MODULE - III

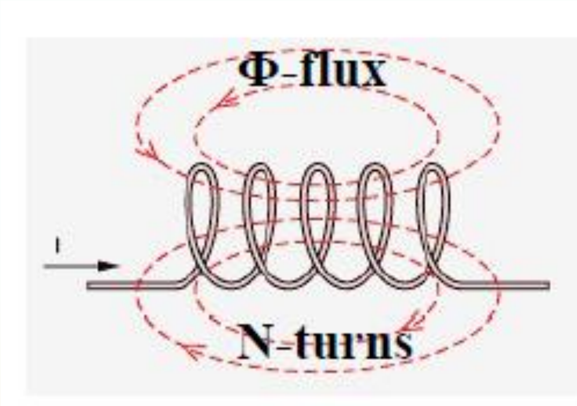
Magnetic Circuits

Contents

- Magnetic field
- Toroidal core
- Flux density and Flux linkage
- Magnetic circuit with airgap
- Reluctance in series and parallel circuits
- Self and mutual inductance
- Transformer: turn ratio determination

Electromagnetic Induction

- **Flux linkages (λ):**
- It is defined as the product of magnetic flux and the number of turns in a given coil.
- Flux linkages $\lambda = N\phi$



- Flux linkages is defined as linking of the magnetic field with the conductors (turns) of a coil when the magnetic field is passing through the coil.
- Units of flux linkages are Wb.Turns

t

- **Faraday's Law of Electromagnetic Induction:**
- When flux linking with the conductor changes, an emf is induced in the conductor. The amount of induced emf is equal to rate of change of flux.

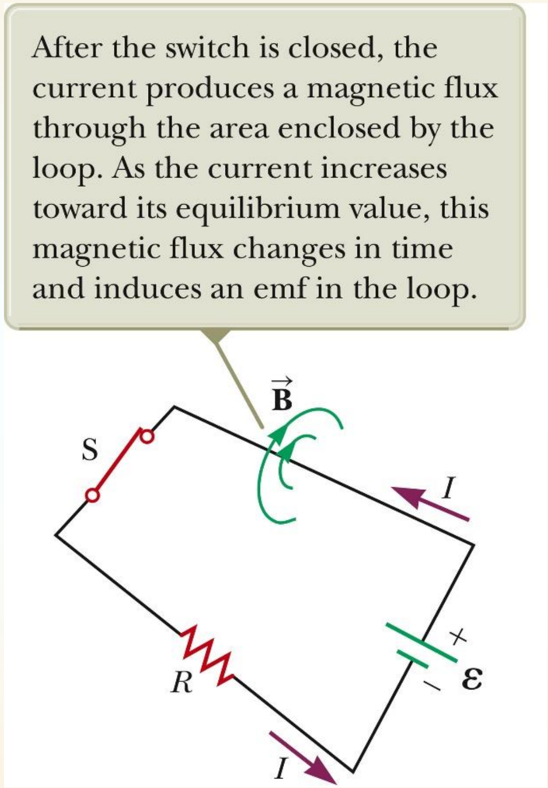
- **Lenz' law**
- The induced of induced emf is always in a direction that opposes the cause created it.

- Incorporating these laws into equation
- $e = -\frac{d\lambda}{dt} = -\frac{d(N\emptyset)}{dt} = -N\frac{d\emptyset}{dt}$

t

Self Inductance

- When the switch is closed, the current does not immediately reach its maximum value.
- As the current increases with time, the magnetic flux through the circuit loop due to this current also increases with time.
- This increasing (Changing) flux creates an induced emf in the circuit (Faraday's Law)
- The direction of the induced emf is such that it would cause an induced current in the loop which would establish a magnetic field opposing the change in the original magnetic field (LenzLaw)



Self Induced EMF

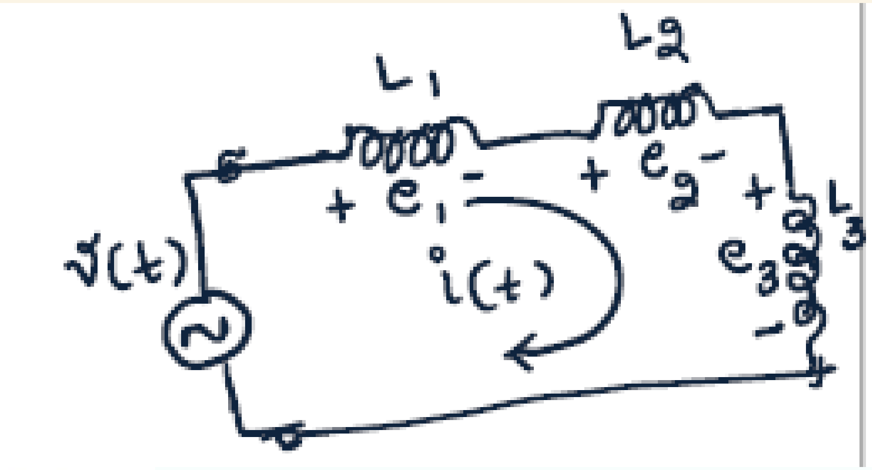
- This results in a gradual increase in the current to its final equilibrium value. This effect is called self – inductance L. The inductance is a measure of the opposition to change in current and is the ratio of flux linkages in the circuit per unit current in the same circuit
- $L = \frac{\lambda}{I} = \frac{N\varphi}{I}$ also $\varphi = \frac{NI}{\mathcal{R}}$
- So combining we get $L = \frac{N^2}{\mathcal{R}}$
- Flux linkages = $\lambda = N\phi = LI$

$$e = -\frac{d\lambda}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

- This emf is called self induced emf

Inductance in series

- When Inductances are connected in series, voltage divides and hence as per Kirchoff's Law we can write



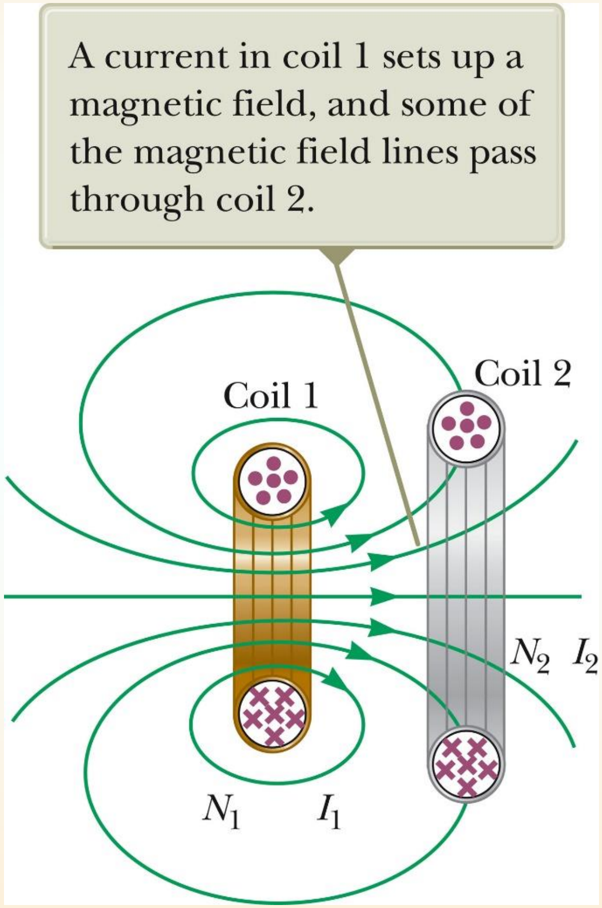
$$v(t) = e_1(t) + e_2(t) + e_3(t)$$
$$= L_1 \frac{d}{dt} i + L_2 \frac{d}{dt} i + L_3 \frac{d}{dt} i$$
$$= (L_1 + L_2 + L_3) \frac{d}{dt} i$$
$$= L_{eq} \frac{d}{dt} i$$

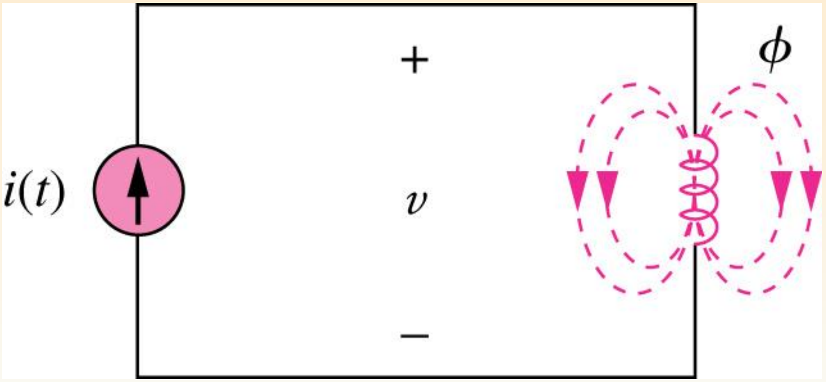
$$\Rightarrow L_{eq} = L_1 + L_2 + L_3$$

Note: Here we are assuming that there is no mutual inductance between the coils

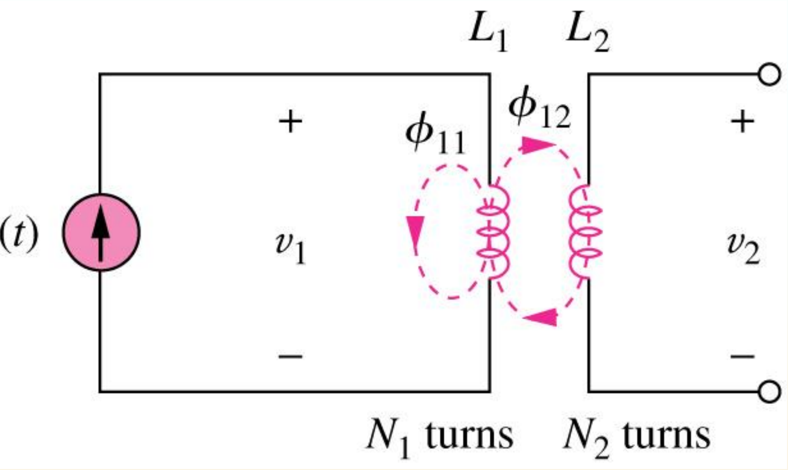
Mutual Inductance

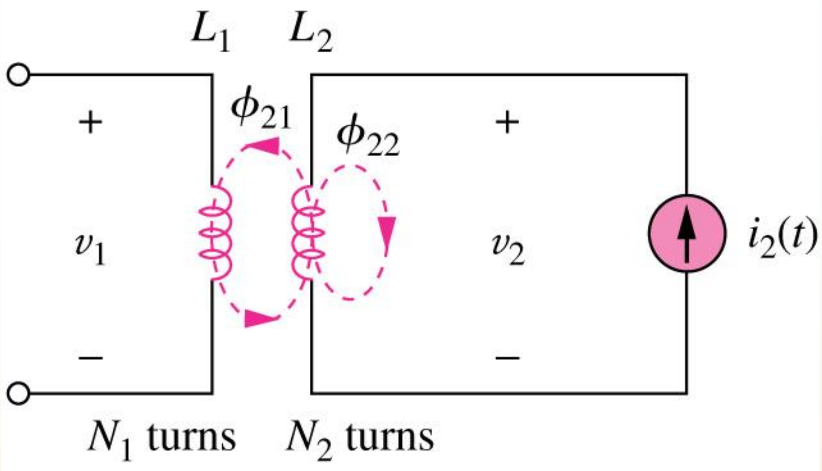
- The magnetic flux through the area enclosed by a circuit often varies with time because of time-varying currents in nearby circuits.
- This process is known as mutual induction because it depends on the interaction of two circuits
- The current in coil 1 sets up a magnetic field and Some of the magnetic field lines pass through coil 2.
- Coil 1 has a current I_1 and N_1 turns.
- Coil 2 has a current I_2 and N_2 turns.





a) Magnetic flux produced by a single coil.





b) Mutual inductance M21 of coil 2 with respect to coil 1.

c) Mutual inductance of M12 of coil 1 with respect to coil 2.

- The Mutual inductance M_{12} of coil 1 with respect to coil 2 is

$$M_{12} = \frac{\text{Flux linkages in one circuit}}{\text{current in another circuit}} = \frac{\lambda_{21}}{i_2} = \frac{N_1 \varphi_{21}}{i_2}$$

$$M_{12} = \frac{N_1 K \varphi_2}{i_2} = \frac{K N_1 N_2}{\mathcal{R}}$$

- The Mutual inductance M_{21} of coil 2 with respect to coil 1 is

$$M_{21} = \frac{\text{Flux linkages in one circuit}}{\text{current in another circuit}} = \frac{\lambda_{12}}{i_1} = \frac{N_2 \varphi_{12}}{i_1}$$

$$M_{21} = \frac{N_2 K \varphi_1}{i_1} = \frac{K N_2 N_1}{\mathcal{R}}$$

Mutually Induced EMF

- If current I_2 varies with time, the emf in coil 1 induced by the current in coil 2 is
- $e_1 = -\frac{d(N_1\varphi_{21})}{dt} \Longrightarrow e_1 = -M_{12}\frac{di_2}{dt}$
- If current I_1 varies with time, the emf in coil 2 induced by the current in coil 1 is
- $e_2 = -\frac{d(N_2\varphi_{12})}{dt} \Longrightarrow e_2 = -M_{21}\frac{di_1}{dt}$

Coefficient of Coupling

$$L_1 = \frac{Flux\ linkages\ in\ a\ circuit}{Current\ in\ that\ circuit} = \frac{\lambda_{11}}{i_1} = \frac{N_1\varphi_1}{i_1}$$

$$L_2 = \frac{Flux\ linkages\ in\ a\ circuit}{Current\ in\ that\ circuit} = \frac{\lambda_{22}}{i_2} = \frac{N_2\varphi_2}{i_2}$$

$$M_{12} = \frac{N_1K\varphi_2}{i_2} \qquad M_{21} = \frac{N_2K\varphi_1}{i_1}$$

And $M_{12} = M_{21}$

Multiplying these we get

$$M^2 = \frac{N_1K\varphi_2}{i_2} \times \frac{N_2K\varphi_1}{i_1}$$

or

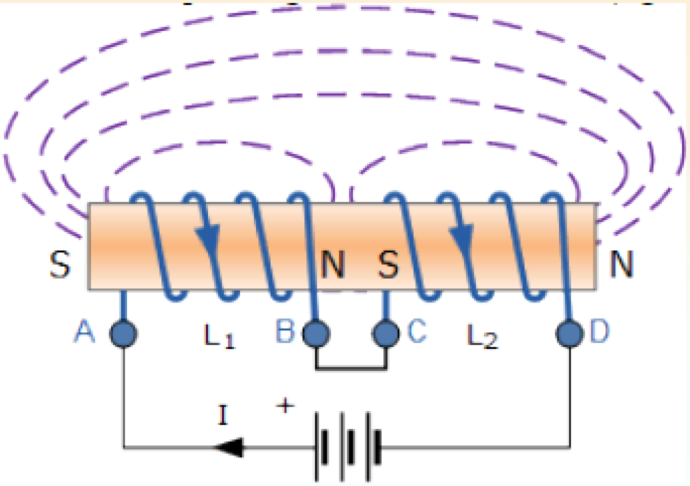
$$\frac{M^2}{\frac{N_2\varphi_2}{i_2} \times \frac{N_1\varphi_1}{i_1}} = K^2$$

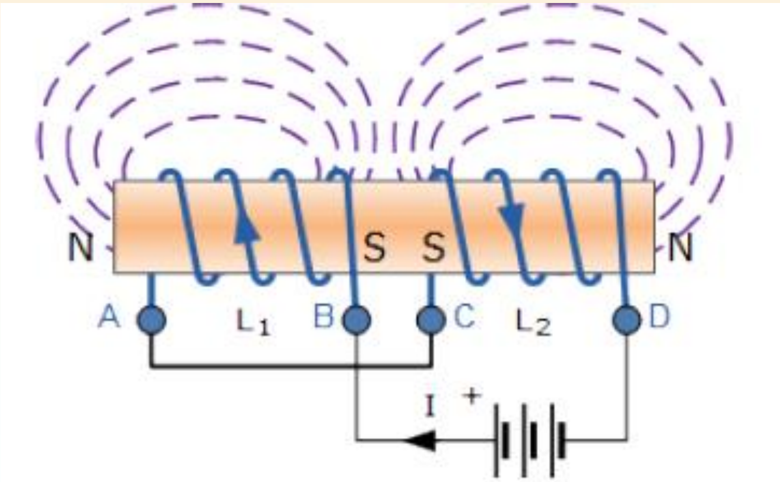
or

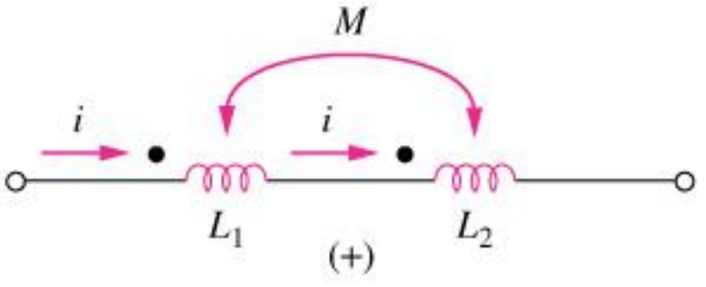
$$K = \frac{M}{\sqrt{L_1L_2}}$$

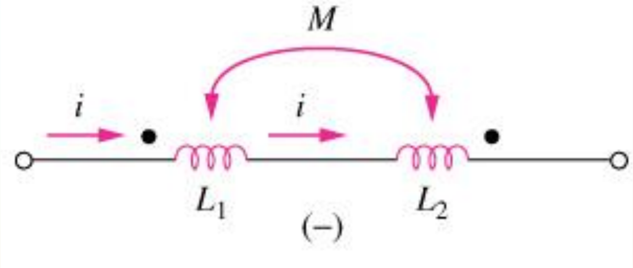
Where K is the coupling coefficient

Conductively Coupled Coils in series









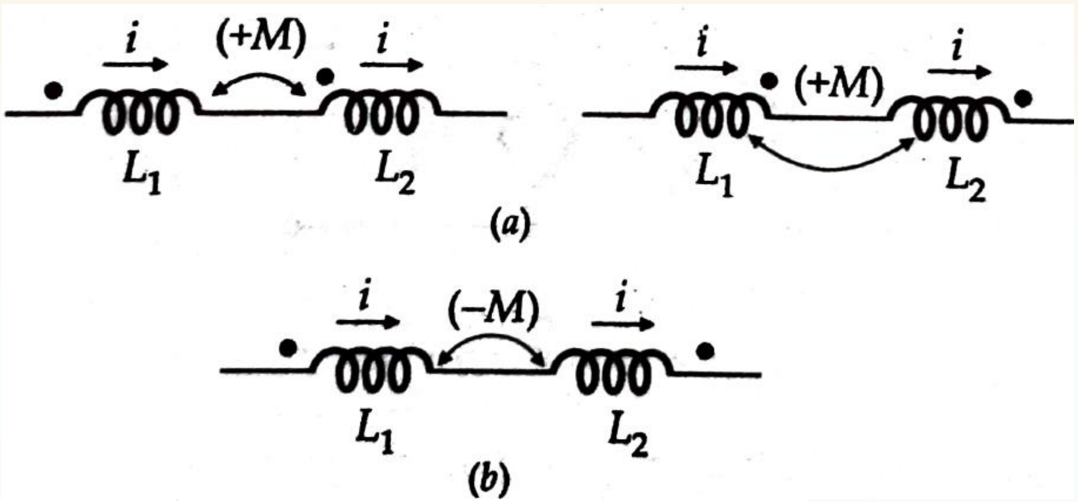
a)Series-aiding connection.

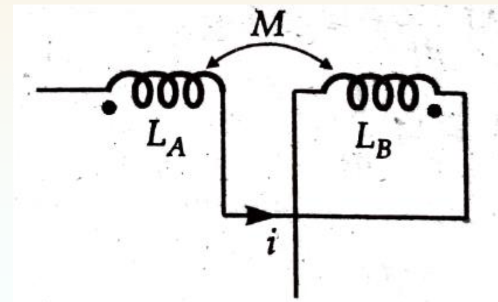
$L=L_1+L_2+2M$

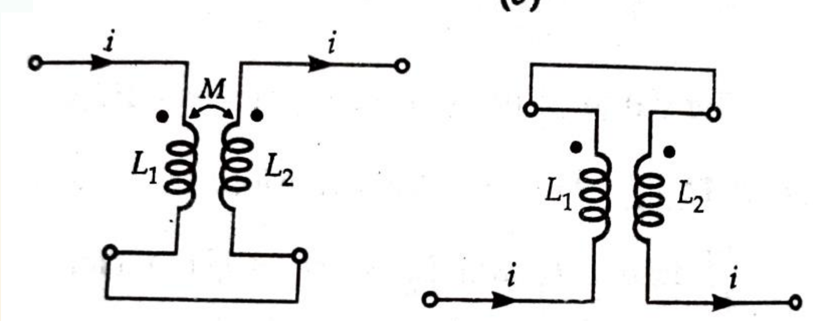
b) Series-opposing connection.

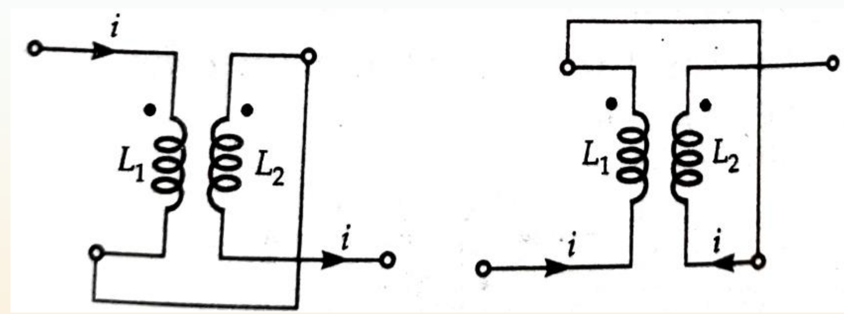
$L=L_1+L_2-2M$

The total inductance of two coupled coils in series depend on the placement of the dotted ends of the coils. The mutual inductances may add or subtract.





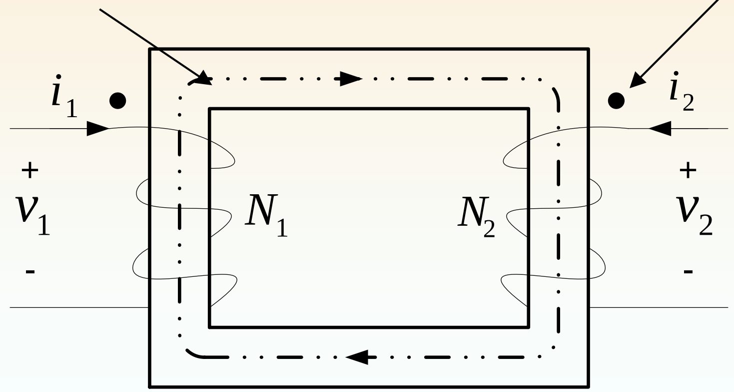




Inductively coupled coils

Magnetic flux, ϕ, in webers, Wb.

Current entering "dots" produce fluxes that add.



ϕ_{11} = flux in coil 1 produced by current in coil 1

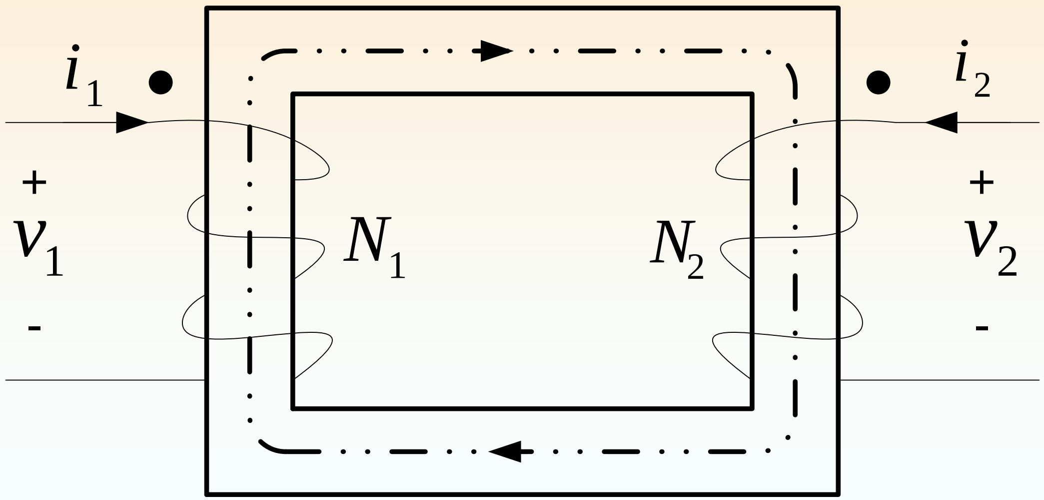
ϕ_{12} = flux in coil 2 produced by current in coil 1

ϕ_{21} = flux in coil 1 produced by current in coil 2

ϕ_{22} = flux in coil 2 produced by current in coil 2

ϕ_1 = total flux in coil $1 = \phi_{11} + \phi_{21}$

ϕ_2 = total flux in coil $2 = \phi_{21} + \phi_{12}$



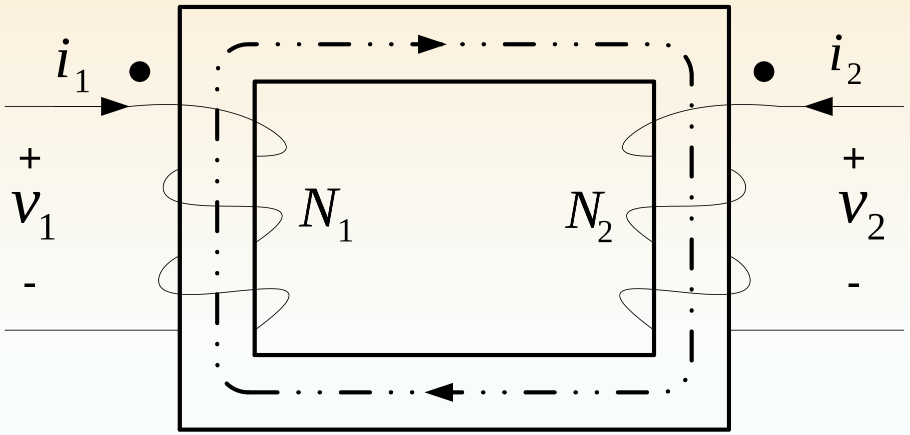
Faraday's Law

$$v_1(t) = N_1 \frac{d\phi_1}{dt} = N_1 \frac{d\phi_{11}}{dt} + N_1 \frac{d\phi_{21}}{dt}$$

In linear range, flux is proportional to current

$$V_1(t) = L_{11} \frac{di_1}{dt} + M_{12} \frac{di_2}{dt}$$

self-inductance **mutual inductance**



$$V_1(t) = L_{11}\frac{di_1}{dt} + M_{12}\frac{di_2}{dt} \quad \Rightarrow \quad v_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2(t) = L_{22}\frac{di_2}{dt} + M_{21}\frac{di_1}{dt} \quad \Rightarrow \quad v_2(t) = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$$

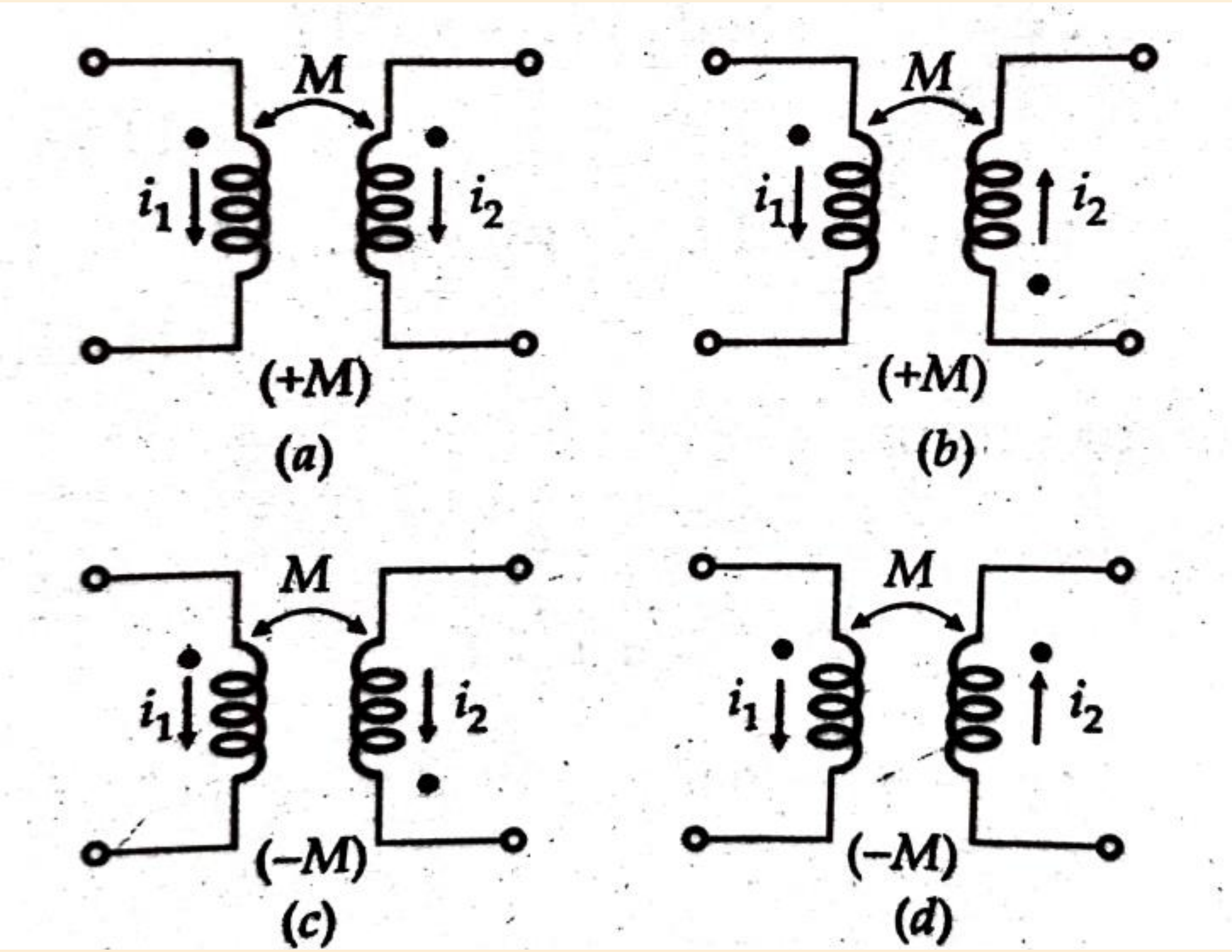
where

$$L_1 = L_{11} \qquad \mathrm{L}_1 = \frac{N_1\varphi_1}{\mathrm{i}_1}$$

And $M_{12} = M_{21} = M$

$$L_2 = L_{22} \qquad \mathrm{L}_2 = \frac{\mathrm{N}_2\varphi_2}{i_2}$$

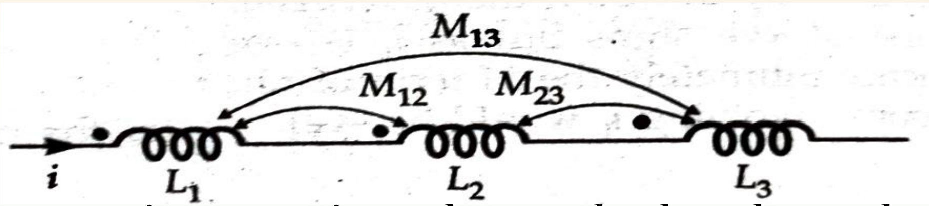
$$\mathrm{M} = \frac{N_1\varphi_{21}}{\mathrm{i}_2} = \frac{N_2\varphi_{12}}{\mathrm{i}_1}$$



M
i_1
i_2
(+M)
(a)
M
i_1
i_2
(+M)
(b)
M
i_1
i_2
(−M)
(c)
M
i_1
i_2
(−M)
(d)

Problems

P 11 Find the total inductance of the three series connected coupled coils , Given : L_1=1H, L_2=2H, L_3=5H,M_{12}=0.5H, M_{23}=1H, M_{13}=1H)



Coils are in series and current is entering through the dotted terminal in all the coils hence Mutual inductance is positive.

Inductance for coil 1 is $L_1+M_{12}+M_{13}$

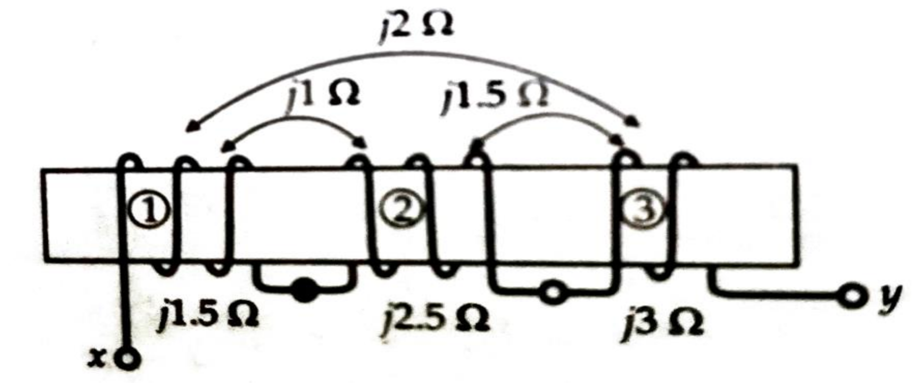
For Coil 2 is $L_2+M_{23}+M_{12}$ and For Coil 3 is $L_3+M_{23}+M_{13}$

Hence total inductance is $L_1+L_2+L_3+2M_{12}+2M_{23}+2M_{13}$

Total Inductance = 13H.

P 12 The resultant inductance of the inductors configuration shown in figure is 40mH, then find the mutual inductance of coils.

P 13 Find the equivalent circuit and the net inductance of the iron cored coupled coils in series connection. Current is flowing from terminal x to terminal y



- Observation, through coil one current is in one direction and through coil 2 current is in opposite direction while in coil 3, the current direction is different from coil2 whereas same as in coil 1. So M_{12} = – ve, M_{23} = – ve whereas M_{13} = + ve
- L_1 = j1.5, L_2=j2.5, L_3=j3, M_{12}=j1, M_{23}=j1.5, M_{31}=j2
- Net inductance = $L_1+L_2+L_3-2M_{12}-2M_{23}+2M_{13}$ = j6

P 14 The combined inductance of two coils connected in series is 0.6H or 0.1H depending on the relative directions of the currents in the coils. If one of the coils when isolated has a self-inductance of 0.2H, calculate (a)mutual inductance and (b) coupling coefficient. (c) the two possible values of the induced emf in coil 2 when the current is increasing at 500A/s in series combination.

Solution. (*i*) $L = L_1 + L_2 + 2M$ or $0.6 = L_1 + L_2 + 2M$

and $0.1 = L_1 + L_2 - 2M$

(*a*) From (*i*) and (*ii*) we get, M = **0.125 H**

Let $L_1 = 0.2$ H, then substituting this value in (*i*) above, we get $L_2 = 0.15$ H

(*b*) Coupling coefficient $k = M/\sqrt{L_1 L_2} = 0.125/\sqrt{0.2 \times 0.15} =$ **0.72**

- Two possible values of Induced Emf in second coil
- $e_2 = (L_2 + M)di/dt = (0.15 + 0.125)*500 = 137.5$ V
- $e_2 = (L_2 - M)di/dt = (0.15 - 0.125)*500 = 12.5$ V

P 15 Two similar coils have a coupling coefficient of 0.25. When they are connected in series cumulatively, the total inductance is 80mH. Calculate the inductance of each coil. Also calculate the total inductance when the coils are connected in series differentially.

Solution. If each coil has an inductance of L henry, then $L_1 = L_2 = L$; $M = k\sqrt{L_1 L_2} = k\sqrt{L \times L} = kL$

When connected in series comulatively, the total inductance of the coils is

$$= L_1 + L_2 + 2M = 2L + 2M = 2L + 2kL = 2L(1 + 0.25) = 2.5L$$

$$\therefore \quad 2.5\,L = 80 \quad \text{or} \quad L = \mathbf{32\ mH}$$

When connected in series differentially, the total inductance of the coils is

$$= L_1 + L_2 - 2M = 2L - 2M = 2L - 2kL = 2L(1 - k) = 2L(1 - 0.25)$$

$$\therefore \quad 2L \times 0.75 = 2 \times 32 \times 0.75 = \mathbf{48\ mH}.$$

P 16 Two coils have a mutual inductance of 0.2H. If the current in one coil is changed from 10A to 4A in 10ms. Calculate (a) the induced emf in second coil (b) the change of flux linked with the second coil if it is wound with 500 turns.

P 17 Two coils with a coefficient of coupling of 0.5 between them, are connected in series so as to magnetise (a) in the same direction (b) in the opposite direction. The corresponding values of total inductances are for (a) 1.9H and for (b) 0.7H. Find these lf inductances of the two coils and the mutual inductance between them.

Solution. (*a*) $L = L_1 + L_2 + 2M$ or $1.9 = L_1 + L_2 + 2M$ ...(*i*)

(*b*) Here $L = L_1 + L_2 - 2M$ or $0.7 = L_1 + L_2 - 2M$ ...(*ii*)

Subtracting (*ii*) from (*i*), we get

$$1.2 = 4M \qquad \therefore \qquad M = 0.3 \text{ H}$$

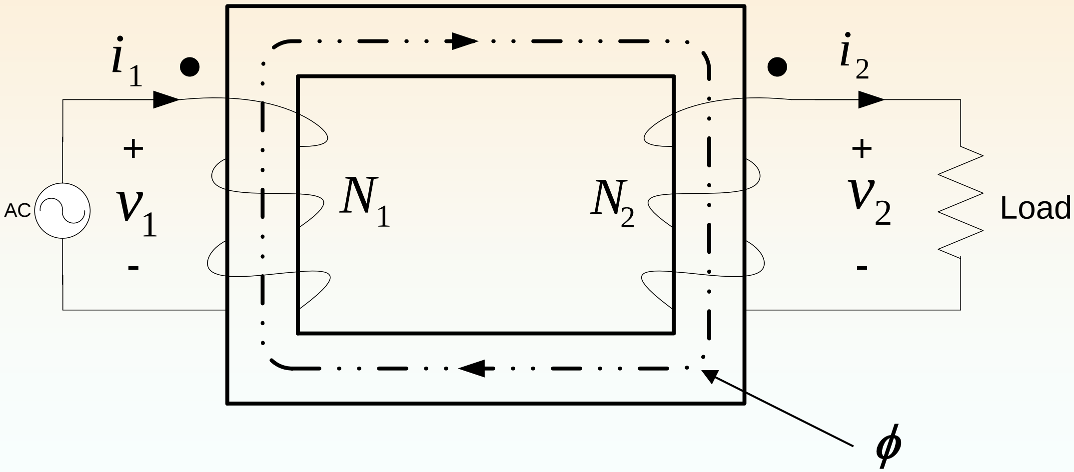
Putting this value in (*i*) above, we get $L_1 + L_2 = 1.3$ H ...(*iii*)

We know that, in general, $M = k\sqrt{L_1 L_2}$

$$\therefore \qquad \sqrt{L_1 L_2} = \frac{M}{k} = \frac{0.3}{0.5} = 0.6 \qquad \therefore \quad L_1 L_2 = 0.36$$

P 18 A 750 turns coil of inductance 3H carries a current of 2 A. Calculate the flux, linking with the coil, and the emf induced in the coil when the current collapses to zero in 20ms.

Ideal Transformer - Voltage



$$v_1(t) = N_1 \frac{d\phi}{dt}$$

$$\phi = \frac{1}{N_1} \int v_1(t) dt$$

This changing flux through coil 2 induces a voltage, v_2 across coil 2

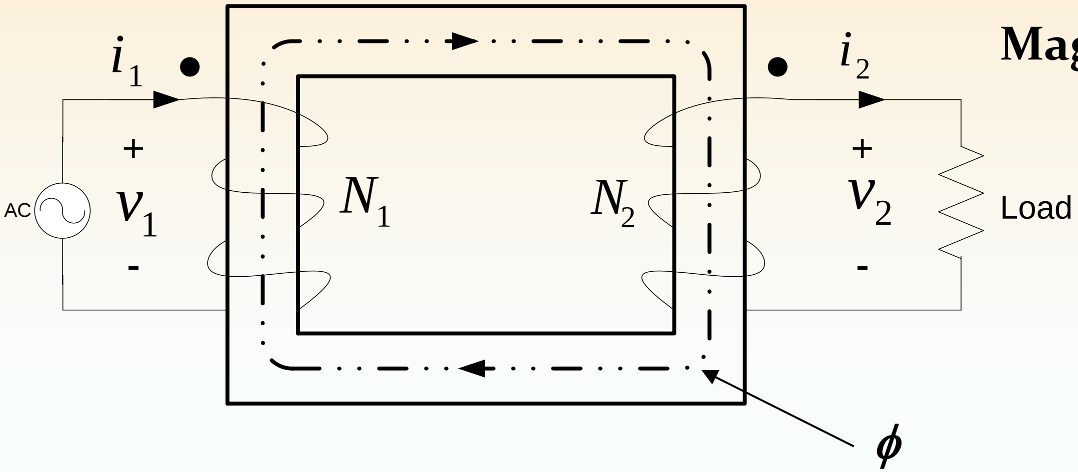
$$v_2(t) = N_2 \frac{d\phi}{dt}$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \frac{\frac{d\phi}{dt}}{\frac{d\phi}{dt}} = \frac{N_1}{N_2}$$

$$v_2 = \frac{N_2}{N_1} v_1$$

Turns ratio $n = \dfrac{N_2}{N_1}$

Ideal Transformer - Current



Magnetomotive force, mmf

$$\mathcal{F} = Ni$$

The total mmf applied to core is

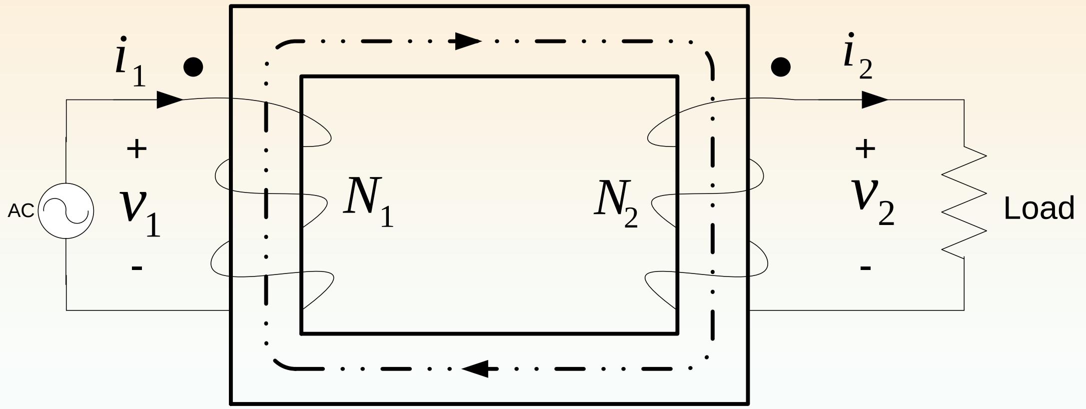
$$\mathcal{F} = N_1 i_1 - N_2 i_2 = \mathcal{R}\phi$$

For ideal transformer, the reluctance $\mathcal{R}$ is zero.

$$N_1 i_1 = N_2 i_2$$

$$i_2 = \frac{N_1}{N_2} i_1$$

Ideal Transformer - Power



$$P = vi$$

Power delivered to primary

$$P_1 = v_1 i_1$$

Power delivered to load

$$P_2 = v_2 i_2$$

$$v_2 = \frac{N_2}{N_1} v_1 \qquad i_2 = \frac{N_1}{N_2} i_1$$

$$P_2 = v_2 i_2 = v_1 i_1 = P_1$$

Power delivered to an ideal transformer by the source is transferred to the load.

Transformer Summary

1. Transformers will work only with AC supply. There is no rotating parts. Only the flux is changing (alternating) and the conductor is stationary. Hence it is statically induced emf (self induced emf and mutual induced emf)

2. We assumed that all of the flux links all of the windings of both coils. Thus, the voltage across each coil is proportional to the number of turns on the coil.

$$v_2(t) = \frac{N_2}{N_1} v_1(t)$$

3. We assumed that the reluctance of the core is negligible, so the total mmf of both coils is zero.

$$i_2(t) = \frac{N_1}{N_2} i_2(t)$$

4. A consequence of the voltage and current relationships is that all of the power delivered to an ideal transformer by the source is transferred to the load.

$$P_1 = P_2$$

Problems

P 19 From the following data, find the self and mutual inductance of the two windings 1 and 2of an ideal transformer operating in a linear zone.

- N_1=500 turns, N_2=750 turns, I_1=2A,
- ϕ_1=10mwb, ϕ_{12}=6mwb , k = 0.6

$$L_1 = \frac{N_1 \varphi_1}{I_1} = \frac{500 \text{ X } 0.01}{2} = 2.5\text{H}$$

$$M = \frac{N_2 \varphi_{12}}{i_1} = \frac{750 \; X \; 0.006}{2} = 2.25H$$

$$M = K\sqrt{L_1 L_2}$$

$$L_2 = \frac{M^2}{K^2 L_1} = \frac{2.25 \; X \; 2.25}{0.6 \; X \; 0.6 \; X \; 2.5} = 5.625H$$

P 20 In an ideal transformer, the mutual inductance = 10H, No of turns in primary and secondary are 50 and 200 respectively. Obtain the value of primary current to produce 0.5 wb flux to link the secondary coil

$$\mathrm{M} = \frac{N_2 \varphi_{12}}{\mathrm{i}_1} = \frac{200\ X\ 0.5}{i_1} = 10$$

So I_1 = 10A

P 21 An ideal transformer, connected to 240 V at primary, supplies a 12 V, 150 W lamp at the secondary. Calculate the transformer turns ratio and the current taken from the supply

2b) $P_2 = 150 \quad V_2 = 12 \quad V_1 = 240$

$$\frac{N_2}{N_1} = \frac{V_2}{V_1} = \frac{12}{240} = 0.05$$ – 2 Mark

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} \qquad I_2 = \frac{P_2}{V_2} = \frac{150}{12} = 12.5A$$ 1 Mark

$$\therefore I_1 = I_2 \times \frac{N_2}{N_1} = 12.5 \times 0.05 = 0.625A.$$ – 1 Mark.