Final Assessment Test – November/December 2023

VIT® Vellore Institute of Technology

Course: BMAT205L - Discrete Mathematics and Graph Theory

Class NBR(s): 2103 / 2104 / 2105 / 2106 / 2107 / 2109 / 2110 / 2111 / 2112 / 2113 / 2114 / 2115 / 2117 / 2126 / 2140 / 2141 / 2142 / 2155 / 3927 / 3929 / 3935 / 3940 / 3943 / 3951 / 3957

Slot: A2+TA2+TAA2

Time: Three Hours

Max. Marks: 100

KEEPING MOBILE PHONE/SMART WATCH, EVEN IN "OFF" POSITION, IS TREATED AS EXAM MALPRACTICE

Answer any TEN Questions

(10 X 10 = 100 Marks)

1. If A works hard, then either B or C will enjoy themselves.
   If B enjoys himself, then A will not work hard.
   If D enjoys himself, then C will not enjoy himself.
   Therefore, if A works hard, then D will not enjoy himself.
   Show that the above statements constitute a valid argument.

2. Use indirect method to prove that the conclusion
   $(\exists z)\, Q(z)$ follows from the premises $(\forall x)(P(x) \rightarrow Q(x))$ and $(\exists y)\, P(y)$.

3. Show that a non-empty subset H of G is a subgroup of a group $\langle G,* \rangle$ if and only if for any pair of elements $a, b \in H; a * b^{-1} \in H$.

4. Find all the code words generated by the encoding function $e: B^3 \longrightarrow B^6$ with respect to the following parity check matrix.

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

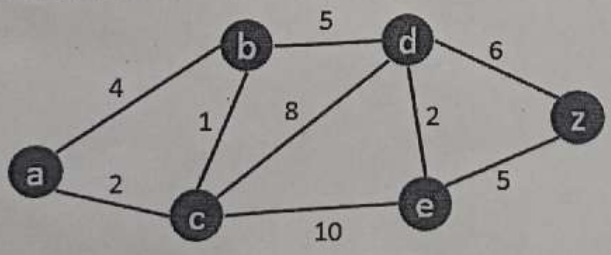
5. Solve the recurrence relation $a_n = 4\,a_{n-1} - 4\,a_{n-2} + 4^n$; $n \geq 2$, with initial conditions $a_0 = 2$ and $a_1 = 8$ using generating functions.

6. Let $\langle L, \leq, *, \oplus \rangle$ be a lattice in which $*$ *and* $\oplus$ denote the operations of meet and join respectively. For any $a, b \in L$, prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.

7. Expand the Boolean expression $(x_1 \oplus x_2)$ into an equivalent sum-of-products and product-of-sums canonical forms in terms of three variables namely x_1, x_2 and x_3.

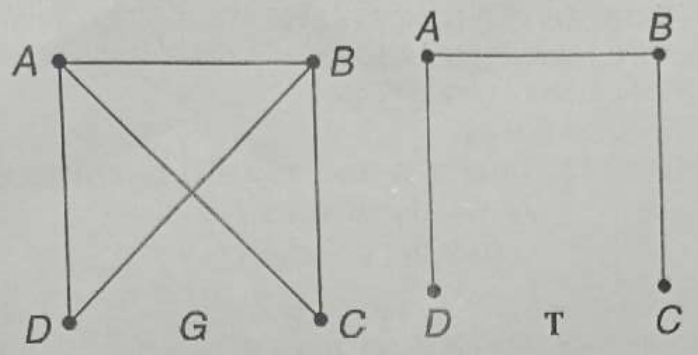
8. Show that the number of vertices of odd degree in a graph is always even.

9. Apply the Dijkstra's algorithm to find the shortest path between the vertices a and z in the following weighted connected graph and hence compute the total weight of the obtained shortest path between the mentioned vertices.

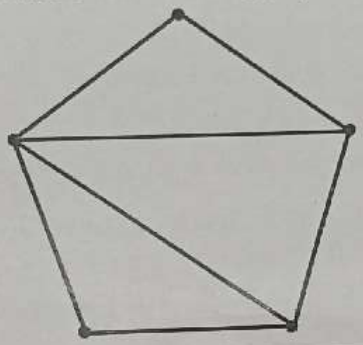


10. Show that a tree with n vertices has exactly $(n-1)$ edges.

11. Find all the possible fundamental circuits for the given graph *G* with respect to the given spanning tree *T*. Also find at least 5 possible spanning trees for the given graph *G*.



12. Find the chromatic polynomial of the following graph.



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