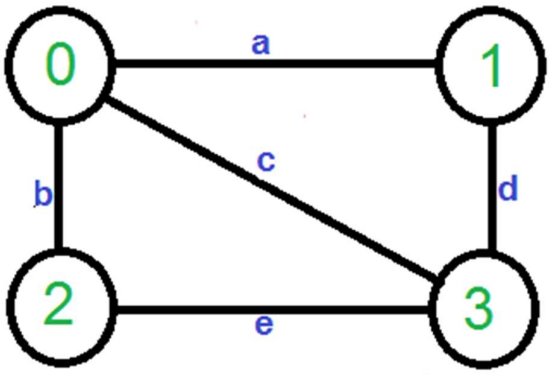
Randomized Global Minimum Cut

- Given a connected graph G = (V, E), compute the smallest set of edges that will make G disconnected.
- That is, find the smallest cut (smallest number of edges that disconnects the graph into two components). The input graph may have parallel edges.
- Minimum cut or min-cut of a graph is a cut that partitions the vertices V of a graph into two disjoint subsets V1 and V2 (no common vertices in V1 and V2) such that V1 U V2 = G

Example:

Consider the following graph, the smallest cut has 2 edges.



Min-Cut for above graph is either **{a, d}** OR **{b, e}**

Karger's randomized min-cut algorithm:

1) Initialize V with set of vertices and E with set of edges in the given graph
2) While there are more than 2 vertices in V.
    a) Pick a random edge (u, v) from E.
    b) Merge u and v into a single vertex and update V.
    c) Remove self-loops from E
3) Return E

V = {0, 1, 2, 3}; E = {a, b, c, d, e)

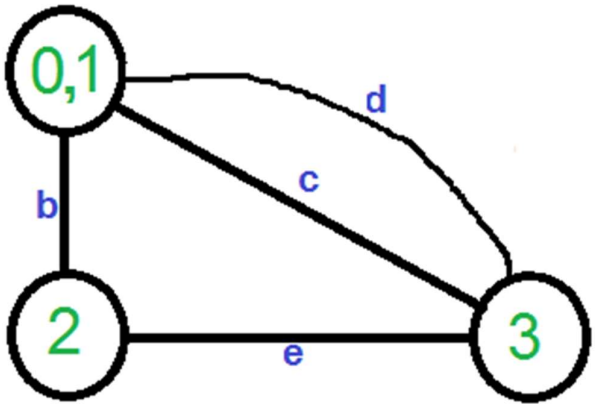
Iteration - 1:

V has more than 2 vertices. While loop condition – True

Let the randomly picked edge be a. It connects vertices 0 and 1. Remove this edge from E.

Merge the vertices 0 and 1 into single vertex.

So that, we get the following graph.



There are no self-loops to remove. Therefore, V = {01, 2, 3} and E = {b, c, d, e)

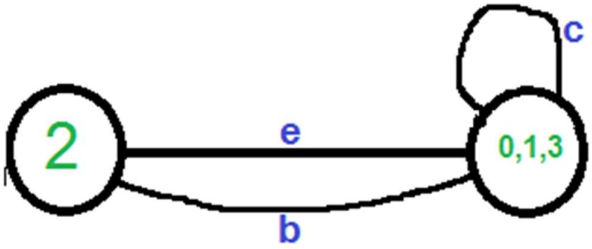
Iteration - 2:

V has more than 2 vertices. While loop condition – True

Let the next randomly picked edge be d. It connects vertices 01 and 3. Remove this edge from E.

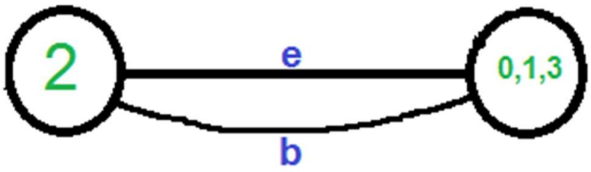
Merge the vertices 01 and 3 into single vertex.

So that, we get the following graph.



There is self-loop. We need to remove self-loops. So, we remove edge ‘c’.

So that, we get the following graph.



Therefore, V = {013, 2} and E = {b, e)

Now again control moves to while loop but condition fails as graph has only two vertices. So, we stop.

Therefore, min-cut for the given graph is {b, e} whose size = 2.

Note: Karger's algorithm is a Monte Carlo algorithm and cut produced by it may not be minimum.

For example, the following diagram shows that a different order of picking random edges produces a min-cut of size 3.