

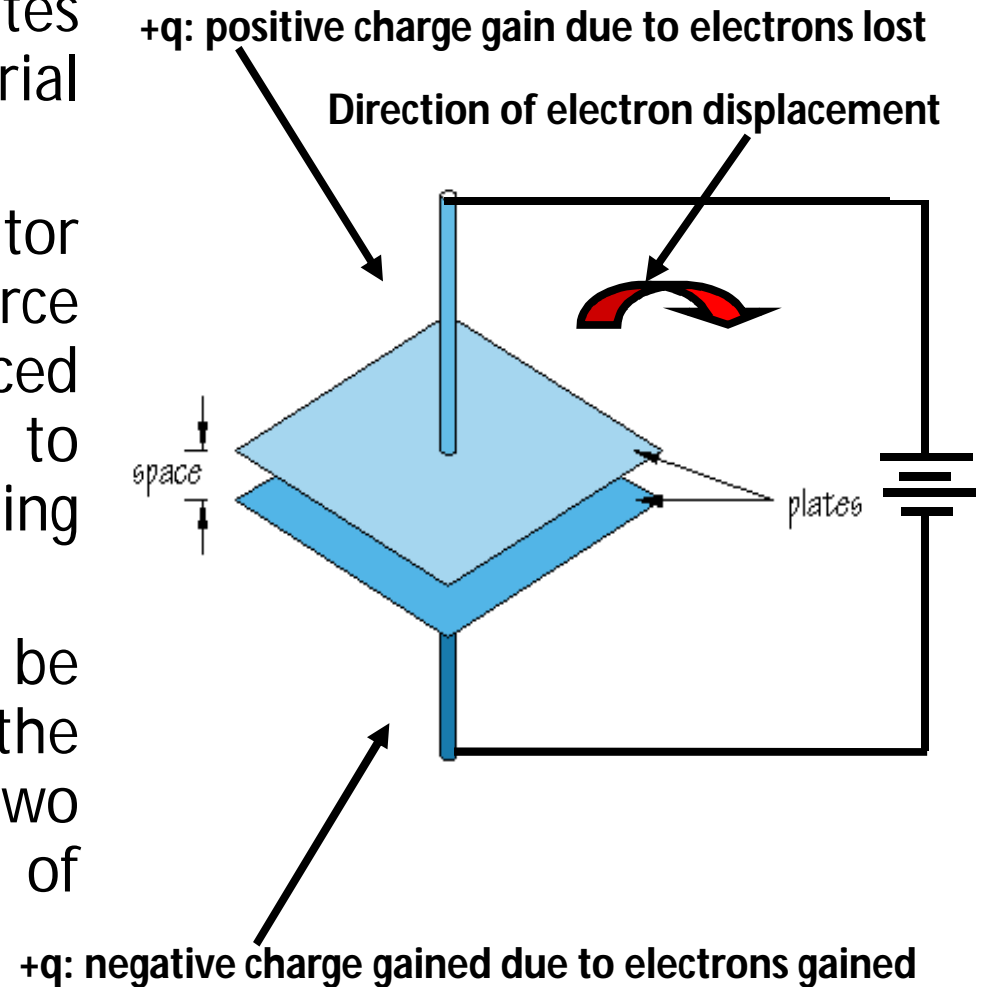
V-I Characteristic of Capacitor and Inductor

Capacitor Concept

- A capacitor is an energy storage element which is analogous to the spring element of mechanical systems.
- It can store electrical pressure (voltage) for periods of time.
 - When a capacitor has a difference in voltage (electrical pressure) across its plate, it is said to be charged.
 - A capacitor is charged by having a one-way current flow through it for a period of time.
 - It can be discharged by letting a current flow in the opposite direction out of the capacitor.

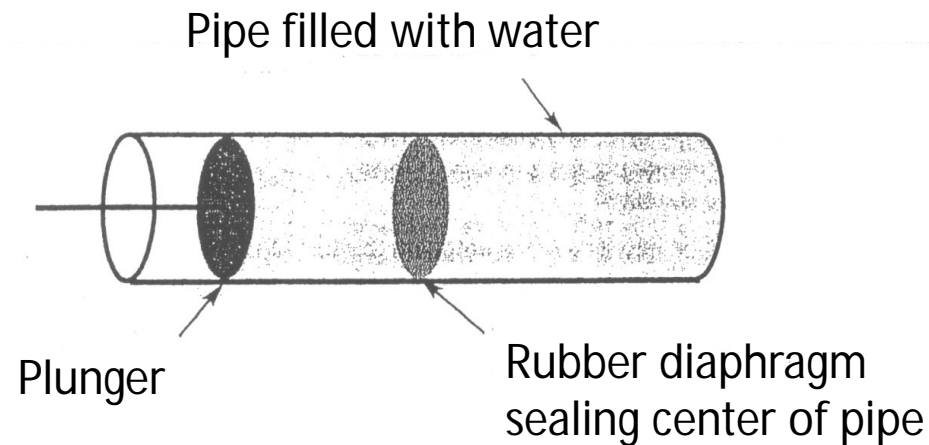
Capacitor Construction

- A capacitor is constructed using a pair of parallel conducting plates separated by an insulating material (dielectric).
- When the two plates of a capacitor are connected to a voltage source as shown, charges are displaced from one side of the capacitor to the other side, thereby establishing an electric field.
- The charges continue to be displaced in this manner until the potential difference across the two plates is equal to the potential of voltage source.



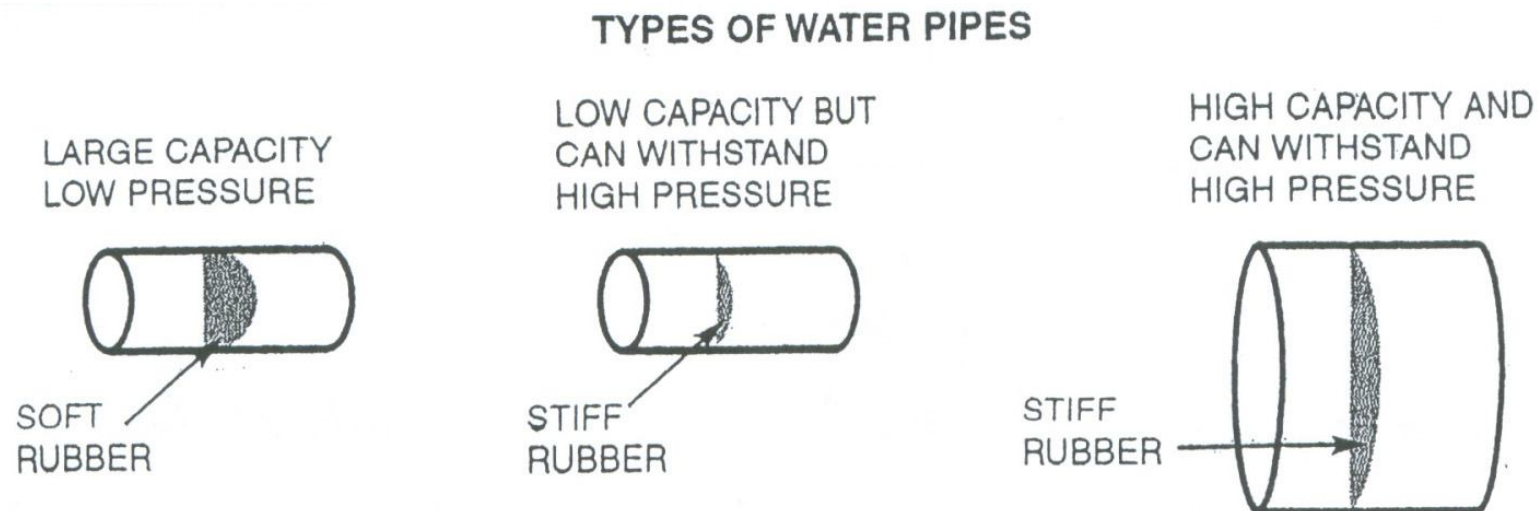
Capacitor Water Pipe Analogy —I

- In the water pipe analogy, a capacitor is thought of as a water pipe:
 - with a rubber diaphragm sealing off each side of the pipe and
 - a plunger on one end.
- When the plunger pushes toward the diaphragm, the water in the pipe forces the diaphragm to stretch until the force of the diaphragm pushing back on the water equals the force on the plunger → pipe is charged!
- If the plunger is released, the diaphragm will push the plunger back to its original position → pipe is discharged.



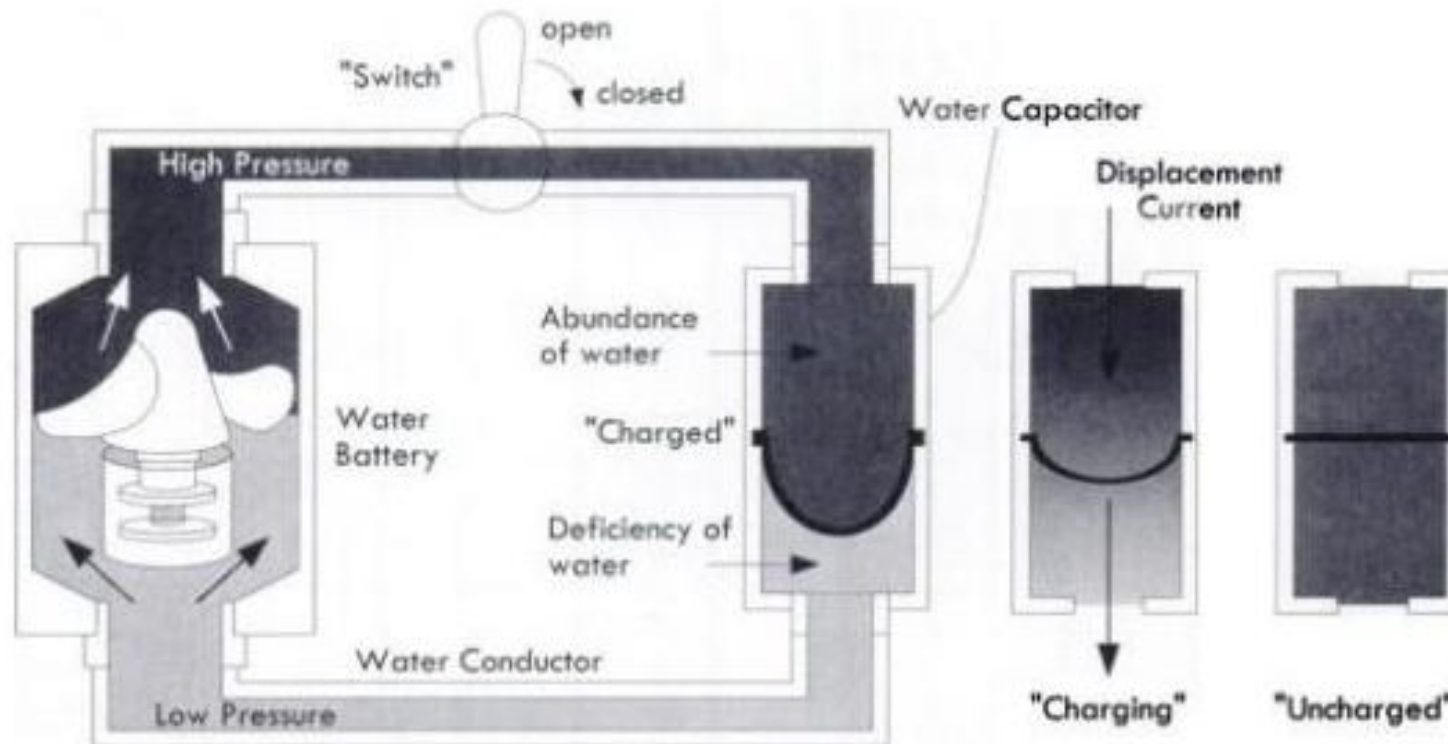
Capacitor Water Pipe Analogy —II

- If the rubber diaphragm is made very soft, it will stretch out and hold a lot of water but will break easily (large capacitance but low working voltage).
- If the rubber diaphragm is made very stiff, it will not stretch far but withstand higher pressure (low capacitance but high working voltage).
- By making the pipe larger and keeping the rubber stiff, we can achieve a device that holds a lot of water and withstand high pressure.
- So the pipe size is determined from the amount of water to be held and the amount of pressure to be handled.



Capacitor Water Pipe Analogy —III

- Water capacitor: a tube with a rubber membrane in the middle
- Rubber membrane analogous to the dielectric, two chambers analogous to two capacitor plates
- When no water pressure is applied on the water capacitor, the two chambers contain same amount of water (uncharged)
- When pressure is applied on the top chamber, the membrane is pushed down causing the water to be displaced from the bottom chamber (appearance of current flow \rightarrow displacement current)



Capacitor V-I Characteristic

- The charge accumulated on capacitor plates is directly proportional to voltage applied across the plates.

$$q \propto V \quad q = CV \quad \longrightarrow$$

where C is the constant of proportionality and is called capacitance (unit: Farad).

- V-I characteristic of a capacitor is obtained by computing

$$\frac{d}{dt}[q = CV] \longrightarrow \frac{dq}{dt} = C \frac{dv}{dt} \longrightarrow I(t) = C \frac{dv}{dt}$$

- Alternatively, integrating the above equation w.r.t. time, and rearranging terms, we get

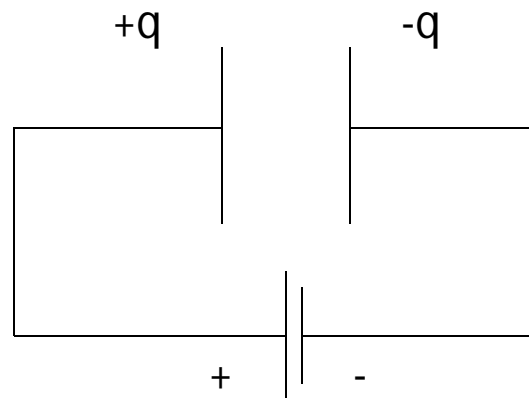
$$V(t) = \frac{1}{C} \int_0^t I(\tau) d\tau$$

Capacitance Formula

- For a parallel capacitor:

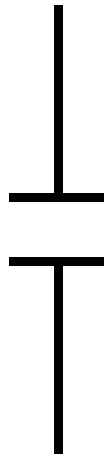
$$C = \frac{\epsilon_0 A}{D}$$

- ϵ_0 = permittivity of free space
 - A = plate area
 - d = separation distance of plate.
- Often, we use $G = A/d$ as geometry factor (for other types of capacitors as well).
 - If a dielectric material with dielectric constant K separates the two plates of the capacitor, then $C = K\epsilon_0 G$, where K = dielectric constant. Usually $K > 1$.

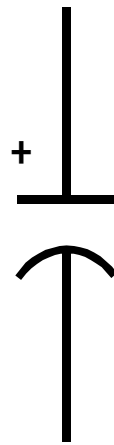


Voltage source

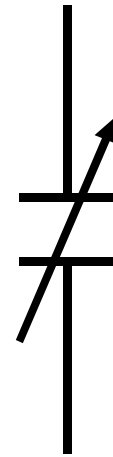
Capacitor Symbols



Fixed capacitor

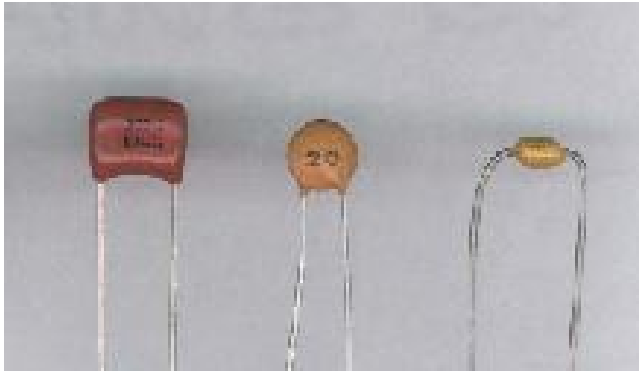


Polarized capacitor



Variable capacitor

Capacitor Variations



- Ceramic capacitors

- very popular nonpolarized capacitor
- small, inexpensive, but poor temperature stability and poor accuracy
- ceramic dielectric and a phenolic coating
- often used for bypass and coupling applications

- Electrolytic

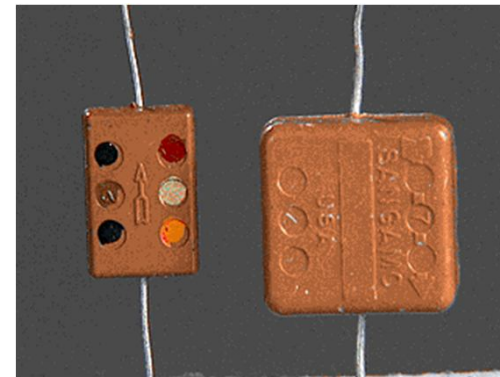
- Aluminum, tantalum electrolytic
- Tantalum electrolytic capacitor has a larger capacitance when compared to aluminum electrolytic capacitor
- Mostly polarized.
- Greater capacitance but poor tolerance when compared to nonelectrolytic capacitors.
- Bad temperature stability, high leakage, short lives

Capacitor Variations



- Mylar

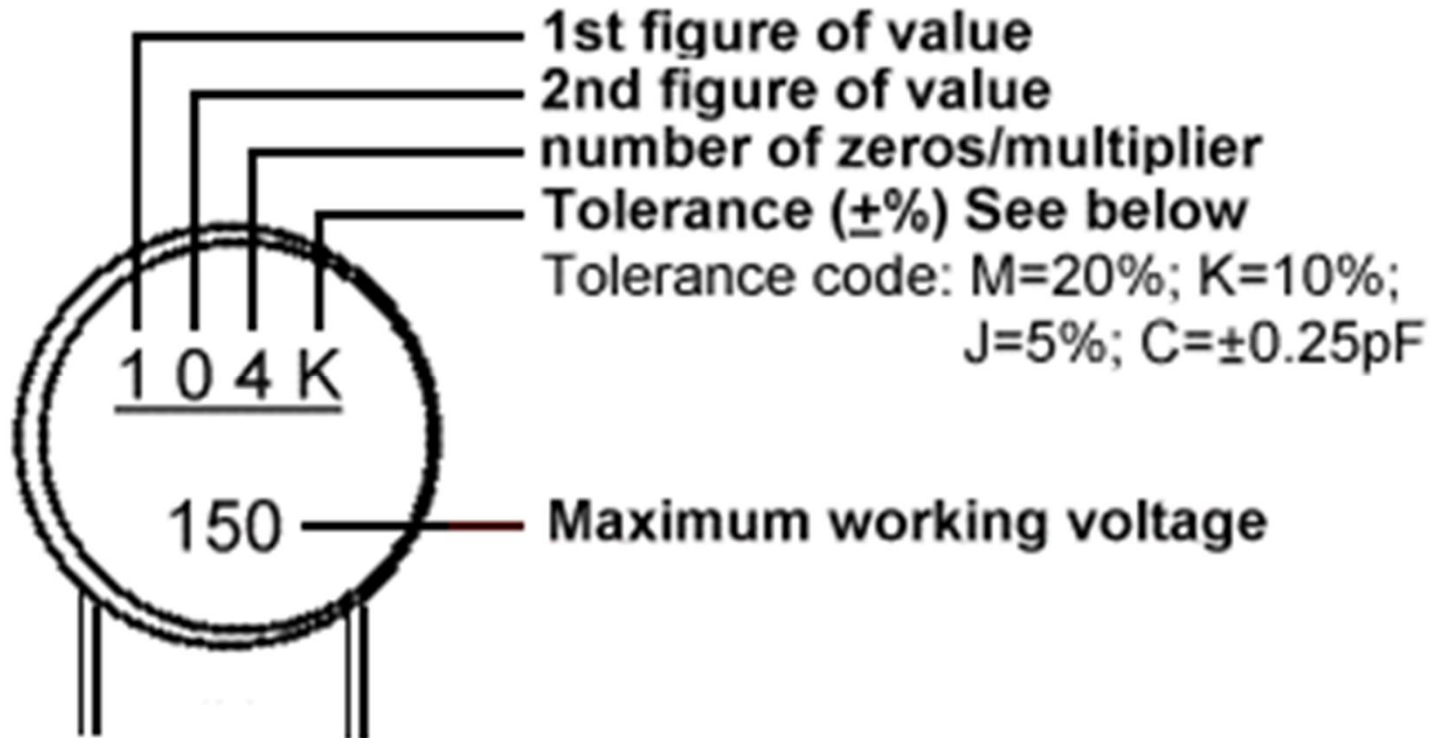
- very popular, nonpolarized
- reliable, inexpensive, low leakage
- poor temperature stability



- Mica

- extremely accurate, low leakage current
- constructed with alternate layers of metal foil and mica insulation, stacked and encapsulated
- small capacitance
- often used in high-frequency circuits (i.e. RF circuits)

Capacitor Reading Example —I

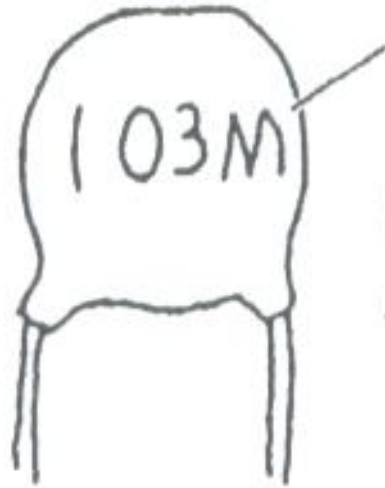


$$10 \times 10^4 \text{ pF} = 10^5 \times 10^{-12} \text{ F} = 10^{-7} \text{ F} = 0.1 \times 10^{-6} \text{ F} = 0.1 \mu\text{F}$$

- Thus, we have a $0.1 \mu\text{F}$ capacitor with $\pm 10\%$ tolerance.

Capacitor Reading Example —II

Ceramic



Label represents
a tolerance

Label says: 103M

Actual value: $0.01\mu\text{F} \pm 20\%$

$$10 \times 10^3 \text{ pF} = 10^4 \times 10^{-12} \text{ F} = 10^{-8} \text{ F} = 0.01 \times 10^{-6} \text{ F} = 0.01 \mu\text{F}$$

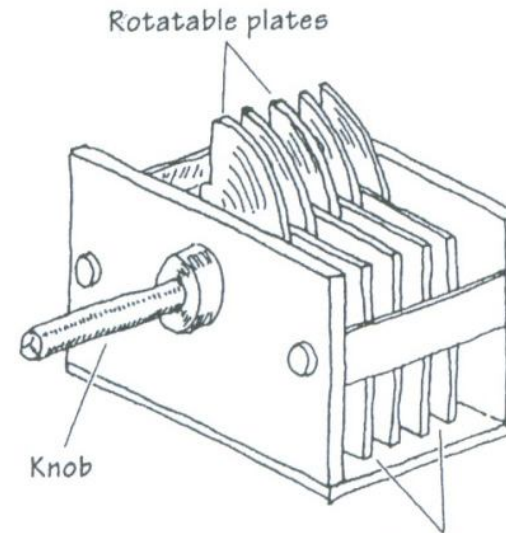
Variable Capacitors

- Devices that can be made to change capacitance values with the twist of a knob.

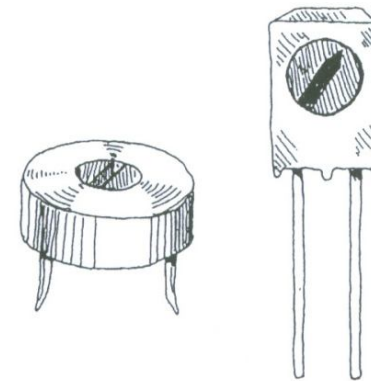
- Air-variable or trimmer forms

- Air-variable capacitor consists of two sets of aluminum plates (stator and rotor) that mesh together but do not touch. Often used in frequently adjusted tuning applications (i.e., tuning communication receivers over a wide band of frequencies).

- A trimmer capacitor is a smaller unit that is designed for infrequent fine-tuning adjustment (i.e., fine-tuning fixed-frequency communications receivers, crystal frequency adjustments, adjusting filter characteristics)



Trimmers



Inductors

- Inductor is a passive energy storage element that stores energy in the form of magnetic field.

- Inductor characteristic is governed by Faraday's law:

$$V(t) = \frac{d\lambda}{dt}$$

- V = voltage induced across an inductor

- λ = magnetic flux (unit: Webers, Wb) through the coil windings (a coil made using resistance-less wires) due to current flowing through inductor.

- For an ideal coil, magnetic flux is proportional to current, so

$$\lambda \propto I \text{ or } \lambda = LI$$

- L is constant of proportionality, called inductance (unit: Henry, Wb/Amp).

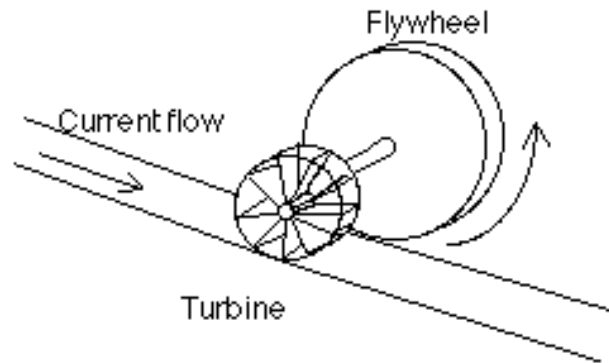
- So, now, the V-I characteristic of an inductor is:

$$V(t) = \frac{d}{dt}(\lambda) = \frac{d}{dt}(LI) = L \frac{dI}{dt}$$

$$I(t) = \frac{1}{L} \int_0^t V(\tau) d\tau$$

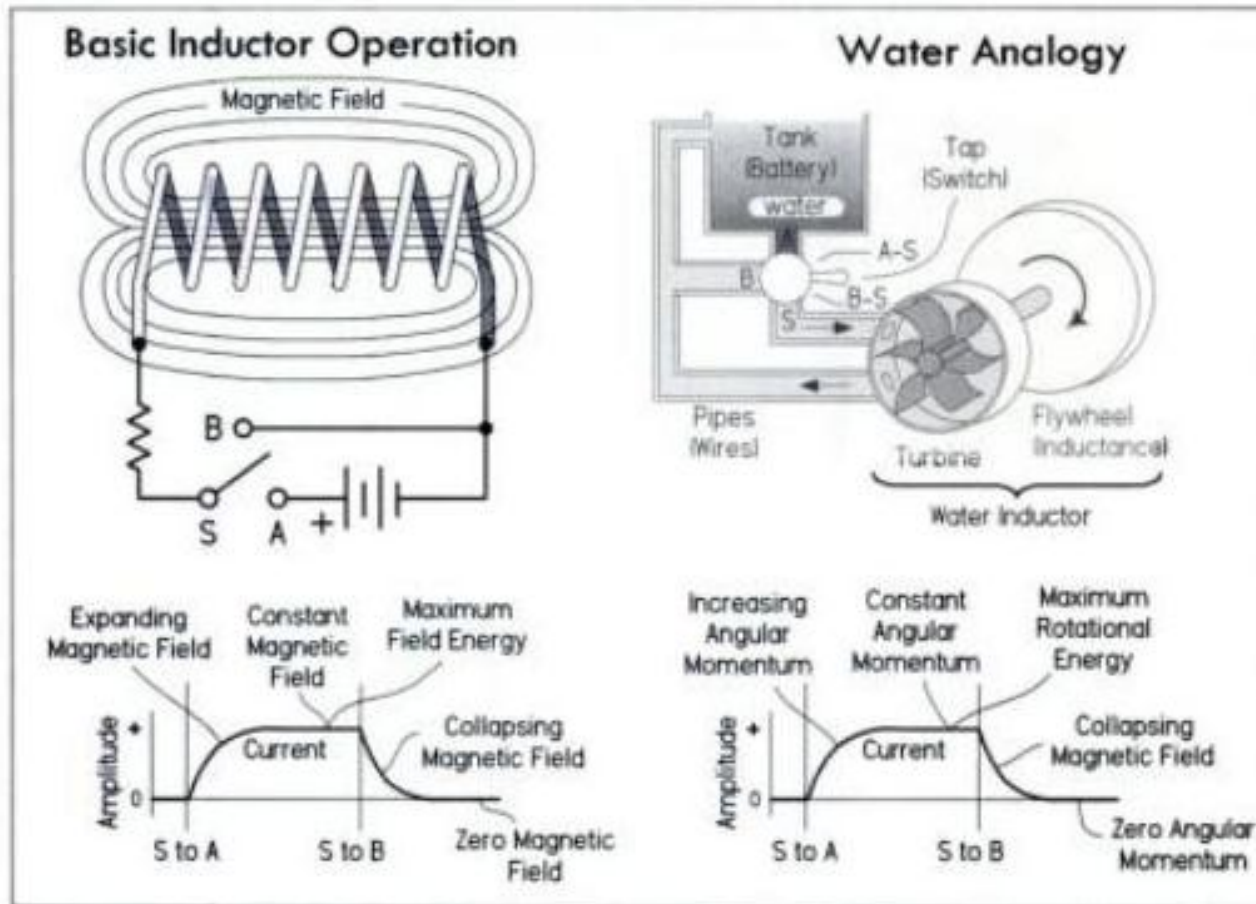
- The above V-I characteristics demonstrate that the current through an inductor can not be altered instantaneously.

Inductor-Water Analogy — I



- Suppose a turbine is hooked up to the flywheel and water is supplied to the turbine. The flywheel will start to move slowly. Eventually, the flywheel will move at the same rate as the current.
- If the current alternates back and forth, the flywheel/turbine will take some time to build up to the initial direction that the water wants to flow.
- As the current moves back and forth, the flywheel creates the extra "resistance" to the change in current flow, but eventually the flywheel/turbine will move in the same direction as the current flow.

Inductor-Water Analogy —II



Mechanical inertia and inductor both resist sudden change in their state

- When switch S contacts A, the field generated by the applied positive voltage creates a reverse induced voltage that initially resists current flow
- Based on the value of inductance, as the magnetic field reaches steady-state, the reverse voltage decays
- A collapsing field is generated when applied voltage is removed (switch S contacts B), creating a forward induced voltage that attempts to keep current flowing
- Based on the value of inductance, as the magnetic field reaches zero steady-state, the forward voltage decays

Inductance of a Cylindrical Coil

$$L = \frac{\mu_0 N^2 \pi r^2}{\ell}$$

- μ_0 = permeability of free space
- N = number of turns in coil
- ℓ = length of resistance-less wire used in coil
- r = radius of coil cross section.

• If number of turns per unit length is " n ", then $N = n\ell$, so:

$$L = \frac{\mu_0 (n^2 \ell^2) \pi r^2}{\ell} = \mu_0 n^2 \ell \pi r^2 = \mu_0 n^2 \ell A$$

- A = cross-sectional area of coil.
- If a magnetizable material forms the core of coil, then permeability μ will be larger than μ_0 .

Inductor Variations —I



Inductor Variations —II



- Antenna coil

- contains an iron core that magnifies magnetic field effects

- used to tune in ultra-high-frequency signals, i.e. RF signals

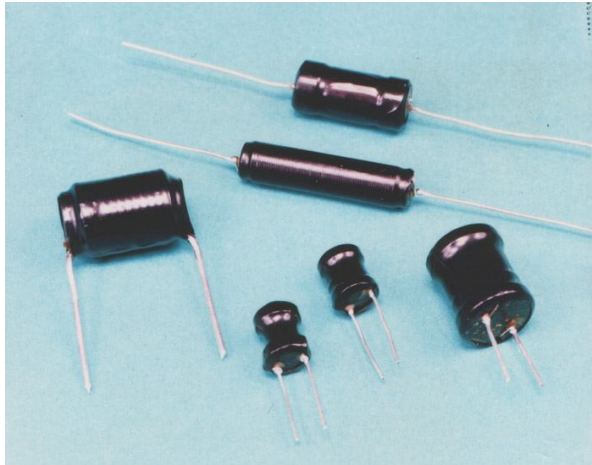


- Tuning coil

- screw-like “magnetic field blocker” that can be adjusted to select the desired inductance value

- used in radio receivers to select a desired frequency.

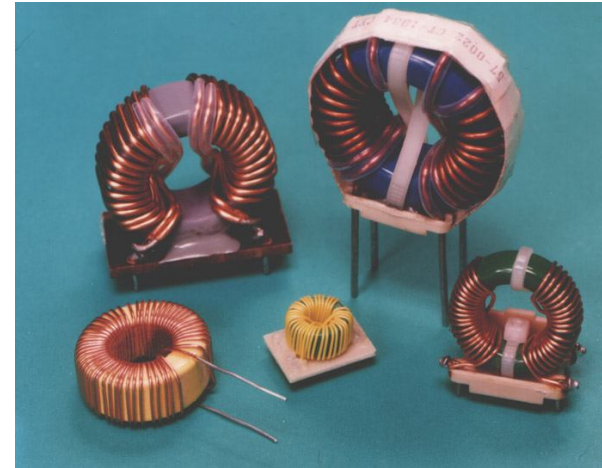
Inductor Variations —III



- Chokes

- general-purpose inductors that act to limit or suppress fluctuating current.

- some use a resistor-like color code to specify inductance values.



- Toroidal coil

- resembles a donut with a wire wrapping

- high inductance per volume ratios, high quality factors, self-shielding, can be operated at extremely high frequencies

Inductor Symbols



Air core



Iron core



Powered-iron
core



Variable
core

Transformer



- Isolation

- acts exclusively as an isolation device; does not increase or decrease the secondary voltage

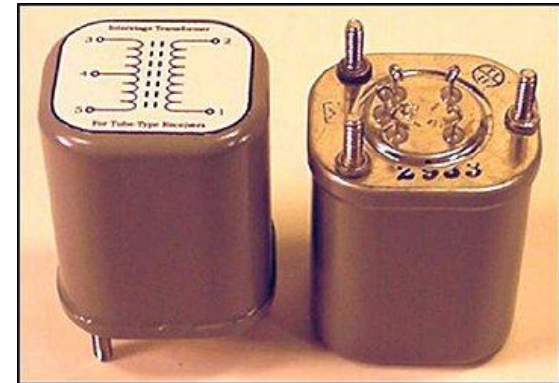
- usually come with an electrostatic shield between the primary and secondary. Often come with a three-wire plug and receptacle that can be plugged directly into a power outlet



- High Frequency

- often come with air or powdered-iron cores

- used for high frequency applications, i.e. matching RF transmission lines to other devices (transmission line to antenna)



- Audio

- used primarily to match impedances between audio devices

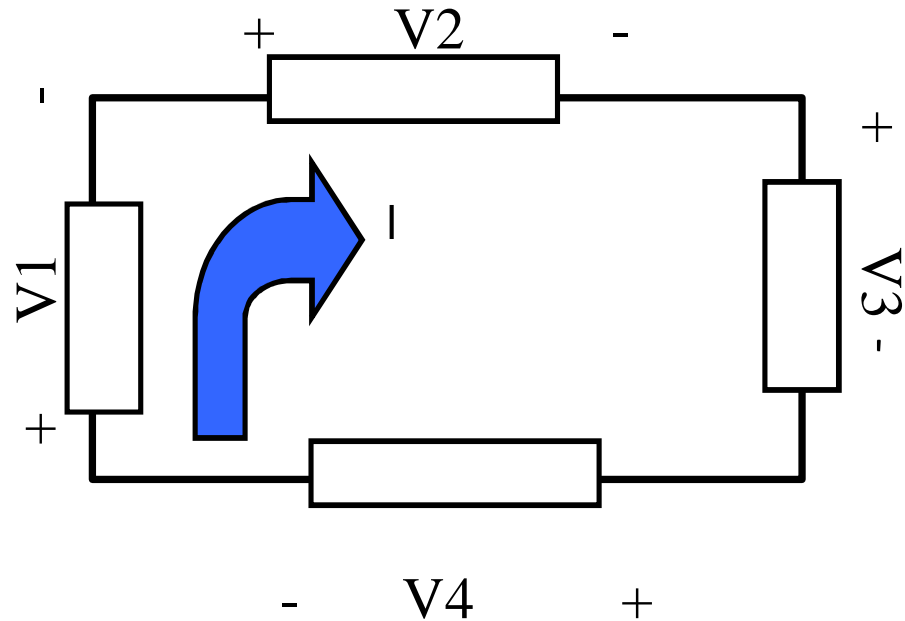
- work best at audio frequencies from 150Hz to 12kHz

- come in a variety of shapes and sizes, typically contain a center tap

Kirchoff's Voltage Law

- The algebraic sum of voltage around a loop is zero.
- Assumption:
 - Voltage drop across each passive element is in the direction of current flow.

$$V_1 + V_2 + V_3 + V_4 = 0$$



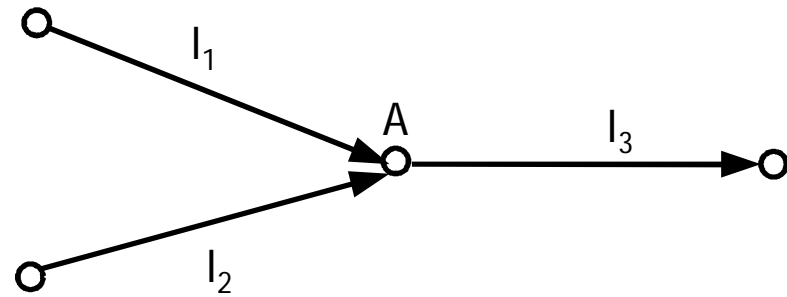
Kirchoff's Current Law

- Algebraic sum of all currents entering and leaving a node is zero.

- At node A:

$$I_1 + I_2 - I_3 = 0$$

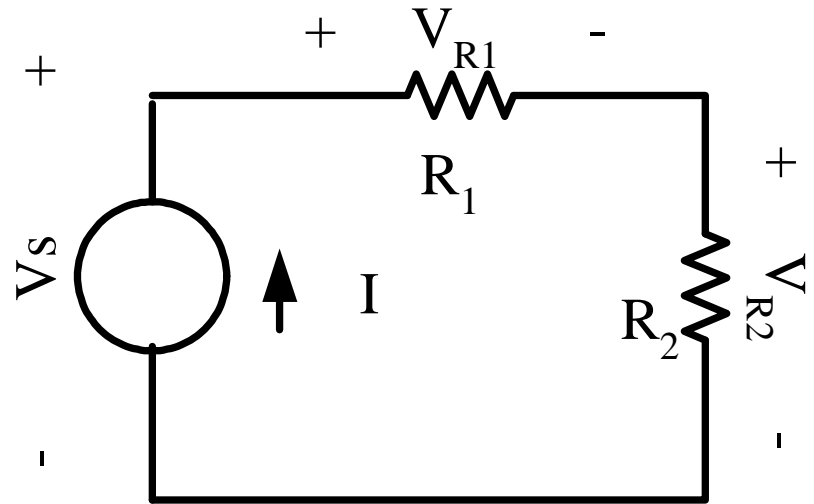
- Current entering a node is assigned positive sign. Current leaving a node is assigned a negative sign.



Law of Voltage division

$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_s$$

$$V_{R_2} = \frac{R_2}{R_1 + R_2} V_s$$



Law of Current division

$$I_{R_1} = \frac{R_2}{R_1 + R_2} I$$

$$I_{R_2} = \frac{R_1}{R_1 + R_2} I$$

