VIT

Vellore Institute of Technology

Regular Arrear Examination – Apr 2022

Course : BMAT101L - Calculus

Time: Three Hours Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE

Answer any FIVE Questions

(5 X 20 = 100 Marks)

1. a) Suppose the temperature (in degrees (Celsius) at a certain location t hours afternoon on a certain day is $\theta = \frac{1}{3}t^3 - 3t^2 + 8t + 10$ for all $t \in [0,5]$. Find the instances of times where the temperature θ is stationary within the interval. Use this information to find the absolute maximum and the absolute minimum temperatures in $[0,5]$. Also, separate the time intervals in which the graph of θ is concave up and in which the graph of θ is concave down, to obtain the point of inflection. [10]

   b) Find the area between the curves $x = 3y - y^2$ and $x + y = 3$ above the x-axis. If the same area is revolved about x-axis find the volume of the solid of revolution [10]

2. a) (i) Find whether the functions $u = x + 2y + z$, $v = x - 2y + 3z$, $w = 2xy - xz + 4yz - 2z^2$ are functionally related. If so find the relation between them. [10]

   (ii) If $u = \log(x^3 + y^3 - x^2y - xy^2)$ then find $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2}$

   b) (i) Show that the functions $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$ and $w = x + y + z$ are functionally related. [10]

   (ii) Calculate $\frac{\partial f}{\partial u}$ when $f(x,y) = x^2 + xy, x(u,v) = u^2v, \; y(u,v) = uv^2$.

3. a) Find the quadratic approximation for $f(x,y) = e^x \sin y$ at $(0,\frac{\pi}{2})$ as a Taylor's series. [10]

   b) Find the maxima of $f(x,y,z) = xy^2z^3$ subject to the condition $x + y + z = 6, x > 0, y > 0, z > 0$, by using Lagrange multiplier method. [10]

4. a) Change the order of integration $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\,dy\,dx$ and hence evaluate it. [10]

   b) Evaluate $\iiint_D xyz\,dx\,dy\,dz$ by changing into cylindrical polar coordinates. Where D is the domain bounded by the planes $x = 0, y = 0, z = 0, z = 1$ and the cylinder $x^2 + y^2 = 1$. [10]

5. a) Using Beta and Gamma function, evaluate [10]

   $\iint [xy(1 - x - y)]^{1/2}\,dx\,dy$, over the area enclosed by the lines $x = 0, y = 0$ and $x + y = 1$ in the positive quadrant

   b) Using Beta and Gamma function, evaluate $\iint x^m y^n\,dx\,dy$, taken over the area $x \geq 0, y \geq 0, x + y \leq 1$, if $m, n > 0$ [10]

6. a) (i) Find the directional derivative of $\emptyset = 2xy + z^2$ at $(1,-1,3)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$. [10]

   (ii) Find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$ where $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$

   b) (i) Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential. [10]

   (ii) Find $div\,curl\vec{F}$ where $\vec{F} = x^2y\vec{i} + xz\vec{j} + 2yz\vec{k}$

7. a) Evaluate $\iint_S \vec{F} \cdot \vec{n}\, ds$ where $\vec{F} = yz\vec{i} + 2y^2\vec{j} + xz^2\vec{k}$ and S is the surface of the cylinder $x^2 + y^2 = 9$ contained in the first octant between the planes $z = 0$ and $z = 2$ [10]

b) Verify Green's theorem in the plane for $\int_C (x^2dx + xydy)$, where C is the curve in the XY plane given by [10]

$x = 0, y = 0, x = a, y = a.\ (a > 0)$

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