



VIT¹⁰

Vellore Institute of Technology
(Deemed to be University under Section 3 of U.O. 1987)

Final Assessment Test – November 2024

Course: BMAT205L - Discrete Mathematics and Graph Theory

Class NBR(s): 2538 / 2559 / 2562 / 2563 / 2564 / 2565 /

2567 / 2569 / 2570 / 2571 / 2572 / 2573 / 2576 / 2579 /

2581 / 2583 / 2586

Slot: C2+TC2+TCC2

Time: Three Hours

Max. Marks: 100

- KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION IS TREATED AS EXAM MALPRACTICE
- DON'T WRITE ANYTHING ON THE QUESTION PAPER

Answer ALL Questions

(10 X 10 = 100 Marks)

1. a) Construct the truth table for

$$\neg(P \vee (Q \wedge R)) \leftrightarrow (P \vee Q) \wedge (P \rightarrow R)$$

[10]

- b) Find the Principal Conjunctive Normal Form of

$$(P \wedge Q) \vee (\neg P \wedge Q) \vee (Q \wedge R) \text{ without using the truth table.}$$

2. Prove or disprove the validity of following argument:

[10]

"Babies are illogical", "Nobody is despised who can manage crocodiles",
"Illogical people are despised". Hence, babies cannot manage crocodiles.

3. a) Let $\{S, *\}$, $\{T, \Delta\}$, and $\{V, \oplus\}$ be semigroups and $g: S \rightarrow T$ and $h: T \rightarrow V$ be semigroup homomorphisms. Then prove that the composition function $h \circ g: S \rightarrow V$ is a semigroup homomorphism from $\{S, *\}$ to $\{V, \oplus\}$.

[5]

b) If Z_6 is the set of equivalence classes generated by the equivalence relation "congruence modulo 6", prove that $\{Z_6, \times_6\}$ is a monoid where the operation \times_6 is defined as

[5]

$$[i] \times_6 [j] = [(i \times j) \pmod{6}], \forall [i], [j] \in Z_6.$$

4. Given the generator matrix $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ corresponding to the

[10]

encoding function $e: B^3 \rightarrow B^6$, find the corresponding parity check matrix and use it to decode the following received words, and hence find the original message.

(i) 110101 (ii) 001111 (iii) 110001 (iv) 111111

5. Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n; n \geq 2$ given that $a_0 = 2$ and $a_1 = 8$ using the method of generating functions.

[10]

6.a) (i) If S_n is the set of all divisors of the positive integer n and D is the relation of the 'division' viz., " aDb " if and only if " a divides b ", verify whether $\{S_{42}, D\}$ is a complemented lattice by finding the complements of all elements.

[10]

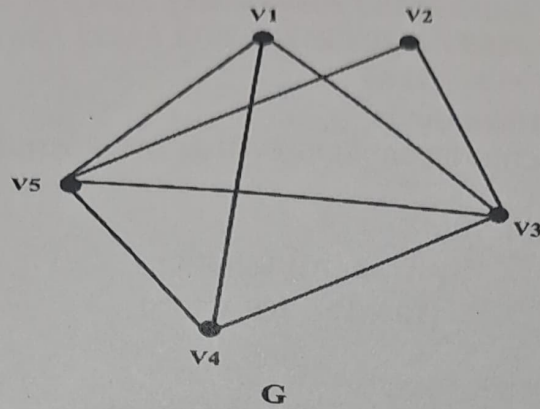
(ii) Simplify the Boolean expressions to a minimum number of literals $(xy'z' + xy'z + xyz + xyz')(x + y)$.

OR

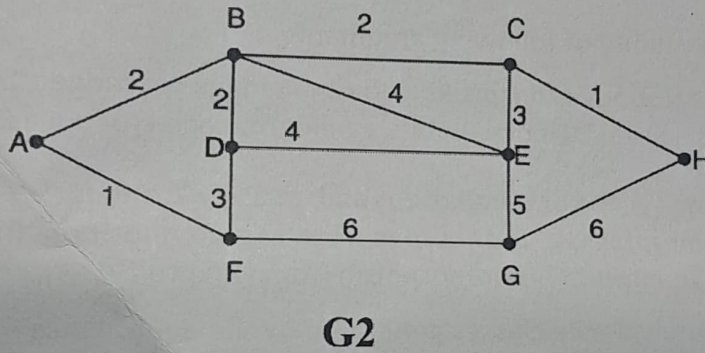
[10]

6.b) Prove that every chain is a distributive Lattice.

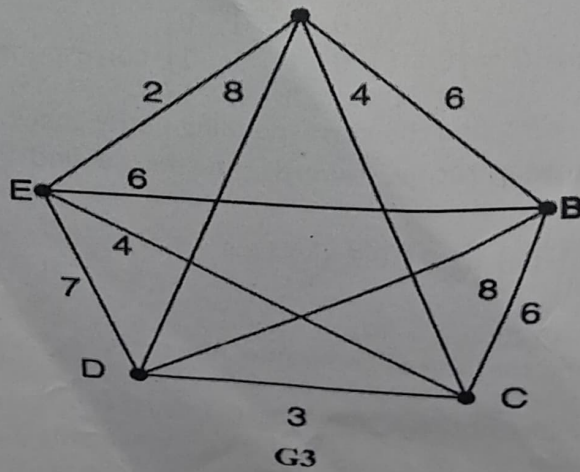
7. (i) Prove that the number of vertices of odd degrees in a graph is always even. [10]
 (ii) Find the adjacency matrix of the following graph G.



8. Use Dijkstra's algorithm to find the shortest path between the vertices A and H in the weighted graph given in G2 [10]



- 9.a) Explain Krushkal's algorithm. Find the minimum spanning tree for the weighted graphs shown in G3 by using Krushkal's algorithm. [10]



OR

9.b) (i) Explain fundamental circuit with example.

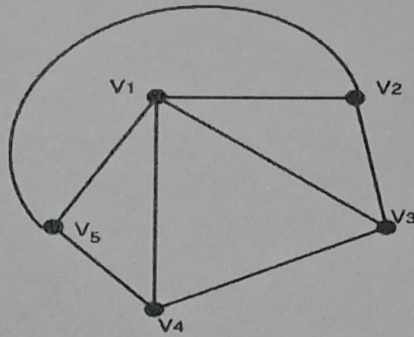
[5]

(ii) Prove that every tree has either one or two centers.

[5]

10. Find the chromatic polynomial of the graph given in G_4 .

[10]



G4

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