Reg. No.:

Name :

# VIT

Vellore Institute of Technology

## Continuous Assessment Test (CAT)- I– October 2022

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|---|---|---|---|
| Programme | : B.Tech. | Semester | : Fall Semester Year I 2022-2023 |
| Course Title | : Calculus | Code | : BMAT101L |
| | | Slot | : E1+TE1 |
| Faculty | : Dr. Saroj Kumar Dash, Dr. Manivannan A, Dr. C. Rajivganthi, Dr. Harshavarthini, Dr. Prosenjit, Dr. Ashis Bera, Dr. Ankit Kumar, Dr. Sandip Saha, Dr. Kriti Arya | Class Nbr | : CH2022231700190, 189, 191, 192, 196, 194, 257, 323, 883 |
| Duration | : 1 ½ Hours | Max. Marks | : 50 |

**Answer all the Questions (50 marks)**

| Q.No. | Question Description | Marks |
|---|---|---|
| 1. | a) Using Mean Value Theorem (MVT) prove that $0 < \frac{1}{x}\log\left(\frac{e^x-1}{x}\right) < 1$ for $x > 0$. | [5] |
| | b) Find the intervals on which the function $f(x) = 3x^2 - 4x^3, x \in \mathbb{R}$ is increasing or decreasing? | [5] |
| 2. | Examine the extreme values of the function $f(x) = x^5 - 5x^4 + 5x^3 + 12, x \in \mathbb{R}$. Also find the intervals on which the function $f(x)$ is concave up and concave down. | [10] |
| 3. | Find the volume of the solid formed by revolving the region enclosed by the parabola $y^2 = 4ax$ and the straight line $y = x$, (i) about $x$-axis, (ii) about $y$-axis. | [10] |
| 4. | Let $f(x, y) = (x^2 + y^2)^{2/3}$. Find $f_x$, $f_y$, $f_{xy}$ and $f_{yx}$ at each point in $\mathbb{R}^2$. | [10] |
| 5. | a) The inductance $L$ (in microhenrys) of a straight nonmagnetic wire in free space is: $L = 0.00021\left[ln\left(\frac{2h}{r}\right) - 0.75\right]$, where '$h$' is the length of the wire in the millimetre and '$r$' is the radius of the circular cross section. Find the maximum possible error of $L$, when $r = 2 \pm \frac{1}{16}$ millimetres and $h = 100 \pm \frac{1}{100}$ millimetres. | [5] |
| | b) Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$, where $u = \cos x \cosh y$, $v = \sin x \cosh y$ and $w = \sinh z$. | [5] |