

Engineering Mechanics

Faculty : Dr. Prashik Gaikwad
School of Mechanical Engineering

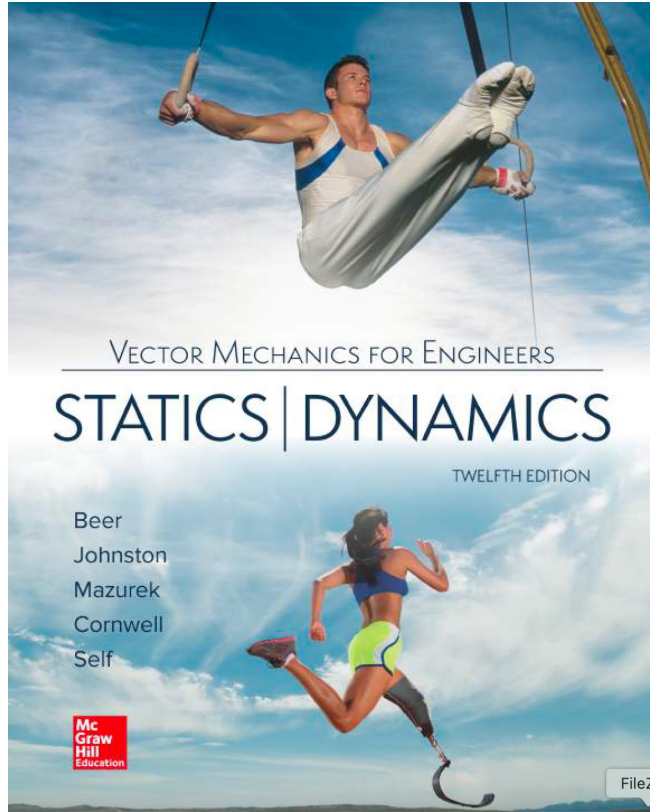
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Cabin: CBMR 304-B

BMEE201L	Engineering Mechanics				L	T	P	C
					2	1	0	3
Pre-requisite	NIL				Syllabus version			
					1.0			
Course Objectives:								
1. To enable students to apply fundamental laws and basic concepts of rigid body mechanics to solve problems of bodies under rest or in motion.								
2. To enable the students to apply conditions of static equilibrium to analyse physical systems.								
3. To compute the properties of areas and bodies.								
Course Outcome:								
Upon successful completion of the course the students will be able to								
1. Compute the resultant and analyse equilibrium (without and with friction) of system of forces acting on particles and rigid bodies in plane and space.								
2. Predict the support-reactions and the internal forces of the members of trusses and frames.								
3. Apply transfer theorems to determine properties of various sections.								
4. Calculate motion parameters of particles and rigid bodies.								
Module:1 Statics of Particles					5 hours			
Fundamental concepts and principles - Resolution of a force -Resultant of forces in a plane- Equilibrium of a particle in a plane; Addition of concurrent forces in space- Equilibrium of a particle in space.								
Module:2 Statics of Rigid Bodies					7 hours			
Equivalent systems of forces- Principle of Transmissibility - Moment of a force about a point and an axis- Couples and force-couple systems- Equilibrium of rigid bodies in two and three dimensions- Types of beams, supports and reactions; Principle of virtual work – System of connected rigid bodies.								
Module:3 Analysis of Structures					5 hours			
Analysis of plane trusses - Method of joints and method of sections- Frames								
Module:4 Friction					5 hours			
The laws of dry friction – Coefficients of Friction- Angles of Friction- Types of Friction Problems - Wedges and Ladder friction- Belt friction.								
Module:5 Properties of Surfaces and Solids					7 hours			
First moments of areas and lines- Centroids of composite areas and lines- - Theorems of Pappus-Guldinus- Second moment of area- Parallel axis theorem- Rectangular and Polar Moments of inertia of composite areas- Radius of Gyration- Product of Inertia- Principal Axes and Principal Moments of Inertia- Mass moments of inertia of thin plates.								
Module:6 Dynamics of Particles					8 hours			
Kinematics of Particles: Displacement, Velocity and Acceleration – Rectilinear motion – Curvilinear motion – Tangential and Normal components – Radial and Transverse components.								
Kinetics of Particles: Newton's Second Law- Energy and Momentum Methods-Principle of Work and Energy-Principle of Impulse and Momentum- Direct Central Impact								
Module:7 Dynamics of Rigid Bodies					8 hours			
Kinematics of rigid bodies: Translation and fixed-axis rotation- General plane motion: velocity- Instantaneous centre of rotation- General plane motion: acceleration.								
Kinetics of rigid bodies:Equations of motion -Angular momentum- Plane motion of a rigid body- Principle of work and energy for rigid bodies- Principle of impulse and momentum for rigid bodies.								
Total Lecture hours:					45 hours			

Textbook for this course



Beer, Johnston, Cornwell, David Mazurek,
and Sanghi, Vector Mechanics for
Engineers: Statics and Dynamics, 12th
Edition, McGraw-Companies

Module 6

Kinematics of Particles

Module Overview

- Rectilinear motion
- Curvilinear motion
- Kinetics of Particles
- Newton's second law
- Energy and Momentum methods

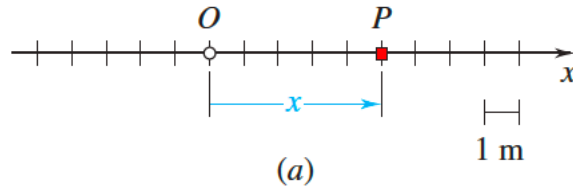
Introduction

Dynamics includes two broad areas of study:

1. **Kinematics**, which is the **study of the geometry of motion**. The principles of kinematics relate the displacement, velocity, acceleration, and time of a body's motion, without reference to the cause of the motion.
2. **Kinetics**, which is the **study of the relation between the forces acting on a body, the mass of the body, and the motion of the body**. We use kinetics to predict the motion caused by given forces or to determine the forces required to produce a given motion.

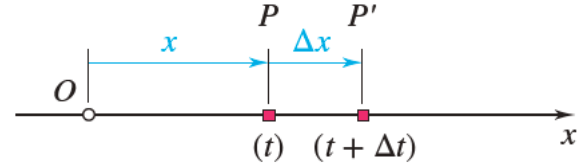
Rectilinear motion

- A particle moving along a straight line is said to be in rectilinear motion.
- The only variables we need to describe this motion are the time, t , and the distance along the line, x , as a function of time.
- With these variables, we can define the particle's position, velocity, and acceleration, which completely describe the particle's motion.
- The distance x , with the appropriate sign, completely defines the position of the particle; it is called the **position coordinate of the particle**



Rectilinear motion

- Consider the position P occupied by the particle at time t and the corresponding coordinate x.
- Consider also the position P' occupied by the particle at a later time t + Δt.



- We define the average velocity of the particle over the time interval Δt as the quotient of the displacement Δx and the time interval Δt as

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

- We can determine the instantaneous velocity v of a particle at the instant t by allowing the time interval Δt to become infinitesimally small

$$\text{Instantaneous velocity} = v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

Rectilinear motion

Velocity of a particle along a line

$$v = \frac{dx}{dt}$$

- Consider the velocity v of the particle at time t and also its velocity $v + \Delta v$ at a later time $t + \Delta t$.
- We define the average acceleration of the particle over the time interval Δt as the quotient of Δv and Δt as

$$\text{Average acceleration} = \frac{\Delta v}{\Delta t}$$

$$\text{Instantaneous acceleration} = a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$

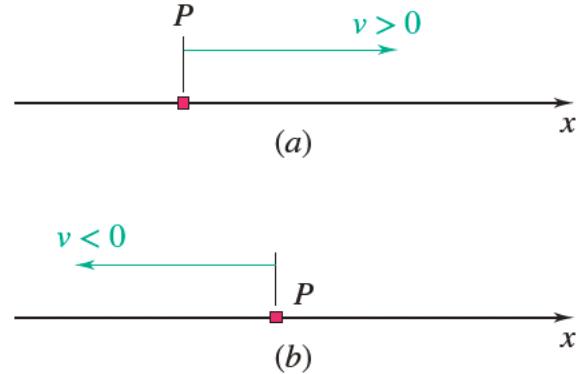
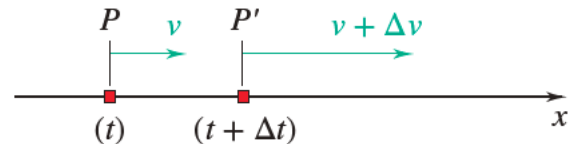


Fig. 11.3 In rectilinear motion, velocity can be only (a) positive or (b) negative along the line.



Rectilinear motion

**Acceleration of a particle
along a line**

$$a = \frac{dv}{dt}$$

$$a = \frac{d^2x}{dt^2}$$

$$a = v \frac{dv}{dx}$$

Determining motion of particle

1. $a = f(t)$. **The Acceleration Is a Given Function of t .** Solving Eq. (11.2) for dv and substituting $f(t)$ for a , we have

$$\begin{aligned} dv &= a dt \\ dv &= f(t) dt \end{aligned}$$

Integrating both sides of the equation, we obtain

$$\int dv = \int f(t) dt$$

This equation defines v in terms of t . Note, however, that an arbitrary constant is introduced after the integration is performed. This is due to the fact that many motions correspond to the given acceleration $a = f(t)$. In order to define the motion of the particle uniquely, it is necessary to specify the **initial conditions** of the motion; that is, the value v_0 of the velocity and the value x_0 of the position coordinate at $t = 0$. Rather than use an arbitrary constant that is determined by the initial conditions, it is often more convenient to replace the indefinite integrals with **definite integrals**. Definite integrals have lower limits corresponding to the initial conditions $t = 0$ and $v = v_0$ and upper limits corresponding to $t = t$ and $v = v$. This gives us

$$\begin{aligned} \int_{v_0}^v dv &= \int_0^t f(t) dt \\ v - v_0 &= \int_0^t f(t) dt \end{aligned}$$

which yields v in terms of t .

We can now solve Eq. (11.1) for dx as

$$dx = v dt$$

and substitute the expression obtained from the first integration for v . Then, we integrate both sides of this equation via the left-hand side with respect to x from $x = x_0$ to $x = x$ and the right-hand side with respect to t from $t = 0$ to $t = t$. In this way, we obtain the position coordinate x in terms of t ; the motion is completely determined.

2. $a = f(x)$. **The Acceleration Is a Given Function of x .** Rearranging Eq. (11.4) and substituting $f(x)$ for a , we have

$$\begin{aligned} v dv &= a dx \\ v dv &= f(x) dx \end{aligned}$$

Because each side contains only one variable, we can integrate the equation. Denoting again the initial values of the velocity and of the position coordinate by v_0 and x_0 , respectively, we obtain

$$\begin{aligned} \int_{v_0}^v v dv &= \int_{x_0}^x f(x) dx \\ \frac{1}{2} v^2 - \frac{1}{2} v_0^2 &= \int_{x_0}^x f(x) dx \end{aligned}$$

which yields v in terms of x . We now solve Eq. (11.1) for dt , giving us

$$dt = \frac{dx}{v}$$

and substitute for v the expression just obtained. We can then integrate both sides to obtain the desired relation between x and t . However, in most cases, this last integration cannot be performed analytically, and we must resort to a numerical method of integration.

3. $a = f(v)$. **The Acceleration Is a Given Function of v .** We can now substitute $f(v)$ for a in either Eqs. (11.2) or (11.4) to obtain either

$$\begin{aligned} f(v) &= \frac{dv}{dt} & f(v) &= v \frac{dv}{dx} \\ dt &= \frac{dv}{f(v)} & dx &= \frac{v dv}{f(v)} \end{aligned}$$

Integration of the first equation yields a relation between v and t ; integration of the second equation yields a relation between v and x . Either of these relations can be used in conjunction with Eq. (11.1) to obtain the relation between x and t that characterizes the motion of the particle.

Numerical

The position of a particle moving along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in feet and t in seconds. Determine (a) the time at which the velocity is zero, (b) the position and distance traveled by the particle at that time, (c) the acceleration of the particle at that time, (d) the distance traveled by the particle from $t = 4$ s to $t = 6$ s.

$$x = t^3 - 6t^2 - 15t + 40 \quad (1)$$

$$v = \frac{dx}{dt} = 3t^2 - 12t - 15 \quad (2)$$

$$a = \frac{dv}{dt} = 6t - 12 \quad (3)$$

a. Time When $v = 0$. Set $v = 0$ in Eq. (2) for

$$3t^2 - 12t - 15 = 0 \quad t = -1 \text{ s} \quad \text{and} \quad t = +5 \text{ s} \quad \blacktriangleleft$$

Only the root $t = +5$ s corresponds to a time after the motion has begun: for $t < 5$ s, $v < 0$ and the particle moves in the negative direction; for $t > 5$ s, $v > 0$ and the particle moves in the positive direction.

b. Position and Distance Traveled When $v = 0$. Substitute $t = +5$ s into Eq. (1), yielding

$$x_5 = (5)^3 - 6(5)^2 - 15(5) + 40 \quad x_5 = -60 \text{ ft} \quad \blacktriangleleft$$

The initial position at $t = 0$ was $x_0 = +40$ ft. Because $v \neq 0$ during the interval $t = 0$ to $t = 5$ s, you have

$$\text{Distance traveled} = x_5 - x_0 = -60 \text{ ft} - 40 \text{ ft} = -100 \text{ ft}$$

$$\text{Distance traveled} = 100 \text{ ft in the negative direction} \quad \blacktriangleleft$$

c. Acceleration When $v = 0$. Substitute $t = +5$ s into Eq. (3) for

$$a_5 = 6(5) - 12 \quad a_5 = +18 \text{ ft/s}^2 \quad \blacktriangleleft$$

d. Distance Traveled from $t = 4$ s to $t = 6$ s. The particle moves in the negative direction from $t = 4$ s to $t = 5$ s and in the positive direction from $t = 5$ s to $t = 6$ s; therefore, the distance traveled during each of these time intervals must be computed separately.

$$\text{From } t = 4 \text{ s to } t = 5 \text{ s:} \quad x_5 = -60 \text{ ft}$$

$$x_4 = (4)^3 - 6(4)^2 - 15(4) + 40 = -52 \text{ ft}$$

$$\begin{aligned} \text{Distance traveled} &= x_5 - x_4 = -60 \text{ ft} - (-52 \text{ ft}) = -8 \text{ ft} \\ &= 8 \text{ ft in the negative direction} \end{aligned}$$

$$\text{From } t = 5 \text{ s to } t = 6 \text{ s:} \quad x_5 = -60 \text{ ft}$$

$$x_6 = (6)^3 - 6(6)^2 - 15(6) + 40 = -50 \text{ ft}$$

$$\begin{aligned} \text{Distance traveled} &= x_6 - x_5 = -50 \text{ ft} - (-60 \text{ ft}) = +10 \text{ ft} \\ &= 10 \text{ ft in the positive direction} \end{aligned}$$

$$\text{Total distance traveled from } t = 4 \text{ s to } t = 6 \text{ s is } 8 \text{ ft} + 10 \text{ ft} \quad = 18 \text{ ft}$$

Numerical

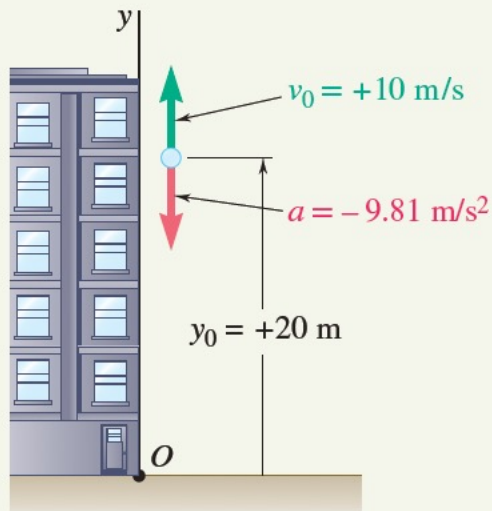


Fig. 1 Acceleration, initial velocity, and initial position of the ball.

You throw a ball vertically upward with a velocity of 10 m/s from a window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s^2 downward, determine (a) the velocity v and elevation y of the ball above the ground at any time t , (b) the highest elevation reached by the ball and the corresponding value of t , (c) the time when the ball hits the ground and the corresponding velocity. Draw the v - t and y - t curves. 

a. Velocity and Elevation. Choose the y axis measuring the position coordinate (or elevation) with its origin O on the ground and its positive sense upward. The value of the acceleration and the initial values of v and y are as indicated in Fig. 1. Substituting for a in $a = dv/dt$ and noting that when $t = 0$, $v_0 = +10 \text{ m/s}$, you have

$$\begin{aligned}\frac{dv}{dt} &= a = -9.81 \text{ m/s}^2 \\ \int_{v_0=10}^v dv &= - \int_0^t 9.81 dt \\ [v]_{10}^v &= -[9.81t]_0^t \\ v - 10 &= -9.81t\end{aligned}$$

$$v = 10 - 9.81t \quad (1) \quad \blacktriangleleft$$

Numerical

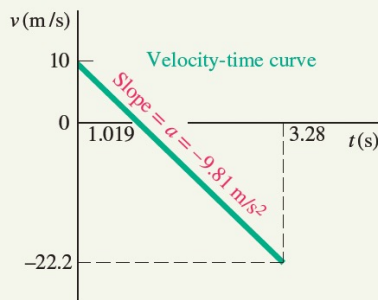


Fig. 2 Velocity of the ball as a function of time.

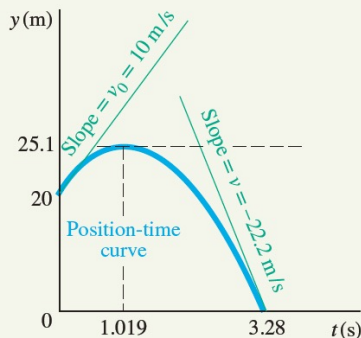


Fig. 3 Height of the ball as a function of time.

Substituting for v in $v = dy/dt$ and noting that when $t = 0$, $y_0 = 20$ m, you have

$$\begin{aligned}\frac{dy}{dt} &= v = 10 - 9.81t \\ \int_{y_0=20}^y dy &= \int_0^t (10 - 9.81t) dt \\ [y]_{20}^y &= [10t - 4.905t^2]_0^t \\ y - 20 &= 10t - 4.905t^2 \\ y &= 20 + 10t - 4.905t^2 \quad (2) \quad \blacktriangleleft\end{aligned}$$

Graphs of these equations are shown in Figs. 2 and 3.

b. Highest Elevation. The ball reaches its highest elevation when $v = 0$. Substituting into Eq. (1), you obtain

$$10 - 9.81t = 0 \quad t = 1.019 \text{ s} \quad \blacktriangleleft$$

Substituting $t = 1.019$ s into Eq. (2), you find

$$y = 20 + 10(1.019) - 4.905(1.019)^2 \quad y = 25.1 \text{ m} \quad \blacktriangleleft$$

c. Ball Hits the Ground. The ball hits the ground when $y = 0$. Substituting into Eq. (2), you obtain

$$20 + 10t - 4.905t^2 = 0 \quad t = -1.243 \text{ s} \quad \text{and} \quad t = +3.28 \text{ s} \quad \blacktriangleleft$$

Only the root $t = +3.28$ s corresponds to a time after the motion has begun. Carrying this value of t into Eq. (1), you find

$$v = 10 - 9.81(3.28) = -22.2 \text{ m/s} \quad v = 22.2 \text{ m/s} \downarrow \quad \blacktriangleleft$$

Rectilinear motion

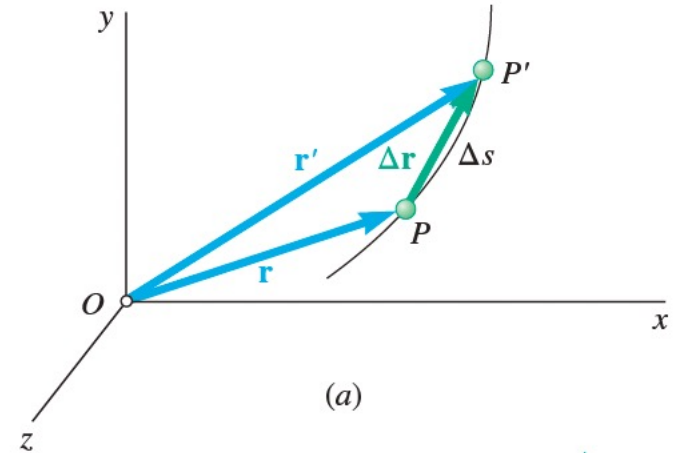
We can summarize these relationships in Table 11.1.

Table 11.1

If. . . .	Kinematic Relationship	Integrate
$a = a(t)$	$\frac{dv}{dt} = a(t)$	$\int_{v_0}^v dv = \int_0^t a(t)dt$
$a = a(x)$	$v \frac{dv}{dx} = a(x)$	$\int_{v_0}^v v \, dv = \int_{x_0}^x a(x)dx$
$a = a(v)$	$\frac{dv}{dt} = a(v)$	$\int_{v_0}^v \frac{dv}{a(v)} = \int_0^t dt$
	$v \frac{dv}{dx} = a(v)$	$\int_{x_0}^x dx = \int_{v_0}^v \frac{v dv}{a(v)}$

Curvilinear motion

- When a particle moves along a curve other than a straight line, we say that the particle is in **curvilinear motion**.
- To define the position P occupied by a particle in curvilinear motion at a given time t , we select a fixed reference system, such as the x, y, z axes shown in Fig. a, and draw the vector $\mathbf{r}$ joining the origin O and point P .
- The vector $\mathbf{r}$ is characterized by its magnitude r and its direction with respect to the reference axes, so it completely defines the position of the particle with respect to those axes. We refer to vector $\mathbf{r}$ as the **position vector** of the particle at time t .



Curvilinear motion

Average velocity

$$\frac{\Delta \mathbf{r}}{\Delta t}$$

Average acceleration

$$\frac{\Delta \mathbf{v}}{\Delta t}$$

Instantaneous velocity

$$\mathbf{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

Instantaneous acceleration

$$\mathbf{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \mathbf{v}}{\Delta t}$$

Velocity

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

Acceleration vector

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

Curvilinear motion

In the case of the motion of a projectile, we can show that the components of the acceleration are

$$a_x = \ddot{x} = 0 \quad a_y = \ddot{y} = -g \quad a_z = \ddot{z} = 0$$

if the resistance of the air is neglected.

Denoting initial coordinates by x_0 , y_0 , and z_0 and the components of the initial velocity v_0 of the projectile by $(v_x)_0$, $(v_y)_0$, and $(v_z)_0$, we can integrate twice in t and obtain

$$\begin{array}{lll} v_x = \dot{x} = (v_x)_0 & v_y = \dot{y} = (v_y)_0 - gt & v_z = \dot{z} = (v_z)_0 \\ x = x_0 + (v_x)_0 t & y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2 & z = z_0 + (v_z)_0 t \end{array}$$

If the projectile is fired in the xy plane from the origin O , we have $x_0 = y_0 = z_0 = 0$ and $(v_z)_0 = 0$, so the equations of motion reduce to

$$\begin{array}{lll} v_x = (v_x)_0 & v_y = (v_y)_0 - gt & v_z = 0 \\ x = (v_x)_0 t & y = (v_y)_0 t - \frac{1}{2}gt^2 & z = 0 \end{array}$$

Numerical

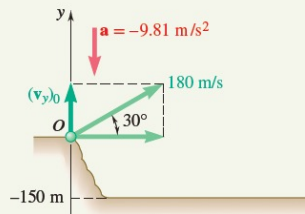
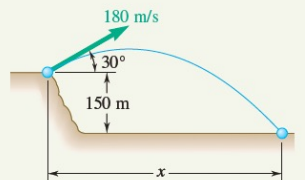


Fig. 1 Acceleration and initial velocity of the projectile in the y direction.

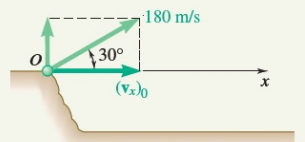


Fig. 2 Initial velocity of the projectile in the x direction.

A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find (a) the horizontal distance from the cannon to the point where the projectile strikes the ground, (b) the greatest elevation above the ground reached by the projectile.

MODELING and ANALYSIS: Model the projectile as a particle and neglect the effects of air resistance. The vertical motion has a constant acceleration. Choosing the positive sense of the y axis upward and placing the origin O at the cannon (Fig. 1), you have

$$(v_y)_0 = (180 \text{ m/s}) \sin 30^\circ = +90 \text{ m/s}$$

$$a = -9.81 \text{ m/s}^2$$

Substitute these values into the equations for motion with constant acceleration. Thus,

$$v_y = (v_y)_0 + at \quad v_y = 90 - 9.81t \quad (1)$$

$$y = (v_y)_0 t + \frac{1}{2} at^2 \quad y = 90t - 4.90t^2 \quad (2)$$

$$v_y^2 = (v_y)_0^2 + 2ay \quad v_y^2 = 8100 - 19.62y \quad (3)$$

The horizontal motion has zero acceleration. Choose the positive sense of the x axis to the right (Fig. 2), which gives you

$$(v_x)_0 = (180 \text{ m/s}) \cos 30^\circ = +155.9 \text{ m/s}$$

Substituting into the equation for constant acceleration, you obtain

$$x = (v_x)_0 t \quad x = 155.9t \quad (4)$$

a. Horizontal Distance. When the projectile strikes the ground,

$$y = -150 \text{ m}$$

Substituting this value into Eq. (2) for the vertical motion, you have

$$-150 = 90t - 4.90t^2 \quad t^2 - 18.37t - 30.6 = 0 \quad t = 19.91 \text{ s}$$

Substituting $t = 19.91 \text{ s}$ into Eq. (4) for the horizontal motion, you obtain

$$x = 155.9(19.91) \quad x = 3100 \text{ m} \quad \blacktriangleleft$$

b. Greatest Elevation. When the projectile reaches its greatest elevation, $v_y = 0$; substituting this value into Eq. (3) for the vertical motion, you have

$$0 = 8100 - 19.62y \quad y = 413 \text{ m}$$

$$\text{Greatest elevation above ground} = 150 \text{ m} + 413 \text{ m} = 563 \text{ m} \quad \blacktriangleleft$$

Newton second law of motion

If the resultant force acting on a particle is not zero, the particle has an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

Newton's second law

$$\mathbf{F} = m\mathbf{a}$$

Newton's second law,
multiple forces:

$$\Sigma \mathbf{F} = m\mathbf{a}$$

Suppose we replace the acceleration a in Eq. (12.2) by the derivative dv/dt .

We have $\Sigma \mathbf{F} = m (dv/dt)$

Because the mass m of the particle is constant, we can write this as

$$\Sigma \mathbf{F} = d/dt (m\mathbf{v})$$

The product $m\mathbf{v}$ is called the **linear momentum, or simply the momentum, of the particle.**

The resultant of the forces acting on the particle is equal to the rate of change of the linear momentum of the particle.

Newton second law of motion

Linear momentum

$$\mathbf{L} = m\mathbf{v}$$

Newton's second law,
momentum form

$$\Sigma \mathbf{F} = \dot{\mathbf{L}}$$

- If the resultant force acting on a particle is zero, the linear momentum of the particle remains constant in both magnitude and direction.
- This is the **principle of conservation of linear momentum for a particle**.

Numerical

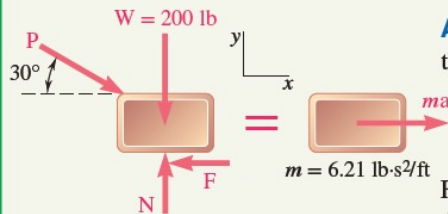
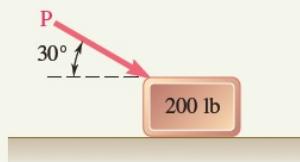


Fig. 1 Free-body diagram and kinetic diagram for the block.

A 200-lb block rests on a horizontal plane. Find the magnitude of the force P required to give the block an acceleration of 10 ft/s^2 to the right. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$. 

STRATEGY: You are given an acceleration and want to find the applied force. Therefore, you need to use Newton's second law.

MODELING: Pick the block as your system and model it as a particle. Drawing its free-body and kinetic diagrams, you obtain Fig. 1.

ANALYSIS: Before using Fig. 1, it is convenient to determine the mass of the object.

$$m = \frac{W}{g} = \frac{200 \text{ lb}}{32.2 \text{ ft/s}^2} = 6.21 \text{ lb}\cdot\text{s}^2/\text{ft}$$

From Fig. 1, it is clear that the forces acting on the block shown in the free-body diagram need to be equal to the vector ma , as shown in the kinetic diagram. Using these diagrams and recognizing that $F = \mu_k N$, you can write

$$\begin{aligned} \rightarrow \Sigma F_x = ma: \quad & P \cos 30^\circ - 0.25N = (6.21 \text{ lb}\cdot\text{s}^2/\text{ft})(10 \text{ ft/s}^2) \\ & P \cos 30^\circ - 0.25N = 62.1 \text{ lb} \end{aligned} \tag{1}$$

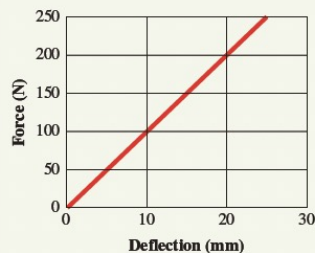
$$+\uparrow \Sigma F_y = 0: \quad N - P \sin 30^\circ - 200 \text{ lb} = 0 \tag{2}$$

Solving Eq. (2) for N and substituting the result into Eq. (1), you obtain

$$N = P \sin 30^\circ + 200 \text{ lb}$$

$$P \cos 30^\circ - 0.25(P \sin 30^\circ + 200 \text{ lb}) = 62.1 \text{ lb} \quad P = 151 \text{ lb} \quad \blacktriangleleft$$

Numerical

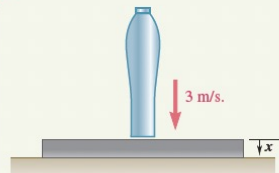


A 0.5-kg fragile glass vase is dropped onto a thick pad that has a force-deflection relationship as shown. Knowing that the vase has a speed of 3 m/s when it first contacts the pad, determine the maximum downward displacement of the vase.

STRATEGY: Use Newton's second law to find the acceleration of the vase and then integrate it to find the displacement.

MODELING: Choose the vase to be your system and model it as a particle. Because the force is a linear function of displacement, you can write the force acting on the vase as

$$F_P = \frac{200 \text{ N}}{0.02 \text{ m}}x = (10\,000 \text{ N/m})x$$



Draw its free-body diagram and kinetic diagram (Fig. 1).

ANALYSIS: You can obtain a scalar equation by applying Newton's second law in the vertical direction. Thus,

$$+\downarrow \Sigma F_x = ma \quad W - (10\,000)x = ma$$

Substituting in values and solving for a gives

$$a = 9.81 - 20\,000x$$

Maximum Displacement. Now that you have the acceleration as a function of displacement, you need to use the basic kinematic relationships to find the maximum compression of the pad. Substituting $a = 9.81 - 20\,000x$ into $a = v dv/dx$ gives

$$a = 9.81 - 20\,000x = \frac{v dv}{dx}$$

Separating variables and integrating, you find

$$v dv = (9.81 - 20\,000x)dx \rightarrow \int_{v_0}^0 v dv = \int_0^{x_{\max}} (9.81 - 20\,000x)dx$$

$$0 - \frac{1}{2}v_0^2 = 9.81x_{\max} - 10\,000x_{\max}^2 \quad (1)$$

Substituting $v_0 = 3 \text{ m/s}$ into Eq. (1) and solving for $x_{\max}$ using the quadratic formula gives $x_{\max} = 0.0217 \text{ m}$.

$$x_{\max} = 21.7 \text{ mm} \quad \blacktriangleleft$$

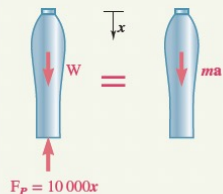


Fig. 1 Free-body diagram and kinetic diagram for the vase.