2-Link planar Robot

Inverse Kinematics

(To find θ_2)

Orientation.

$$
= \begin{array}{c}
\left[\begin{array}{ccc|c}
C\theta_1\theta_2 & -S\theta_1\theta_2 & 0 & l_1 C\theta_1 + l_2 \cos\theta_1\theta_2 \\
S\theta_1\theta_2 & C\theta_1\theta_2 & 0 & l_1 S\theta_1 + l_2 S\theta_1\theta_2 \\
0 & 0 & 1 & 0 \\
\hline
0 & 0 & 0 & 1
\end{array}\right]
\end{array}
\begin{array}{l} P_x \; [\text{Position}] \\ P_y \\ P_z \\ \\ \end{array}
$$

Here, $C\theta_1\theta_2 = \cos(\theta_1+\theta_2)$

$S\theta_1\theta_2 = \sin(\theta_1+\theta_2)$

The above matrix is transformation matrix of 2-link planar. ~~from the above matrix~~ The below equation show's the position of x and y from the above matrix.

$$X = l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \quad \text{——— (1)}$$

$$Y = l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \quad \text{——— (2)}$$

Squaring on both side of eq. (1) and (2), we get

$$X^2 = \left[l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2)\right]^2$$

$$Y^2 = \left[l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2)\right]^2 \qquad [\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$X^2 = l_1^2 \cos^2\theta_1 + l_2^2 \cos^2(\theta_1+\theta_2) + 2 l_1 l_2 \cos\theta_1 \cdot \cos(\theta_1+\theta_2) \quad (3)$$

$$Y^2 = l_1^2 \sin^2\theta_1 + l_2^2 \sin^2(\theta_1+\theta_2) + 2 l_1 l_2 \sin\theta_1 \cdot \sin(\theta_1+\theta_2) \quad (4)$$

Add eq. (1) and (2)

$$X^2+Y^2 = \underline{l_1^2 \cos^2\theta_1} + \underline{l_2^2 \cos^2(\theta_1+\theta_2)} + 2 l_1 l_2 \cos\theta_1 \cdot \cos(\theta_1+\theta_2)$$
$$+ \underline{l_1^2 \sin^2\theta_1} + \underline{l_2^2 \sin^2(\theta_1+\theta_2)} + 2 l_1 l_2 \sin\theta_1 \cdot \sin(\theta_1+\theta_2)$$

$$= l_1^2 [\cos^2\theta_1 + \sin^2\theta_1] + l_2^2 [\cos^2(\theta_1+\theta_2) + \sin^2(\theta_1+\theta_2)]$$
$$+ 2 l_1 l_2 [\cos\theta_1 \cdot \underbrace{\cos(\theta_1+\theta_2)}_{\cos(A+B)}] + 2 l_1 l_2 [\sin\theta_1 \cdot \underbrace{\sin(\theta_1+\theta_2)}_{\sin(A+B)}]$$

$$\boxed{\begin{array}{l} \because \cos^2\theta + \sin^2\theta = 1 \\ \cos(A+B) = \cos A \cos B - \sin A \sin B \\ \sin(A+B) = \sin A \cos B + \cos A \sin B \end{array}}$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 [\cos\theta_1 (\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2)]$$
$$+ 2 l_1 l_2 [\sin\theta_1 (\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2)]$$

Take $2 l_1 l_2$ as common.

$$= l_1^2 + l_2^2 + 2 l_1 l_2 [\cos^2\theta_1 \cos\theta_2 - \cos\theta_1 \cancel{\sin\theta_1} \sin\theta_2 + \sin^2\theta_1 \cos\theta_2$$
$$+ \sin\theta_1 \cancel{\cos\theta_1} \sin\theta_2]$$

$$= l_1^2 + l_2^2 + 2l_1l_2\left[\cos\theta_2\left[\cos^2\theta_1 + \sin^2\theta_1\right]\right]$$

$$\boxed{x^2+y^2 = l_1^2 + l_2^2 + 2l_1l_2\cos\theta_2} \quad \text{— (5)}$$

from, eq (5)

$$\theta_2 = \cos^{-1}\left[\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right] \quad \text{— (6)}$$

We usally avoid using arcsin and arcCos because of inaccuracy. So, we employ the half angle formula.

$$\tan^2\left(\frac{\theta_2}{2}\right) = \frac{\sin^2(\theta/2)}{\cos^2(\theta/2)}$$

$$\left[\because \sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta_2}{2}} \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}\right]$$

$$= \frac{\frac{1-\cos\theta_2}{\cancel{2}}}{\frac{1+\cos\theta_2}{\cancel{2}}} = \frac{1-\cos\theta_2}{1+\cos\theta_2}$$

Sub θ_2 from eq (6)

$$\tan^2\left(\frac{\theta_2}{2}\right) = \frac{1-\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)}{1+\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)}$$

$$\tan^2\left(\frac{\theta_2}{2}\right) = \frac{\dfrac{2l_1l_2 - (x^2+y^2-l_1^2-l_2^2)}{2\cancel{l_1l_2}}}{\dfrac{2l_1l_2 + (x^2+y^2-l_1^2-l_2^2)}{2\cancel{l_1l_2}}}$$

$$\cancel{\tan^2\left(\frac{\theta_2}{2}\right)} = \cancel{((2l_1l_2)-x^2+y^2)+l_1^2+l_2^2)}$$

$$\tan^2\left(\frac{\theta_2}{2}\right) = \frac{2l_1l_2 - x^2 - y^2 + l_1^2 + l_2^2}{2l_1l_2 + x^2 + y^2 - l_1^2 - l_2^2}$$

$$= \frac{l_1^2 + l_2^2 + 2l_1l_2 - (x^2+y^2)}{-(l_1^2+l_2^2-2l_1l_2) + (x^2+y^2)}$$

$[(a+b)^2$ and $(a-b)^2]$

$$\tan^2\left(\frac{\theta_2}{2}\right) = \frac{(l_1+l_2)^2 - (x^2+y^2)}{(x^2+y^2) - (l_1-l_2)^2}$$

$$\boxed{\theta_2 = \pm 2\,\mathrm{atan2}\sqrt{\frac{(l_1+l_2)^2 - (x^2+y^2)}{(x^2+y^2) - (l_1-l_2)^2}}}$$

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