

SCHOOL OF ADVANCED SCIENCES

CONTINUOUS ASSESSMENT TEST - I

Fall SEMESTER 2022-2023

Programme Name & Branch: B.Tech

Course Code: BMAT101L

Course Name: Calculus

Faculty Name : Dr. K.Indhira

Class Number : VL2022230105487

Exam Duration: 90 minutes

Maximum Marks: 50

Answer ALL Questions ($5 \times 10 = 50$)

Sl.No.	Question
1.	Suppose the temperature (in degrees Celsius) at a certain location t hours afternoon on a certain day is $\theta = 13 - 3t^2 + 8t + 10$ for all $t \in [0, 5]$. Find the instances of times where the temperature θ is stationary within the interval. Use this information to find the absolute maximum and the absolute minimum temperatures in $[0, 5]$. Also, separate the time intervals in which the graph of θ is concave up and in which the graph of θ is concave down, to obtain the point of inflection.
2.	Justify that the function $f(x) = x^3 - 4x + 1$ satisfies the conditions of Lagrange's mean value theorem on the closed interval $[0, 3]$. To find the constant $c \in (0, 3)$ such that $3f'(c) = f(3) - f(0)$. Also obtain the equation of the chord of the curve $y = f(x)$ joining the points with abscissa $x = 0$ and $x = 3$. What are the absolute extrema of f on $[0, 3]$?
3.	(i) Find the area of the region enclosed by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ in the first quadrant. (5M) (ii) Find the volume of the solid obtained by rotating the region bounded by $y = x^2 - 2x$ and $y = x$ about the line $y = 4$. (5M)
4.	(i) If $w = xe^{y/z}$, $x = t^2$, $y = 1 - t$, $z = 1 + 2t$, then find $\frac{dw}{dt}$ (5M) (ii) If $H = f(x - y, y - z, z - x)$, then find the value of $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z}$ (5M)
5.	Show that $u = xe^y \sin z$, $v = xe^y \cos z$, $w = x^2 e^{2y}$ are functionally dependent. What is the functional relationship between them?