \sin t]$$

With Initial conditions

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0^-)$$

in time domain

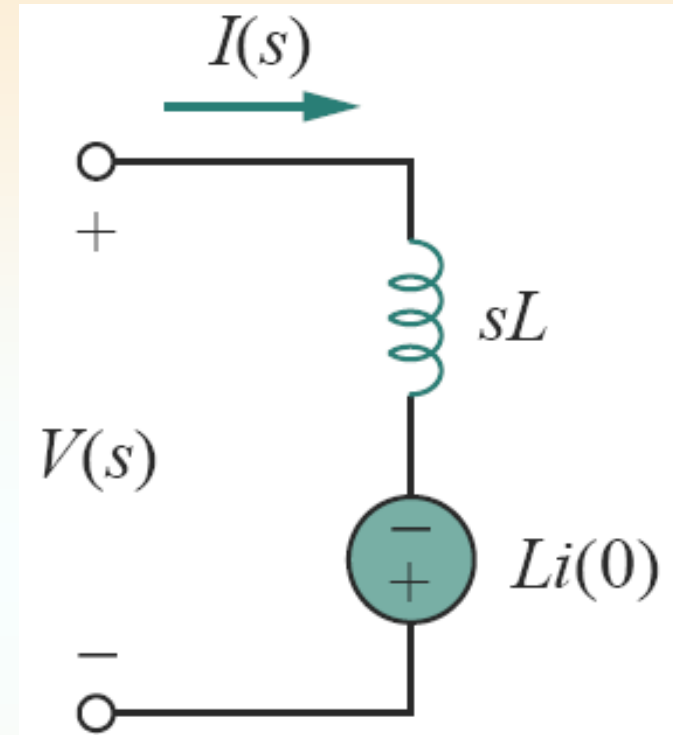


Inductive circuit with initial current

$$\mathcal{L}[v(t)] = \mathcal{L}\left[L \frac{di(t)}{dt}\right]$$

$$v(t) = L \frac{di(t)}{dt}$$

in frequency domain

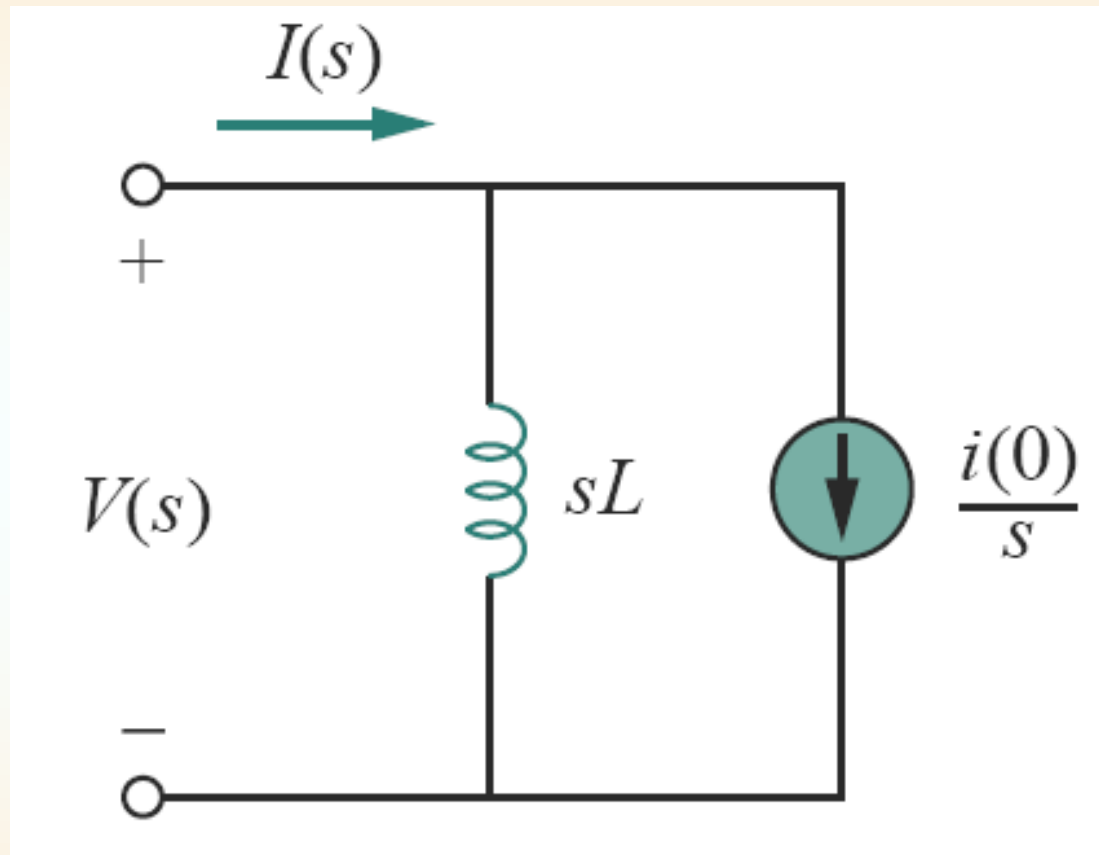


$$V(s) = sLI(s) - Li(0^-)$$

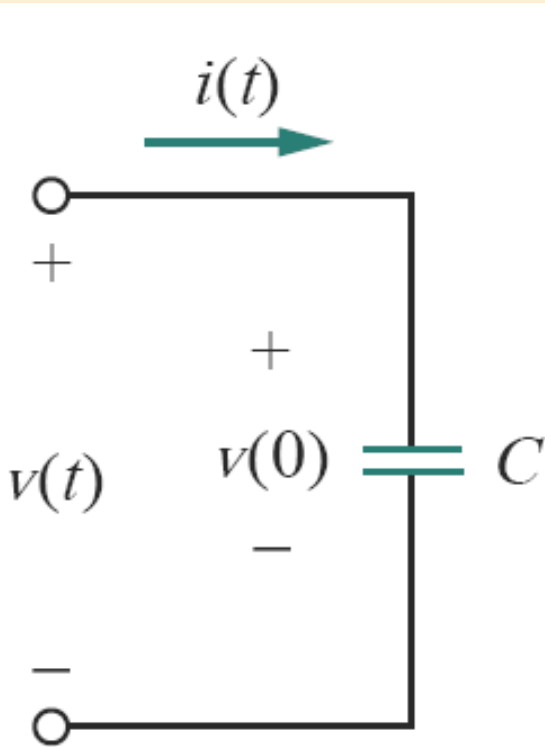
or

$$I(s) = \frac{1}{sL} V(s) + \frac{i(0^-)}{s}$$

Converting voltage to current

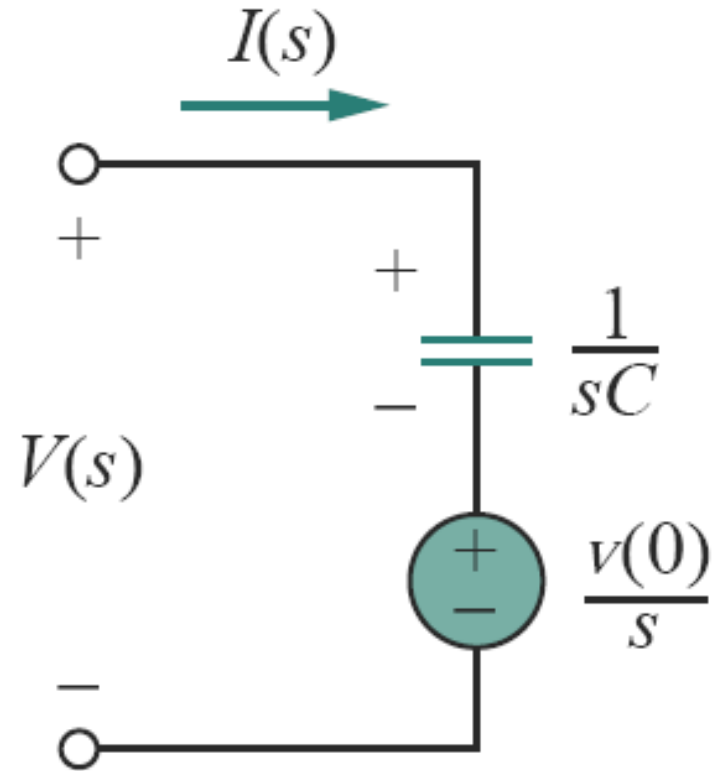


in time domain



Capacitive circuit with initial voltage

in frequency domain



$$\mathcal{L}[i(t)] = \mathcal{L}\left[C \frac{dv(t)}{dt}\right]$$

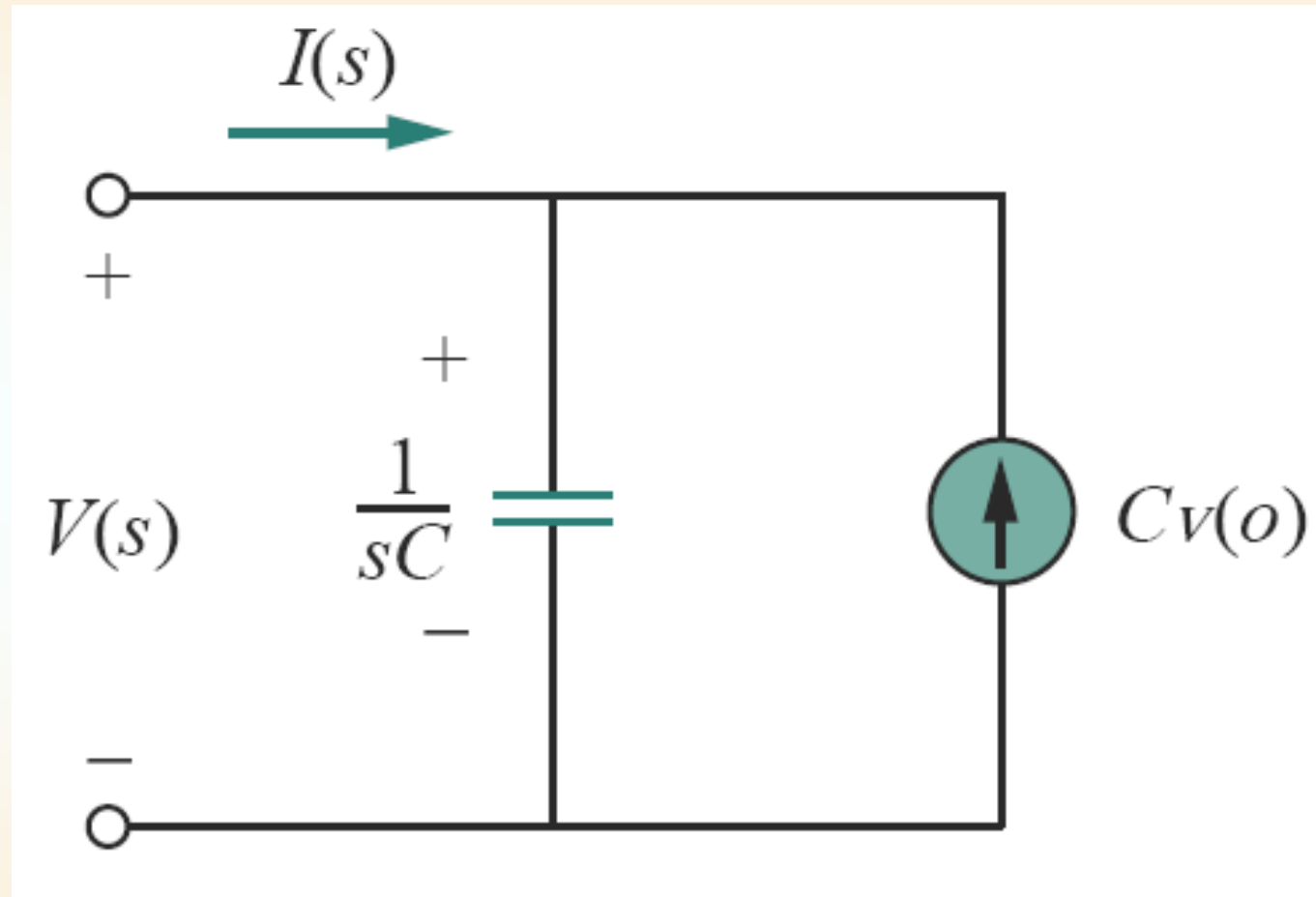
$$i(t) = C \frac{dv(t)}{dt}$$

$$I(s) = sC V(s) - C v(0^-)$$

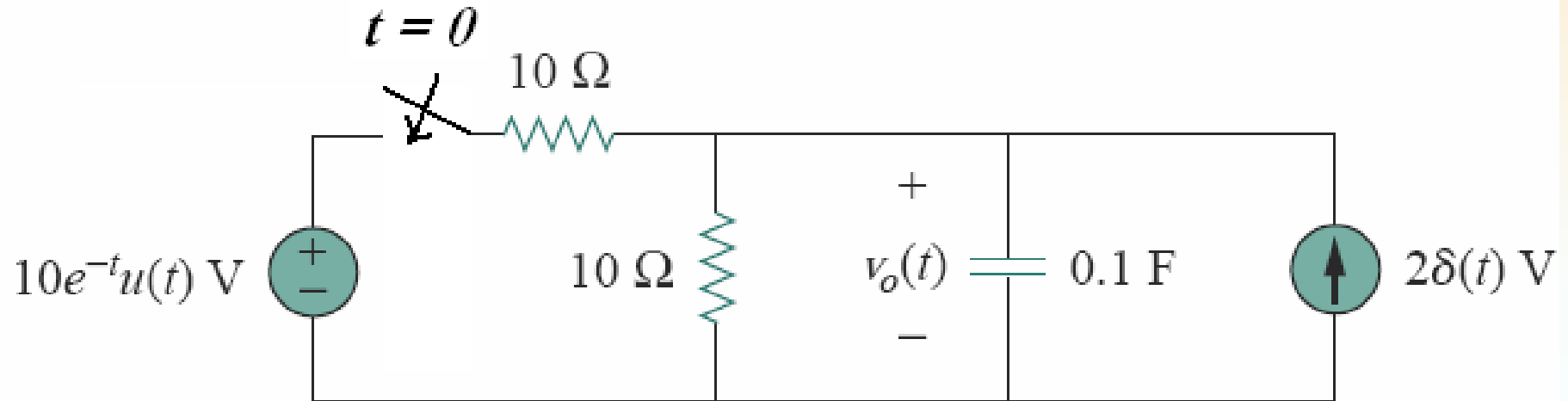
or

$$V(s) = \frac{1}{sC} I(s) + \frac{v(0^-)}{s}$$

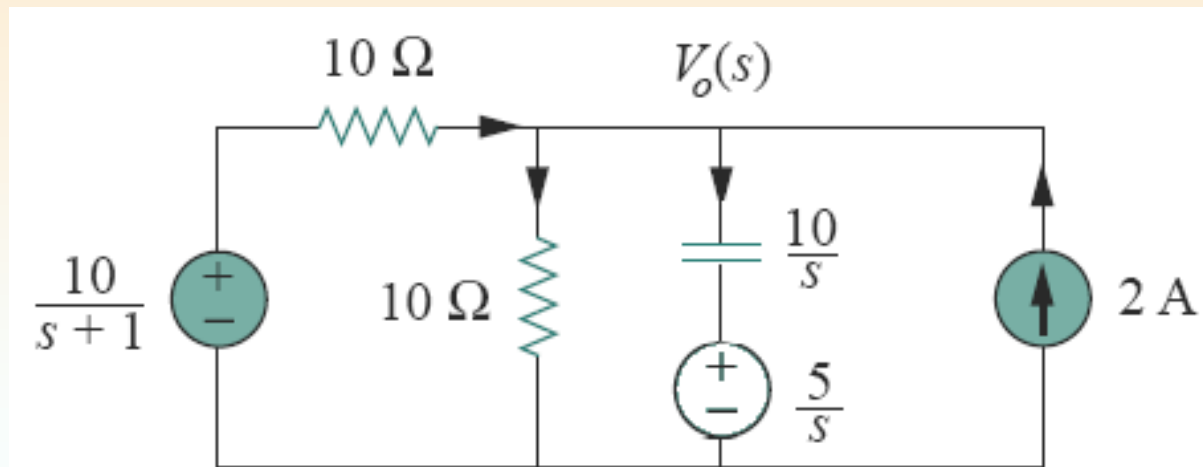
Converting voltage to current



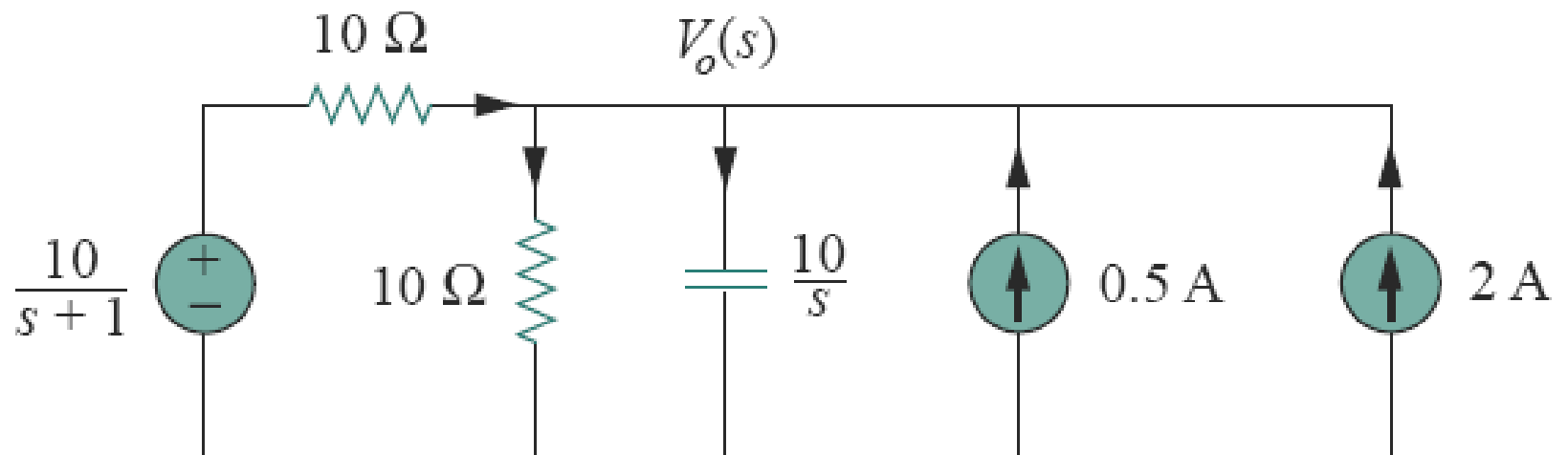
P.1: Find $v_o(t)$ in the circuit in Fig, assuming the switch is closed at $t = 0$ and $v_o(0) = 5$ V.



With capacitor initial voltage



Converting voltage to current



Nodal equation

$$\frac{10/(s+1) - V_o}{10} + 2 + 0.5 = \frac{V_o}{10} + \frac{V_o}{10/s}$$

$$V_o = \frac{25s + 35}{(s+1)(s+2)}$$

Taking partial fraction

$$V_o = \frac{25s + 35}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

where

$$A = (s + 1)V_o(s) \Big|_{s=-1} = \frac{25s + 35}{(s + 2)} \Big|_{s=-1} = \frac{10}{1} = 10$$

$$B = (s + 2)V_o(s) \Big|_{s=-2} = \frac{25s + 35}{(s + 1)} \Big|_{s=-2} = \frac{-15}{-1} = 15$$

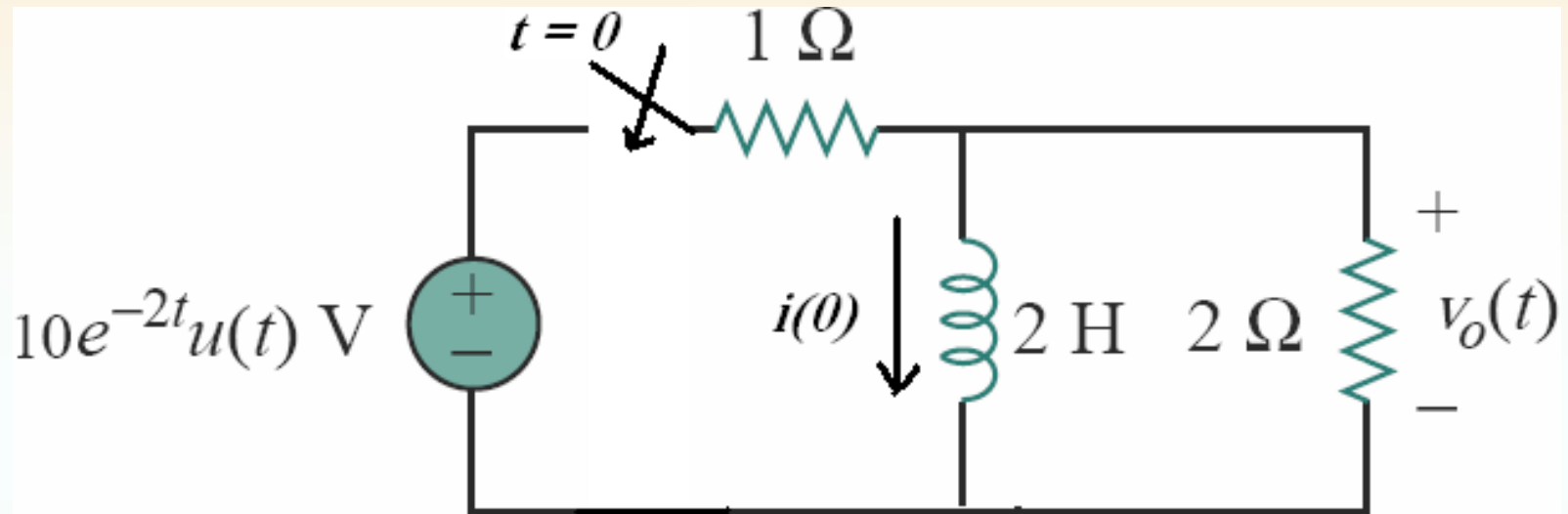
Thus,

$$V_o(s) = \frac{10}{s + 1} + \frac{15}{s + 2}$$

Taking the inverse Laplace transform, we obtain

$$v_o(t) = (10e^{-t} + 15e^{-2t})u(t)$$

P.2: Find $v_o(t)$ in the circuit in Fig, the switch is closed at $t=0$ and initial current through inductor is -2A .

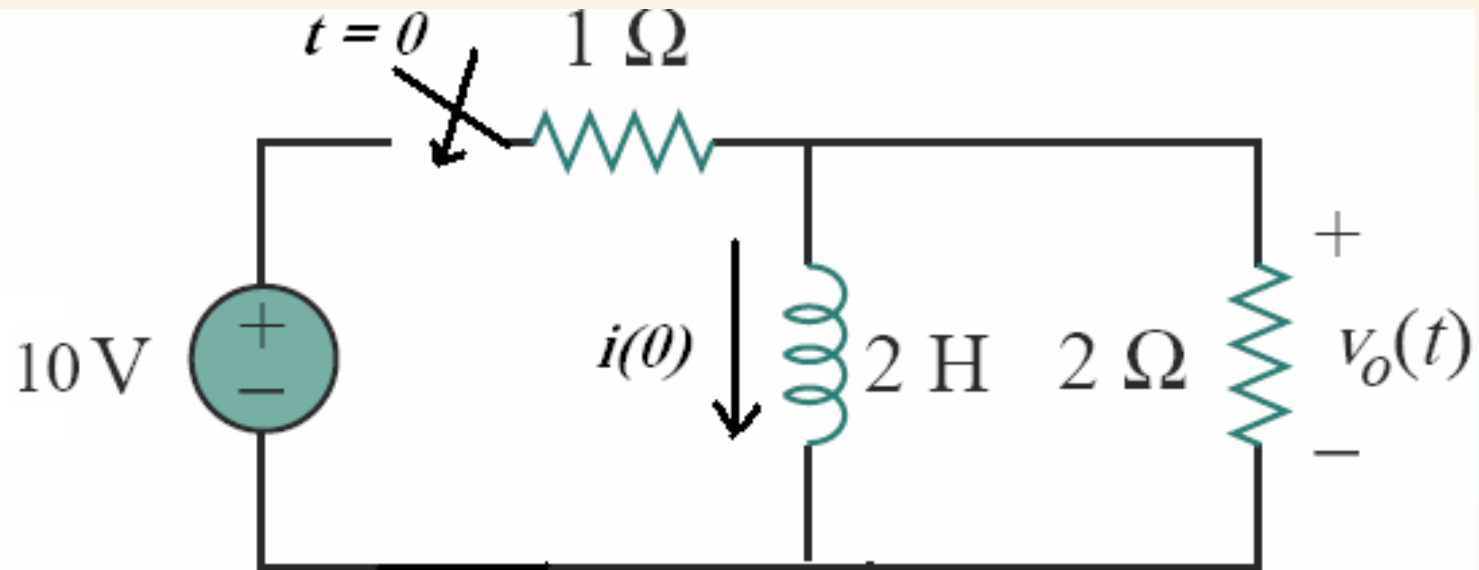


$$v(s) \left[1 + \frac{1}{2s} + \frac{1}{2} \right] = \frac{12s + 4}{s(s + 2)}$$

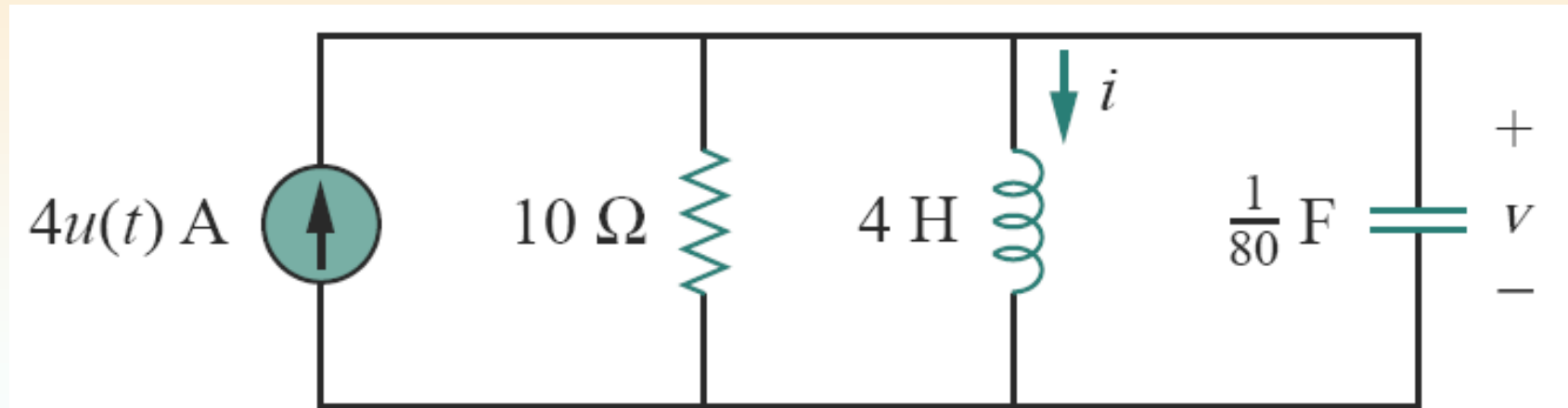
$$v(s) \left[\frac{3s + 1}{2s} \right] = \frac{12s + 4}{s(s + 2)}$$

$$v(s) = \frac{8}{(s + 2)}$$

P.3: Find $v_o(t)$ in the circuit in Fig, the switch is closed at $t=0$ and initial current through inductor is 4 A.



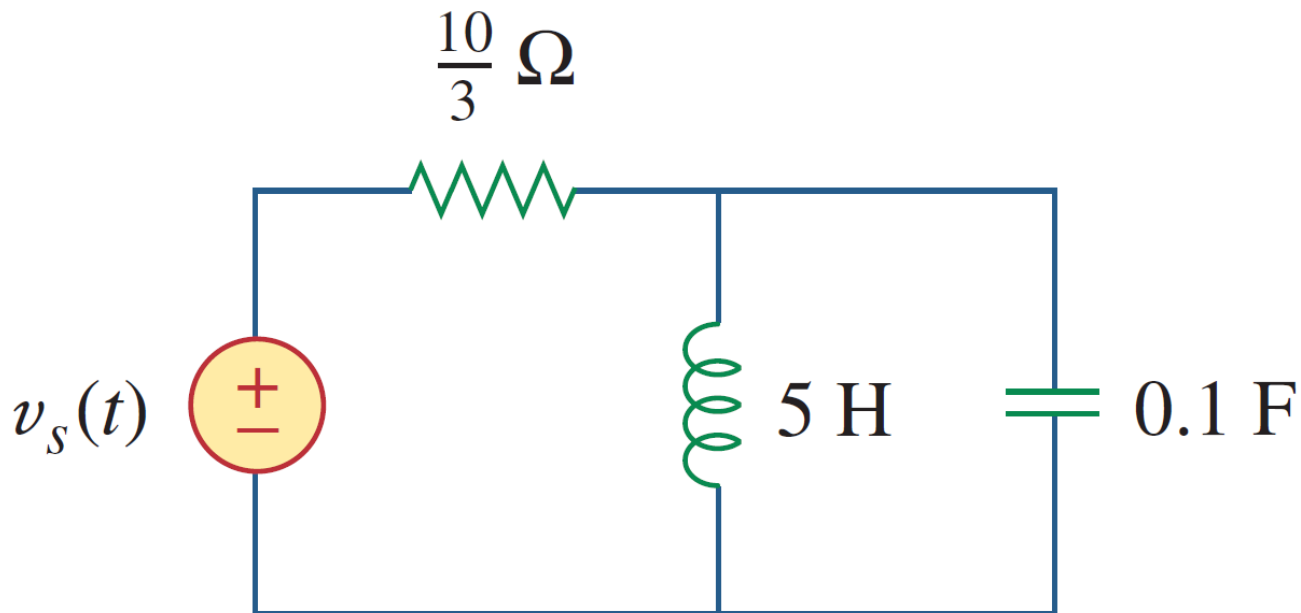
P.4: Find $v(t)$ and $i(t)$ given that $v(0) = 5$ and $i(0) = -2$ A.



$$(5e^{-4t} \cos 2t + 230e^{-4t} \sin 2t)u(t) \text{ V},$$
$$6u(t) - 6e^{-4t} \cos 2t - 11.37e^{-4t} \sin 2t \text{ A}, t > 0$$

Current through inductor alone

P.5: For the circuit, find the value of the voltage across the capacitor assuming that the value of $V_s(t)=10 u(t)$ V and assume that at $t=0$, -1 A flows through the inductor and $+5$ V is across the capacitor.



$$v_1(t) = (35e^{-t} - 30e^{-2t})u(t) \text{ V}$$